

PART 2: Teaching and Learning of Mathematics

Part 2 contains two subsections, each asking you about your thoughts regarding teaching and learning mathematics. The subsections include:

- A. A Classroom Reflection
- B. Views about Math

PART 2 Section A. Classroom Reflection

Instructions: Many people think the classroom is the best context for thinking about teaching and learning. Below is a classroom scenario. Please read the scenario all the way through. Then respond to the questions that follow the scenario:

- What is the teacher doing in this lesson?
- Is it good teaching or not? Why?
- For the third question, one sentence from the scenario is quoted. You are asked how this sentence helped you decide if the teaching in the scenario was good or not.

There are no right or wrong answers here — we are interested in learning your thoughts about what the teacher and the students are doing. Please explain your thinking as thoroughly as possible, so that we can understand your views.

Please turn to the next page.

SCENARIO

This scenario was written by Mrs. Kimberly, the teacher of an Integrated 1 Math class made up of a nearly even split of 8th and 9th graders sprinkled with a few 7th graders. The class was discussing how to define even and odd numbers; what happens when you add even numbers to even numbers, odd numbers to odd numbers, and an even number to an odd number; and how you can prove your answers are true for all pairs of numbers.

Many students began the same way that Ryan, a 9th grader, did.

Ryan: Because if a number can be divided by 2 it's even and 8 can be divided by two.

$$8/2 = 4.$$

Mrs. K: So, I can divide 7 by 2 and it is 3.5. Is 7 an even number?

Ryan and other students: I think we have to add that you get a whole number when you divide by 2. That is really what we mean.

Mrs. K: How would you prove the odd numbers?

Ryan: Any number that ends in 1,3,5,7,9 is odd.

Mrs. K: Ryan, what would you say is a good definition for even and odd numbers?

Ryan: An even number is divisible by 2 and ends in 0,2,4,6,8. An odd ends in 1,3,5,7,9 and cannot be divided by 2.

I thought when I first started my conversation with Ryan that he was showing real insights. His definition seemed weak and I am not sure that the weakness comes from shallow understanding or from the need to finish before class is over. He seemed to be able to tell me the characteristics of an odd number but not the underlying reasons.

I asked Hubert how he would define an even and odd number.

Hubert: An even number is a number divided by two that ends without a remainder. An odd number is a number divided by itself and 1 only. When divided by 2 you get a remainder.

I was encouraged at the beginning of Hubert's definition. His was more general in nature. Hubert's definition seemed more basically able to describe all numbers. His confusion with odd and prime numbers was something that we would need to discuss at a later date. I asked Hubert what he got when he added an even number to an even number. He replied it will always be an even number. I asked him how he could prove that an even plus an even will always be even. Following is the text of his written answer:

Hubert: When you add two even numbers you always get an even number. When you add 2, a small even number, with 6, a larger even number, you get 8, a larger number.

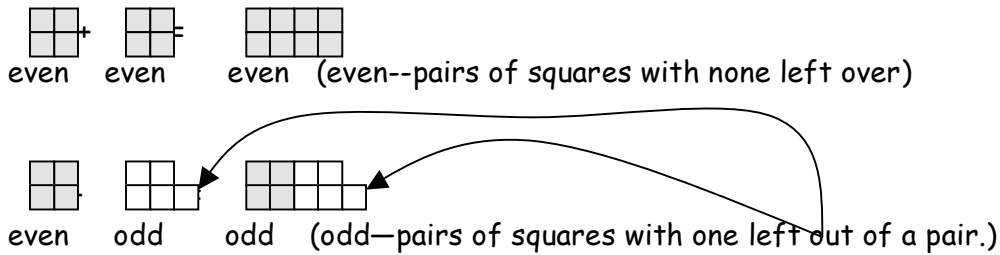
I was not sure at this point what Hubert meant. I asked him what he thought about adding an even number with an odd number. He immediately said that he knew it equaled an odd number. I asked him how he had proved this to be true and he showed me a table that he made:

Hubert:

even		odd		Answer
2	+	1	=	3
4	+	3	=	7
6	+	5	=	11
8	+	7	=	15

Hubert seemed confident about this as a proof. I was not convinced and pushed him for further proof. How could I tell this was true for all even and odd numbers? This again showed evidence of characteristics but I am not convinced for "all numbers".

Ryan wanted to show us at this point the drawings he made for the same proof as Hubert—an odd plus an even always equals an odd number. He was convinced that his really showed why an even plus an odd always ends up with an odd number. Here are his drawings—that begin with an even plus an even equals and even:



Finally, Ryan drew pictures for an odd plus an odd equals an even.



Since there are none of the "sticking out" squares left. It must be an even number.

Even though there were many differences of definitions with my students and many had weak proofs, I felt that the class had dug much deeper than any of us thought possible when we began this simple subject of "Evens and Odds." I was a little worried that so many of my students used multiple addition problems as a proof and they were totally convinced. ($3+4=7$, $2+3=5$, $8+9=17$). I was surprised that the most convincing methods of proof came from drawings rather than words and numbers. I was also surprised that no one in my class tried to write an equation with variables to explain.

Each box below contains a question about the scenario. Please respond to each question, taking into account the context of the entire scenario. Please explain your thinking thoroughly.

2A.1. What was the teacher doing?

2A.2. Was it good teaching or not? Why?

2A.3. The following sentence occurs near the end of the scenario. "Even though there were many differences of definitions with my students and many had weak proofs, I felt that the class had dug much deeper than any of us thought possible when we began this simple subject of 'Evens and Odds.'" How does this statement help you decide whether the teaching in the scenario was good or not?

PLEASE CONTINUE TO THE NEXT PAGE FOR TWO ADDITIONAL QUESTIONS ABOUT THE SCENARIO.

Please respond to the two questions below based on the scenario you have just read.

2A.4. What were the mathematical ideas involved in this classroom scenario?

2A.5. What can students learn in this class?