

*Grades*  
3-5

# Geometry and Measurement

## TABLE OF CONTENTS

<b>1</b>	Shapes .....	2
<b>2</b>	Angles and Turns .....	8
<b>3</b>	Measuring Length .....	15
<b>4</b>	Perimeter .....	20
<b>5</b>	Area . .....	24
<b>6</b>	Cubes, Surface Area, and Volume .....	29
<b>7</b>	Coordinate Systems .....	33

# Shapes

# 1

## Overview

### Mathematical Focus

- ▶ Identify, describe, compare and construct different shapes
- ▶ Classify two dimensional shapes according to their attributes

In this activity, students explore two-dimensional shapes. They describe, compare and classify shapes according to their attributes. Students play several games: they determine a rule for sorting geometric shapes; make shapes with string; draw shape pictures from verbal directions; and use shapes to create new shapes.

### Preparation and Materials

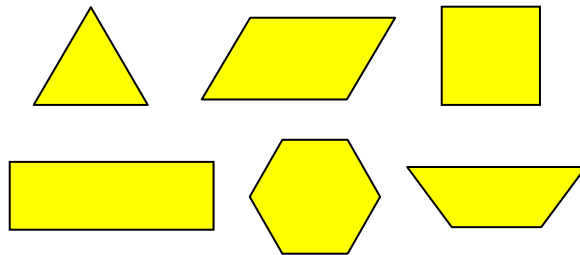
- ▶ Student Page 1: Shapes
- ▶ Student Page 2: Hexagons and Parallelograms
- ▶ 3-foot pieces of string with ends tied together, one for each student

Cut shapes (Student Pages 1 and 2) out of heavy paper ahead of time.

## Shape Attributes and Names

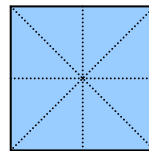
### 1. Identify shapes and describe their attributes.

Give students the shapes cut out from Student Page 1. Ask students to name each shape and then describe its attributes. Explain that *attributes* are the features of the shape that help you tell what kind of shape it is (e.g., the number of sides, the types of angles).



### 2. Determine which shapes are symmetrical.

Give students time to explore the shapes and determine if any of them have lines of symmetry. Explain that if a shape can be folded in half so that one side fits on the other side, then it is symmetrical. Ask students to determine which shapes have more than one line of symmetry. For example, the square has multiple lines of symmetry:






### 3. Use cut-out shapes to create a symmetrical picture.

Have students draw a symmetrical picture, or create a symmetrical picture by combining the cut-out shapes. Students may periodically fold the paper as they work to check that their picture is still symmetrical. Ask students to show all the lines of symmetry in their picture.

**4. Create an attributes chart to record information about different shapes.**

Ask students to fill in the chart for each cut-out shape. Encourage students to create and add additional shapes to the chart. When they have finished, discuss which shapes have common attributes.

Shape	# of Sides	# of Angles (Corners)	# of Square Corners	Lines of Symmetry (yes/no)	Other Info.
					
					
					
...					

**5. Sort shapes according to attribute rules.**

Have students sort the shapes, putting all the shapes with a given attribute in one group and all other shapes in another group, for example: all shapes with four sides in one group and all of the other shapes in another group. Ask: *What sorting rule did you use? What are some other ways you could sort the shapes?*

**6. Play Guess My Rule**

Students take turns being the “sorter.” The sorter begins sorting the shapes according to a secret rule. For example, the sorter’s rule might be: All shapes with four sides in one group and all shapes with more than four sides in the other group. As the sorter is sorting, the “guessers” take turns guessing the rule. The guessers may also try to place additional shapes that fit the rule with the sorter verifying whether the placement is correct. Guessers try to determine the rule in as few guesses as possible.

**7. Use string to make different shapes.**

Give each student a piece of string and tie the ends together. Have them use the string to make the following shapes:

- ♦ Circle
- ♦ Square
- ♦ Rectangle that is not a square
- ♦ Triangle
- ♦ Three other shapes that have not yet been made

Challenge students to create additional shapes with the string. Ask questions such as:

- ♦ *Each time you add a corner, how many sides do you add to the shape?*
- ♦ *Will there always be the same number of sides added when you add a corner?*
- ♦ *If you know the number of corners, or angles in a shape, do you know how many sides are in that shape?*

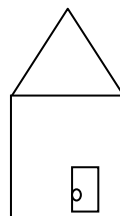
## Describing Shapes

### 1. Draw shapes and pictures from verbal directions.

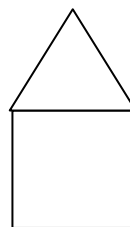
Designate one person as the “describer” and the rest as “recorders.” Using geometric shapes such as those from Student Page 1, the describer draws a picture. The describer must then describe the picture to everyone so that they can draw it. The describer must describe the picture without using the names of the shapes in the picture. Instead he or she uses attributes of the shapes, such as number of sides, types of corners, and lines of symmetry, to describe the picture.

Example Picture:

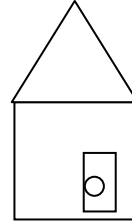
Example directions:



- ♦ *Draw a shape with four right angles, four sides that are the same length, and many lines of symmetry.*
- ♦ *On top of that shape, place a shape with three angles and three sides that are the same.*



- ♦ *Inside the first shape, at the bottom of and off to the right a bit, draw a shape with four sides and four square corners, with two sides longer than the other two sides—the long sides are standing up.*
- ♦ *Inside the shape you just drew, a little to the left of the middle of the shape, draw a very small shape with no corners and many lines of symmetry.*



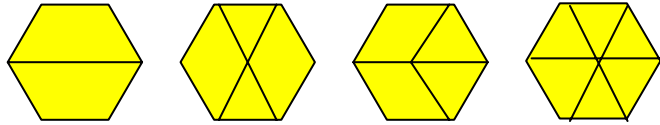
The recorders try to draw the picture that is being described. If it is too difficult for the participants to draw the picture as they hear the instructions, the describer can write the directions on the board. When all of the recorders have completed the picture, they should compare their pictures to the describer's picture to see if the shapes are the same. Repeat this activity several times, giving each student a chance to be the describer. Ask students:

- ♦ *What was difficult about describing how to draw a picture that others cannot see?*
- ♦ *What was difficult about drawing a picture from verbal directions?*

## Shape Construction

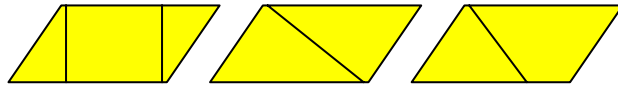
### 1. Combine shapes to make a hexagon.

Give students shapes cut out from Student Page 2. Help them to identify the hexagon and the parallelogram. Ask students to describe the attributes of each shape. Make sure students understand the attribute of parallelism. Challenge students to combine two or more of the shapes in different ways to make hexagons. There are many possible ways to make a hexagon.

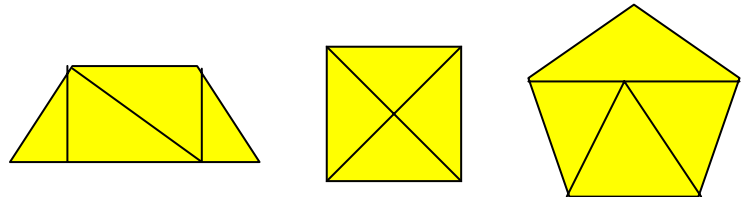


Ask:

- ♦ *What shapes did you use to construct your hexagon?*
- ♦ *Can you make a parallelogram by combining shapes?*



- ♦ *What shapes did you use to construct your parallelogram?*
- ♦ *Are there other shapes you could make by combining shapes?*



# Angles and Turns

## Overview

### Mathematical Focus

- ▶ Estimate and measure angles
- ▶ Explore the relationships between angles and turns

In this activity, students explore angles. They use right angles as benchmarks for estimating angle measurements and use a protractor or square corners to check their accuracy. They use also use right angles to explore changes in orientation. Students play games to increase their familiarity with angle and turn size and how angles can fit together to form different shapes.

### Preparation and Materials

**Before the session, gather the following materials:**

- ▶ Student Page 1: Shapes (1)
- ▶ Student Page 3: Protractor Circle, 2 copies
- ▶ Student Page 4: Protractor
- ▶ Student Page 5: Angle Cards
- ▶ Student Page 6: Complete the Circle, 2 copies
- ▶ Protractor (optional)
- ▶ Two 6-inch strips of paper
- ▶ Fastener
- ▶ Scissors
- ▶ Ruler
- ▶ Masking tape or string

Cut out shapes and angle cards (Student Pages 1 and 5) ahead of time. Cut out all of the fraction pieces from one copy of Student Page 6.

In this activity, students will measure angles. For younger students, you may want to deal only with right angles. You can make comparisons between angles that are larger or smaller than a right angle, without actually measuring those angles.

## Activity

## Square Corners

### 1. Use a protractor to measure $90^\circ$ angles.

Show students a square and ask them to identify its corners. Ask if they know a special name for the angles in a square. Each angle is a *right angle*. Give students either a copy of the protractor from Student Page 4 or an actual protractor. Explain that a protractor is a tool that can be used to measure angles, the way rulers measure lengths. Protractors measure angles in numbers of degrees. Square corners are angles of 90 degrees, which can be written as  $90^\circ$  (write “ $90^\circ$ ” on a piece of paper so that they can see the degree symbol). Demonstrate how to use the protractor to measure right angles – such as the corner of a piece of paper or a book.

### 2. Create an angle-maker and use it to show angles of varying degrees.

Give students two 6-inch strips of paper and a fastener. Have them attach the ends of the paper strips together with the fastener so that it can be used to make different size angles. Ask: *Can you use the angle-maker to show a  $90^\circ$  angle? An angle less than  $90^\circ$ ? An angle greater than  $90^\circ$ ?*

[Insert graphic of angle-maker.]

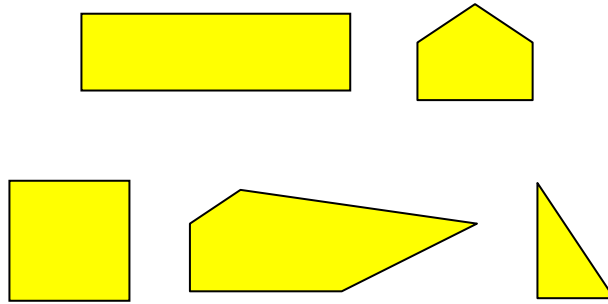
### 3. Identify angles of varying degrees within the room

Challenge students find several  $90^\circ$  angles around the room (suggest that they use the corner of a piece of paper to check the angle). Ask students to find some angles that are less than 90 degrees and some angles that are larger than 90 degrees on objects in the room, such as staplers, open books, or clock hands.

### 4. Draw several shapes that have at least one right angle.

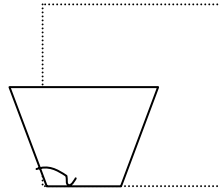
Draw several shapes that include at least one right angle. Have students point out the right angles in each shape and tell whether the other angles in the shape are bigger or smaller than a right angle.

Examples:

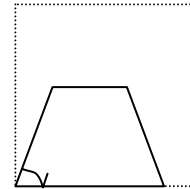


**5. Sort shapes according to the size of their angles.**

Have students look through the shapes cut from Student Page 1. Ask them to identify right angles and angles that are larger or smaller than right angles. Have them use a square corner to check their answers. For example, a trapezoid has two angles that are greater than a right angle and two angles that are smaller than a right angle:

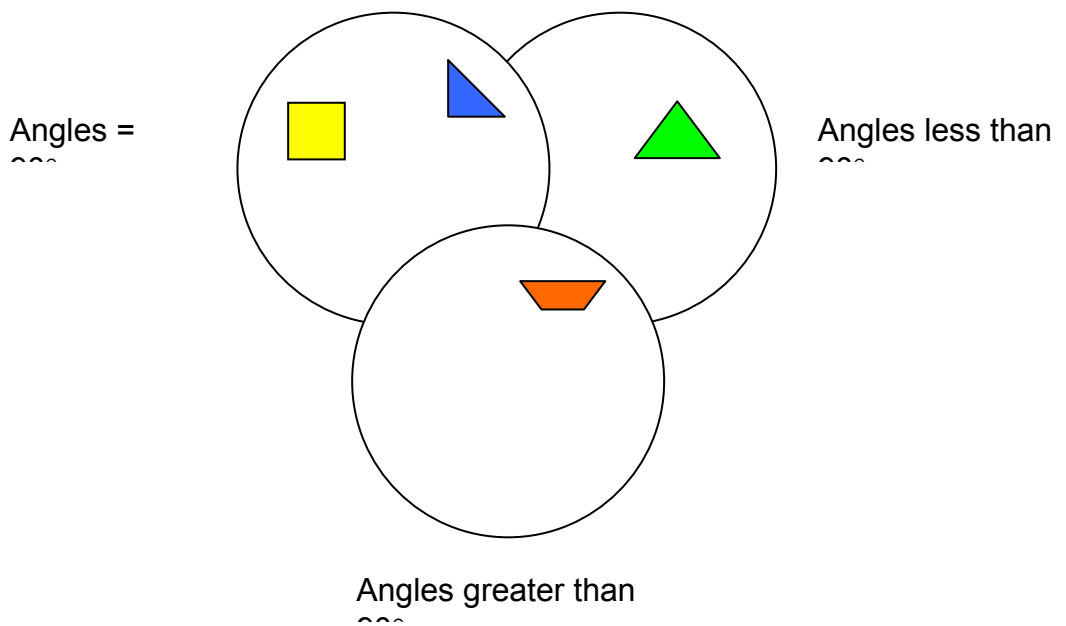


Bigger than  
a Right



Smaller  
than a Right

Draw three overlapping circles on a piece of paper. Label the diagram as shown below. Have students sort the cut-out paper shapes into the appropriate categories. Ask: *Is there a shape in every section of the diagram? If not, can you draw a shape for the section(s) in which there are no shapes?*



### 6. Use a door to explore the size of angles.

Open the door a little and ask students whether it is open 90 degrees. Have students show what the door would look like if it were open 90 degrees. Put the door at different angles and ask students whether its angle is a right angle, bigger than a right angle, or smaller than a right angle.

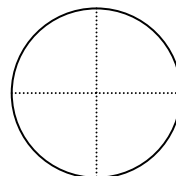
## Protractors and Angles

*Teaching Tip*

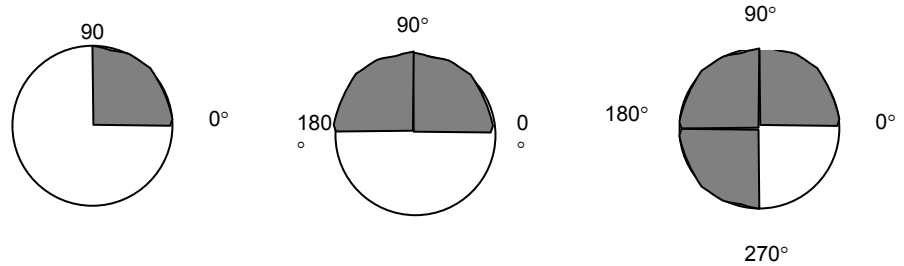
Some students may not be ready for measuring angles. If students had difficulty in understanding what an angle is or which angles are larger or smaller than a right angle, skip Protractors and Angles.

### 1. Add four $90^\circ$ angles in a circle to reach a total of $360^\circ$ .

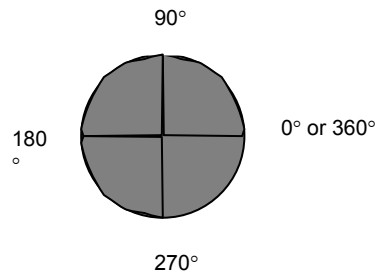
Give students two copies of Student Page 3. Have them fold then cut one of the circles into four quarters.



Look at the four angles formed by each quarter of the circle and use the protractor to verify that the size of each angle is  $90^\circ$ . Ask students to place one of the circle pieces onto their other copy of Student Page 3 so that it fits between the 0-degree line and the 90-degree line. Place the second piece next to the first piece in the circle. *Ask: What is the total number of degrees in the two angles?* ( $90^\circ + 90^\circ = 180^\circ$ ). Write “180°” outside the circle.



Have students put the third piece inside the circle and add the three 90-degree angles together. *Ask: How many degrees are there in the entire circle?* Have students place the last piece in the circle, add the final 90-degree to 270 degrees. The full circle makes a  $360^\circ$  angle.



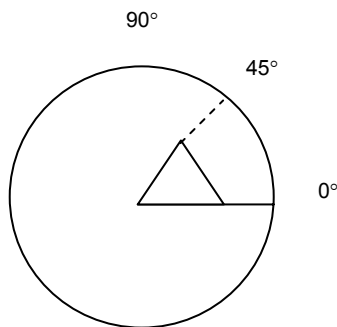
## 2. Relate the degree measure of quarter circles to the degree measures on a protractor.

Give students a copy of Student Page 4: Protractor. Show students that 0 degrees, 90 degrees, 180 degrees, and 270 degrees are marked in the same way as they marked them on the blank copy of the protractor. Point out that other numbers of degrees are labeled in between these points on the protractor.

## 3. Use a protractor to measure the angles of shapes.

Have students place the shape to be measured so that it touches the center of the circle and one of its sides falls along the line pointing at the 0-degree mark. Using a straight edge draw a line that extends from the side of the shape to form an angle to the edge of the circle. *Ask: Is the angle formed by the  $0^\circ$  line and the line you just drew less than, equal to or greater than  $90^\circ$ ?*

Experiment with the different shapes from Student Page 1. Ask students to predict the size of the different shapes' angles, and then measure the angles with the protractor. Ask questions, such as: *Which angles were the easiest to predict? Which were more difficult? Why do you think that is?*



## Angle Games

### 1. Emphasize sight recognition of angles by playing Largest Angle Wins.

Deal angles cards (Student Page 5) face down to all players. Players turn over their top card; the player who has the largest angle wins all the top cards from this round, and adds them to his or her pile. Note: If one of the Angle Cards displayed is a right angle, that player gets to take a card from each player's pile (even if the player with the right angle did not win the round). If two players display the same angle, then they have an angle contest. Each player places two cards face down and then a third card face up. The player who displays the largest angle wins, and that player gets all six cards. (Right-angle cards only work when they are played face up.) The player with the most cards at the end of the game is the winner.

### 2. Play Complete the Circle to practice combining angle measures to make $360^\circ$ .

Give students a copy of Student Page 6 and the cut-out pieces from another copy of Student Page 6. Challenge students to write the size of each angle on the piece. Encourage them to figure out ways to determine the angle size without having to measure each angle. When all of the pieces are labeled, have students experiment with various ways to complete the circle. When students are comfortable with the sizes of the different angles and how they can be used to fill in the circle, play Complete the Circle. Mix up the pile of circle pieces, and have students take turns drawing one piece from the pile. As each player draws a new piece, he or she places it inside the circle. The goal is to be the one who

places the piece that completes the circle. If a piece cannot be used, it is placed at the bottom of the deck. The first player to completely fill his or her circle wins.

## Making Turns

### 1. Explore the relationship between angles and turns by walking along the shapes on the floor.

Take a piece of masking tape or string and create a shape outline on the floor. Ask students to walk along the edges of the shape, identifying the corners of the shape as they walk. Ask:

- ♦ *How many corners are in this shape?*
- ♦ *How much do you turn your body at each of the corners?*
- ♦ *What size are the angles at the places where you turn?*

Create several more shapes, each time having students walk along the edges of the shape and describe the corners they encounter. Encourage students to determine whether the angle at each corner is bigger or smaller than a right angle. Students may notice that they have to turn more at corners that are smaller than right angles and less at corners that are larger than right angles.

# Measuring Length

# 3

## Overview

### Mathematical Focus

- ▶ Estimate and measure length
- ▶ Use standard units of measurement

In this activity, students estimate, measure, and compare the lengths of everyday objects. Students trace their hands and feet. They estimate the length of their hands, fingers, and feet, and then make measurements to check their estimates. Students also explore the concept of distance.

### Preparation and Materials

**Before the session, gather the following materials:**

- ▶ Student Page 7: Neighborhood Map
- ▶ Student Page 8: Distance Cards
- ▶ Student Page 9: Length Hunt Cards
- ▶ Rulers or tape measures
- ▶ Scratch Paper

Cut out distance cards (Student Page 8) and Length Hunt Cards (Student Page 9) ahead of time. This activity is written using the English system, but can be adapted to use the metric system if necessary.

# Activity

## How Long Is It?

### 1. Estimate the lengths of various objects; check estimates using a ruler.

Give each student a ruler, and ask:

- ♦ *Where and when do you use a ruler?*
- ♦ *Look at the divisions on the ruler; how are the numbers placed?*
- ♦ *What are the division marks between the numbers?*
- ♦ *How many inches are on this ruler?*
- ♦ *What kinds of objects do you measure in inches? When do you use feet instead?*

### 2. Use benchmark measurements of one inch and one foot to estimate length.

Ask students to draw a line that is one inch long. Have students use the benchmark inch they just drew to estimate the length of the paper they are writing on. Using a ruler, they can then measure the paper to check their estimate. Ask students to look around the room:

- ♦ *Do you see an object that is about one inch long?*
- ♦ *Do you see an object that is about one foot long?*

Have students measure the objects and check their estimates to see how close they were. Continue in the same way asking students to identify objects in the room that are *about* 2 inches long, 6 inches long, 2 feet long and 6 feet long. Have them measure the objects to check their estimates.

### 3. Create a chart to record estimates and measurements of length.

Have students start a chart in which they record the names of objects, estimates of the lengths of those objects, and the actual measured lengths. Include objects such as a door, a pencil, an eraser, a desk or table, a tile on the floor.

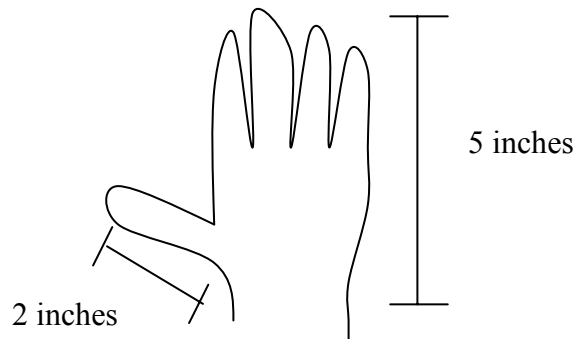
Object	Estimate	Length
Paper	10 inches	11 ½ inches

## Note

Students may need help placing a ruler correctly, or understanding where to measure from. Also discuss with students how to handle fractional parts of inches or feet. When measuring in feet, if the length is not an exact number, the remainder can be reported in inches. When measuring in inches, fractions of an inch can be rounded up or down to the nearest inch (or, if students are already familiar with fractions, they can report fractions of an inch).

### 4. Estimate, then measure the lengths of hands, fingers and feet.

Have students to trace their hands and feet on a piece of paper. Ask them to estimate the lengths of their hands (from wrist to tip of longest finger), feet, and fingers, and then use a ruler to measure the actual lengths. Have them write the lengths next to each body part on the piece of paper.



Challenge students to estimate the length of *your* hands, feet, and fingers. Then let them trace your hands and feet and make measurements to check their estimates.

## How Far Is It?

### 1. Discuss the ways in which people describe and measure distance.

Ask:

- ♦ *How far do you think it is from here to the door?*
- ♦ *How could you estimate the distance?*
- ♦ *What could you use to measure the distance?*

Students measure the distance with a ruler, yardstick, or tape measure. They can also look for more than one path to the door, and check how the distances differ.

Continue to explore the concept of distance by giving the students challenges. For example:

- ♦ *Find an object that is five feet away from you.*
- ♦ *Measure the distance to the nearest empty chair.*
- ♦ *Estimate the distance between a given object and the door, then measure the actual distance.*
- ♦ *Find a path from one point to another that's three feet long.*
- ♦ *Describe a path from where you are sitting to three different locations in the room. Estimate the length of each path. Which is longest? Measure the length of each path.*

## **2. Measure distances on a map.**

Give students a copy of Student Page 7: Neighborhood Map. Explain that on this map, the distance from one point to another is measured along the roads. Point out the scale on the map: 1 inch = 1 mile. Discuss what the scale means. Ask students to use a ruler to help them answer the following questions about distances in the neighborhood:

- ♦ *How many inches on the map is it from the red house to the blue house? How many miles does this distance represent?*
- ♦ *Can another path be taken from the red house to the blue house? How long is that path?*
- ♦ *Using the sidewalks, what are two ways to get from the red house to the store? How long is each?*
- ♦ *Can you find two paths that are about the same distance from the red house to the post office?*
- ♦ *Can you find two paths that are about the same distance from the red house to the green house?*
- ♦ *How far is the green house from the red house, if you did not have to use the sidewalks?*

Encourage students to come up with other questions about distances between locations on the map.

# Distance and Length Challenges

## 1. Hold a Distance Race.

Use the cut-out cards from Student Page 8: Distance Cards. Before beginning the Distance Race, designate a starting line and a finish line (about 10 feet apart). Have players line up on the starting line, each with a stack of Distance Cards and a ruler. Each player turns over the top card and follows its directions, using the ruler to measure how far forward or backward to move. That card then goes to the bottom of the pile. The first player to reach the finish line is the winner.

## 2. Have a Length Hunt.

Use the cut-out cards from Student Page 9. Ask students to find objects that match the different criteria on the cards. Encourage them to make up new cards for the length hunt.

# Perimeter

## Overview

### Mathematical Focus

- ▶ Estimate, measure and compare perimeters
- ▶ Development a formula for finding the perimeter of rectangles

In this activity, students estimate, measure and compare the perimeters of different objects. They use both non-standard units of measurement (footsteps) and standard units of measurement (e.g. inches and feet) to measure the perimeters of various shapes and objects. They develop a formula for finding the perimeter of a rectangle.

### Preparation and Materials

**Before the session, gather the following materials:**

- ▶ Student Page 1: Shapes (1)
- ▶ Student Page 10: Perimeters
- ▶ Student Page 11: Graph Paper, several copies
- ▶ Student Page 12: Rectangles
- ▶ String, one 8-foot piece
- ▶ Rulers

Cut out shapes from Student Page 1 ahead of time.

# Activity

## The Distance Around

### 1. Explore the concept of perimeter using non-standard units and then standard units of measurement.

Give students a piece of string to make a shape on the floor. Have students walk the shape and count the number of footsteps it takes to get all the way around. Create a different shape with the string and have students again count the number of steps around the shape. Explain that the distance around a shape (in this case, the number of steps) is called the *perimeter*.

Discuss the difference between telling someone that the perimeter of a shape is 10 steps or that the perimeter is 8 feet (measured with a ruler). Footsteps are a non-standard unit of measurement and feet (12-inches) are a standard unit of measurement. Discuss the difference and ask why it might be okay to describe length in steps and in other situations it is important to use a standard unit of measurement such as feet.

### Teaching Tip

Students should understand that there are different ways to measure length, such as the number of steps or the number of feet, etc., but that it is important to have standard units of measurement that are familiar to everyone. If available from the library, read *How Big Is a Foot?* by Rolf Myller with the students. This short story nicely illustrates the need for standard units of measurement.

### 2. Determine the perimeter of different shapes.

Give students a copy of Student Page 10. Pretend that a little creature walked around the edge of these shapes, and that each step this creature took was the size of one square on the graph paper. Ask students to find and record the number of steps the creature took as it walked around each shape. Draw shapes on graph paper; determine the perimeter of each shape.

Give students a copy of Student Page 11. Have them draw some shapes on the graph paper, making sure that the lines they use to create their shapes are along the lines of the graph paper. Have students first estimate and then determine the perimeter of each shape.

**3. Use a ruler to measure the perimeter of shapes in inches or centimeters.**

Ask: *How else could you measure the perimeter of these shapes.* Remind students that the perimeter is the distance around each shape.

Ask: *Could you measure the perimeter with a ruler?* [yes]. Give students some cut-out shapes cut from Student Page 1 and a ruler. Have them measure and record the perimeter of several shapes in inches.

## Creating Shapes

**1. Use string to make different shapes with the same perimeter.**

Give students the long piece of string again and ask them to measure it, using a ruler. Ask:

- ♦ *If we tied the ends of this string together and made a square, what would the perimeter of the square be?*
- ♦ *How do you know?*
- ♦ *If you made a triangle, would the perimeter be the same length as the perimeter of the square? Explain your thinking. Try it and see.*
- ♦ *What other shapes can you make? Does the perimeter stay the same or does it change for each of these shapes? Do you think the perimeter would be the same for any shape you made with this piece of string? Explain your thinking.*
- ♦ *How could we make a shape with a different perimeter?*

**2. Use graph paper to make shapes and explore the concept of perimeter.**

Give students some graph paper and ask them to make five different shapes, each with a perimeter of eight units. Ask:

- ♦ *How are these shapes the same?*
- ♦ *How are they different?*

Have students make some shapes with different perimeters, such as 5, 6, or 10, and ask them to think about how their shapes are the same and how they are different.

## Finding a Rule

Give students a copy of Student Page 12: Rectangles. For each rectangle, have students use a ruler to find the lengths of all four sides. Have them record the lengths in the chart and then calculate the perimeter of each rectangle.

- ♦ *What do you notice about the lengths of the sides of these rectangles?*
- ♦ *How did you find the perimeters?*
- ♦ *Could you figure out the perimeter if you knew the length of two of the sides of a rectangle? Explain your thinking.*
- ♦ *Which two sides would you need to know?*
- ♦ *If a rectangle had one side that was four inches long and one side that was two inches long, what would the perimeter be?*
- ♦ *If a rectangle was  $x$  inches long and  $y$  inches wide, is there a way to say what the perimeter would be? [The rule for finding the perimeter of a rectangle is  $2x + 2y$ , if  $x$  is the length of the rectangle and  $y$  is the width of the rectangle.]*

# Area

# 5

## Overview

### Mathematical Focus

- Estimate, measure and compare areas
- Develop a formula for finding the area of a rectangle

In this activity, students explore the concept of area by creating a set of pentominoes. They investigate methods of measuring area and determine a rule for finding the area of a rectangle.

### Preparation and Materials

**Before the session, gather the following materials:**

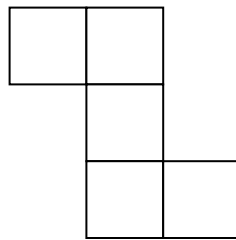
- Student Page 11: Graph Paper, several copies
- Scissors

Cut out five squares from the graph paper ahead of time.  
Save the pentominoes created in Part 1 for use in Activity 6.

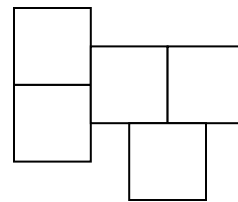
## Pentominoes

### 1. Use five squares to make different shapes: explore the relationship between area and perimeter.

Give students five squares cut out from a piece of graph paper. Ask them to put the five squares together to make one big shape. When squares are placed side-by-side, the sides must line up exactly, as shown below.



Okay



Not Okay

Explain that the area of a shape is the amount of space taken up by the shape. Often area is measured in square units. Each of the small squares on the graph paper is one square unit. The area of the shape is the total number of square units the shape takes up. Have students trace their shape on a piece of graph paper. Ask them to find and record the area and the perimeter of the shape.

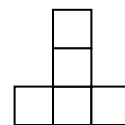
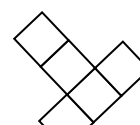
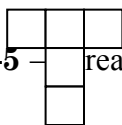
Have students put the squares together in a different way to make another large shape. Trace the second shape on a piece of graph paper, cut it out and record the area and perimeter on the shape. Ask:

- ♦ *What is the area of your new shape?*
- ♦ Explain why this shape has the same area as the first shape.
- ♦ *Do the shapes have the same perimeter?* Explain why this is so.

### 2. How many different shapes can you make with five squares.

Challenge students to find all the shapes they can make with five squares. Each time they find a new shape, they should trace it on graph paper and then cut it out. Ask students to record the area and perimeter of each shape. (Note: Save these cut-out pentominoes, or groups of five squares, for use in Activity 6.)

Have students make sure that each of their shapes is different. Explain



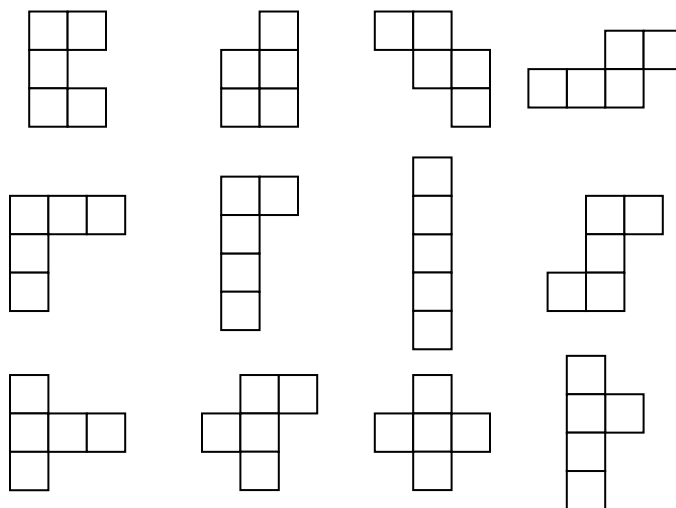
that if a shape can be rotated or flipped so that it fits exactly on another shape, then it does not count as a new shape. For example, the following shapes are all the same because they can be rotated to match up with one another:

### 3. Talk about the pentominoes and the process of making them.

Ask:

- ♦ *How many different shapes made with five squares could you find?*
- ♦ *What are the areas of each?*
- ♦ *What are the perimeters of each?*
- ♦ *Did the areas change or stay the same?*
- ♦ *Did the perimeters change or stay the same?*
- ♦ *Can you make any generalizations about the area and perimeter of shapes made from these five squares?*

There are 12 different distinct combinations of the 5 squares:



### 4. Use pentominoes to cover different rectangles.

Outline three different rectangles on pieces of graph paper: 3 x 20, 4 x 5, and 5 x 12. Challenge students to cover each rectangle with the 12 pentominoes they made and cut out. Have students find the area of each rectangle.

## Areas on Graph Paper

### 1. Find the area of large squares drawn on graph paper.

Have students draw several different-size large squares on a piece of graph paper. Ask:

- ♦ *How many small squares are in by each of the large squares you drew?*
- ♦ *Can you draw a square that covers 16 small squares? 25 small squares? 9 small squares?*
- ♦ *Can you draw a square that covers 6 small squares? Why or why not?*

## 2. Draw shapes on graph paper; give the area in square units.

Ask students to draw some rectangles on the graph paper and find their areas. Students may also try drawing other shapes. If they try to draw shapes such as triangles, explain that to look at these areas they will have to think about cutting squares in half by drawing a line along the diagonal. For each shape that students draw, ask them to give the area in square units.

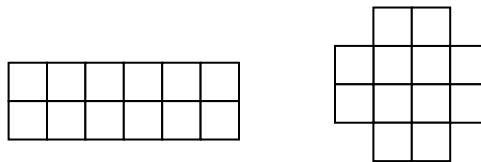
*Teaching Tip*

Talk with students about the importance of standard units of measurement. They are using the small squares from the graph or graph paper as a square unit here, but make sure they understand that a measurement of area could be made in square inches, or square centimeters, or square units of any form. In this way, the area is the number of squares with a length and a width of one unit (inch, centimeter, etc.) that the shape covers.

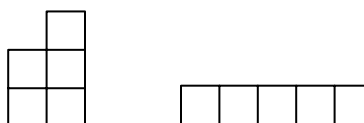
## 3. Experiment with drawing shapes of specified areas.

Challenge students to:

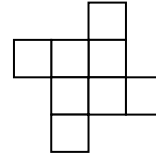
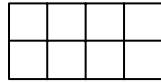
- ♦ Draw two different shapes with an area of 12.



- ♦ Draw two different shapes with an area of five.



- ♦ Draw three different shapes with an area of eight.



## Area Formula for Rectangles

1. **Discuss how the length and width of a rectangle are related to its area.**

Draw a 1 x 5 rectangle, 2 x 5 rectangle, 3 x 5 rectangle, and 4 x 5 rectangle on a piece of graph paper. Have students write the dimensions of each rectangle and the area. Ask:

- ♦ *Are length and width related to area? Can you explain your thinking?*
- ♦ *Can you write a rule for finding the area of the first rectangle?*
- ♦ *Can you write a rule for finding the area of the second rectangle?*
- ♦ *Can you write a rule for finding the area of any rectangle?*
- ♦ *Will the rule always work? Use pictures to explain why.*
- ♦ *What other shapes does this rule work for? Can you use pictures to explain your thinking?*

# Cubes, Surface Area, and Volume

## Overview

### Mathematical Focus

- ▶ Identify three-dimensional shapes from two-dimensional views
- ▶ Estimate, measure and compare volume in cubic units.
- ▶ Estimate, measure and compare surface area

In this activity, students use cubes and nets to explore three-dimensional shapes, surface area and volume. Students construct cubes from nets. They predict the size of the net needed to cover ever-increasing shapes made of cubes and they calculate the volume of objects made from cubes.

### Preparation and Materials

**Before the session, gather the following materials:**

- ▶ Student Page 11: Graph Paper
- ▶ Student Page 15: Cube Net
- ▶ Student Page 16: Cube Packages
- ▶ Small same-size cubes (such as sugar cubes, dice, or blocks), 30
- ▶ Pentominoes made in Activity 5
- ▶ Scissors
- ▶ Tape

## Using Nets

### 1. Experiment with creating a “jacket” for a cube.

Give students a cube (such as a wooden block or small cardboard box), a sheet of graph paper and some scissors. Challenge students to use the paper to create a cover or “jacket” for the cube. Tell them that the jacket must cover the cube completely with no holes or overlaps. Let them experiment for a while, testing the different covers they make by placing them around the cube and seeing if they fit. Ask students to explain their thinking as they work. If students are having trouble, pose the following questions:

- ♦ *How many sides are there on a cube?*
- ♦ *What shape are the sides?*
- ♦ *How are the sides connected to one another?*
- ♦ *What would this cube look like if it were spread out flat on the table, with every square attached to another square by at least one side?*

When students think they have a cover that meets the criteria, ask:

- ♦ *Do you think there are other covers that would also work for this cube?*
- ♦ *How would those covers be the same as or different from this cover?*

### 2. Make a cube from a two-dimensional net.

Give students a cube net cut out from Student Page 15: Cube Net. Ask: *Can you fold this shape into a cube? How is it similar to or different from the covers you made for your cubes?* Explain that these covers for cubes are called *nets*.

### 3. Use graph paper to make nets for cubes.

Challenge students to use graph paper to create some different nets for cubes. Ask questions such as:

- ♦ *How many squares will need to be in any net that can be folded into a cube?*
- ♦ *How many sides does a cube have?*

- ♦ *How many corners does a cube have?*
- ♦ *How many edges does a cube have?*

#### **4. Explore pentominoes as possible nets for boxes without a top.**

Ask students to look back at the pentominoes they created during the last activity. Ask: *Which of the pentominoes could be folded into a box with no top?* Explain your thinking.

## **Surface Area of Cubes**

### **1. Find the surface area of a cube.**

Using a cube that students made from the net, ask: *What is the area of one face on the cube? How do you know?* You may need to remind students of what they talked about in the activity about area. Ask students what possible ways there are to check the area of a square face of the cube. (They could either count the squares on the graph paper, or they could measure the length and the width of the square and multiply them together to find the area.)

Explain that the total area of every surface of a three-dimensional shape, such as a cube, is called its *surface area*. Ask: *What parts of this cube would need to be measured to find its surface area?*

Ask students to calculate the surface area of the cube and to explain their thinking as they work. (Students may calculate the area of each face, and add these areas together. They may also realize that all faces have the same area, so they can multiply that area by the number of faces—in this case, six.)

### **2. Create a scenario in which a company creates covers for small packages.**

Explain that all of the company's packages are made up of some number of cubes. The packaging material comes in flat sheets of square units. Students must determine how much packaging material is needed to cover various types of cube packages. Give students a copy of Student Page 16: Cube Packages, and give students 30 small cubes. For each cube package shown on the student page, have students build the package with their own cubes, and then determine how many square units of packaging material are needed to cover it. Before they begin working on each example, ask them to estimate whether the amount of packaging material needed will be more or less than the amount needed for previous examples.

# Volume of Cubes

## 1. Construct large cubes; determine the volume in cubic units.

Using the cubes, ask students to construct a large cube out of the small cubes, keeping track of how many little cubes they use to make the large cube. Explain that the amount of space a three-dimensional shape takes up is called its volume. In other words, if each small cube represents one cubic unit, then the volume of the big cube is the total number of small cubes (or units) it contains. Have students construct a large cube with a volume of 8 cubic units and then a cube with a volume of 27 cubic units. Ask students to look back Student Page 16 and explain how to find the volume of each package.

# Coordinate Systems

# 7

## Overview

### Mathematical Focus

- ▶ Use coordinate systems to record and communicate positions and distances
- ▶ Explore maps and scale drawings

In this activity, students use ordered pairs to describe the location of houses on a coordinate map. In game-oriented activities students explore how coordinate systems can be used to record and communicate position.

### Preparation and Materials

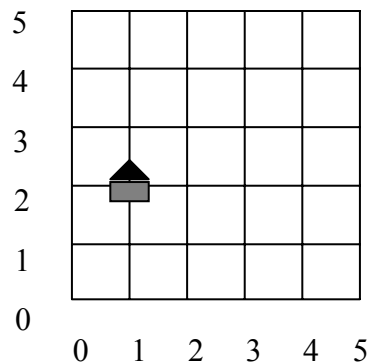
**Before the session, gather the following materials:**

- ▶ Student Page 15: Coordinate Map
- ▶ Student Page 16: Coordinate System, several copies
- ▶ Markers (e.g., pennies, paper clips, slips of paper), 15 per student

## Constructing a Coordinate System

### 1. Use ordered pairs to describe the location of houses on a coordinate grid.

Give students a copy of Student Page 20: Coordinate Map. Explain that the houses on the grid represent homes in a city neighborhood and the lines represent streets. Each house has a unique address that can be described by two numbers, referred to as an ordered pair (the order makes a difference!). To find the address for House A, begin at the star point (0, 0). First determine how many streets you have to go over (one street) – this is the first number in the ordered pair. Then determine how many streets you need to go up (two streets) – this is the second number in the ordered pair. The address for House A is (1, 2). Ask students to find the address for each house on the grid and write the ordered pair beside the house.



Place additional houses on the grid. Describe the location of each house using ordered pairs.

After students have found the addresses for houses A-G, have them add two additional houses, label them “H” and “I” and find their addresses. Ask questions such as the following:

- ♦ *Where would the house with an address of (5, 20) be?*
- ♦ *What is the address for a house located here? (Point to any intersection of lines on the grid.)*
- ♦ *What is the same about the addresses of all the houses in one row? In one column?*

## 2. Connect points on the grid to create a shape.

*Use a pencil to show where a house with the address (3, 20) would be. Without picking up your pencil, make a straight line to the address (3, 10). Still without picking up your pencil, make a straight line to the address (10, 20). Now make a straight line back to (3, 20). What shape have you created?*

## Distances in a Coordinate System

### 1. Find and compare distances between points on the grid.

Ask:

- ♦ *If you can only travel along the streets in the coordinate system, what is the shortest distance from B to G?*
- ♦ *Is there a shorter distance between the two points if you could travel across the empty spaces in the coordinate system? Explain your thinking.*
- ♦ *Have students draw four different paths that could be taken from B to G. Ask: How long is each of these new paths? Are any of the paths from B to G equal in length?*

## Shapes in a Coordinate System

### 1. Use ordered pairs to describe the points of a shape drawn on a coordinate grid.

Give students a copy of Student Page 16: Shapes in a Coordinate System. Ask students what they would do to describe each shape's placement in the coordinate system to someone else. Ask:

- ♦ *What are the important parts of each shape to give a location for?*
- ♦ *Which points would you describe first?*

### 2. Take turns describing the location shapes on a grid.

Have the “describer” draw a shape on a grid and then say the location of the important points of the shape aloud. Have the listener draw a shape to fit into that location and then name the shape. The describer and the listener should then discuss whether they were thinking of the same shape.

### **3. Determine the area and perimeter of shapes.**

Challenge students to determine the perimeter or area of any of the shapes they have drawn in coordinate system during this activity.

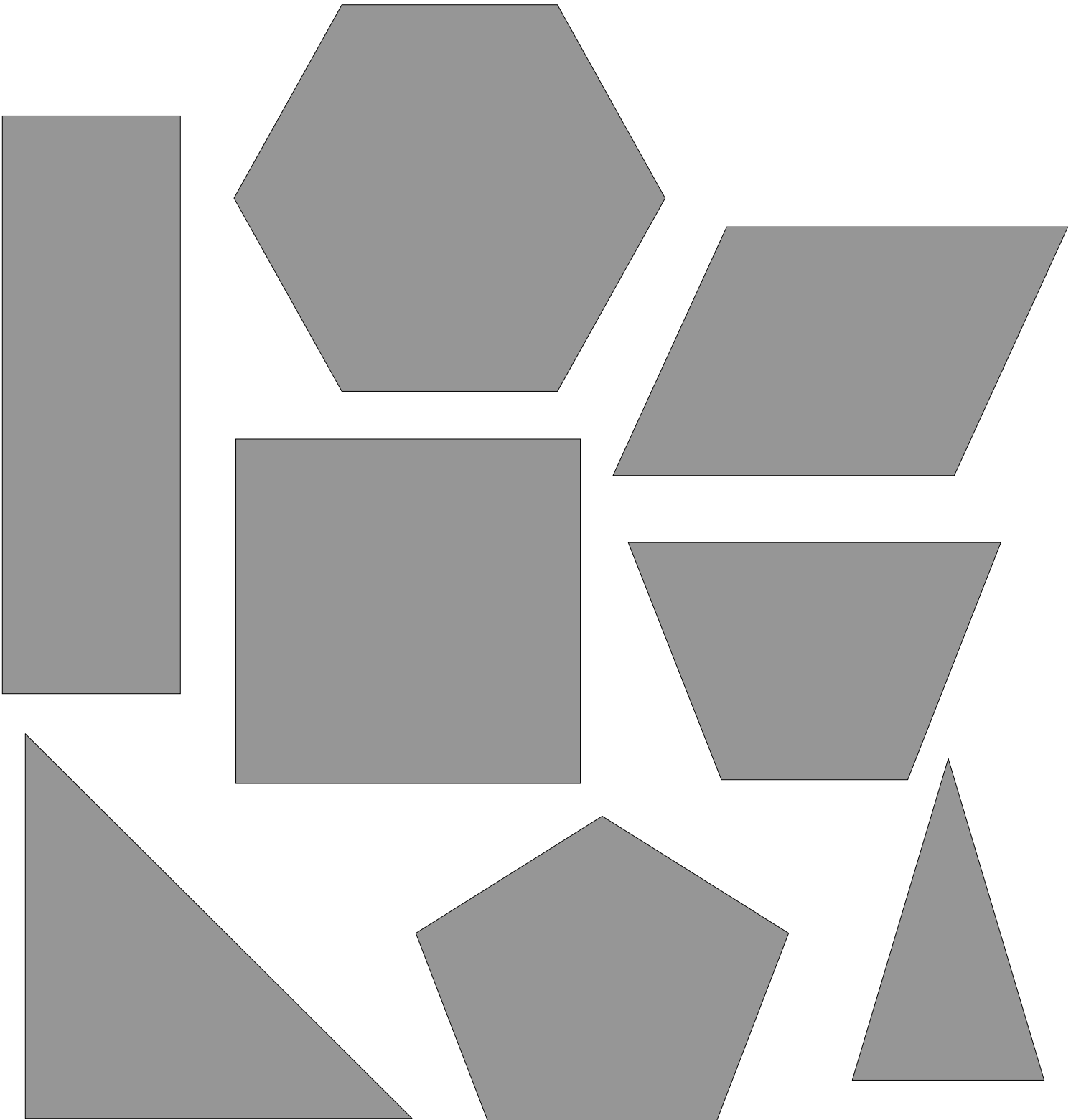
## **Five in a Row in a Coordinate System**

Name the coordinates of points on the grid. Five correctly named points in a row wins the game.

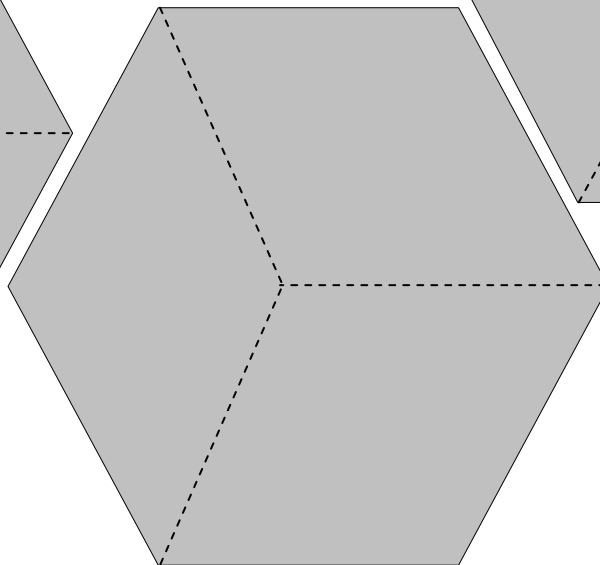
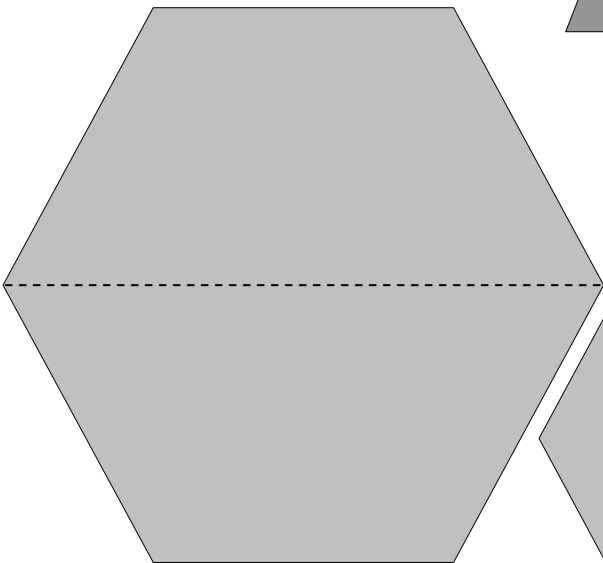
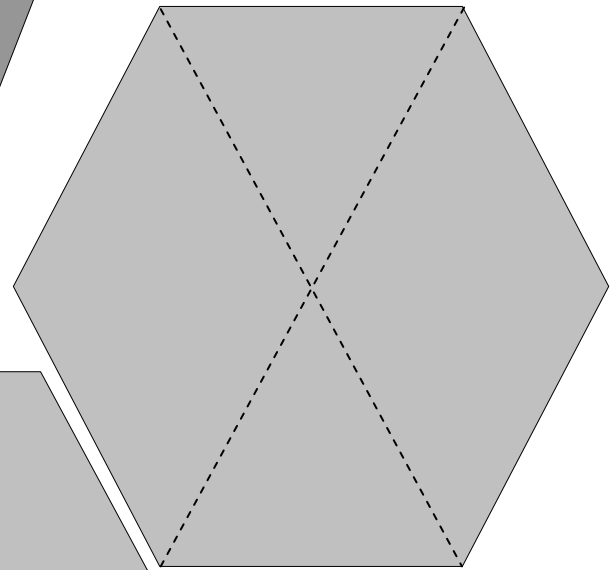
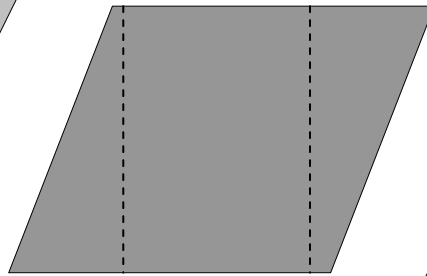
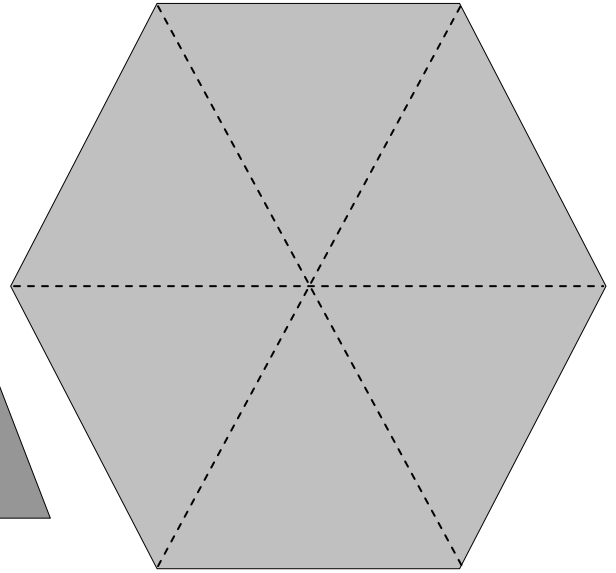
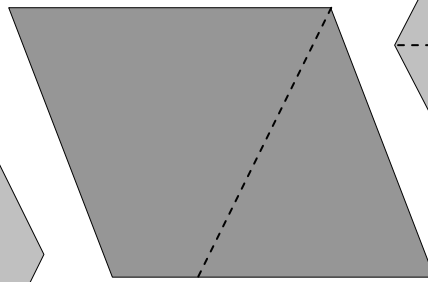
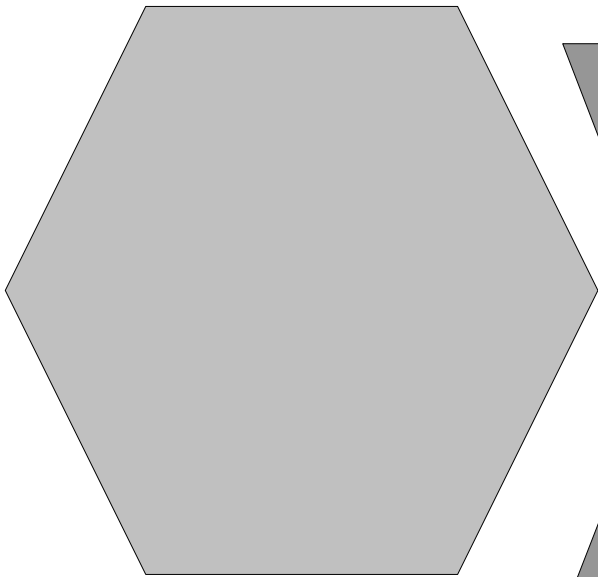
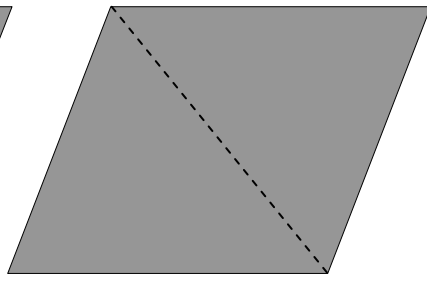
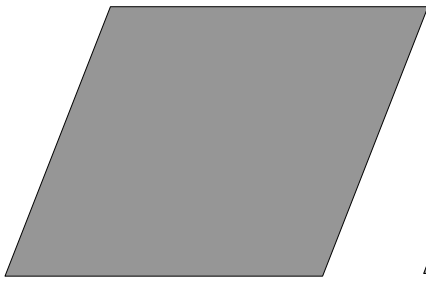
Give students a copy of Student Page 17: Coordinate System. Have each player pick a symbol to mark points on the coordinate system (such as an X, or an O, or an initial). Players take turns naming the intersection they want to mark by naming the coordinates of that point. After naming it, the player puts his or her symbol on the point; and then it is the next player's turn. The object of the game is to get five in a row.

# Shapes

*1*

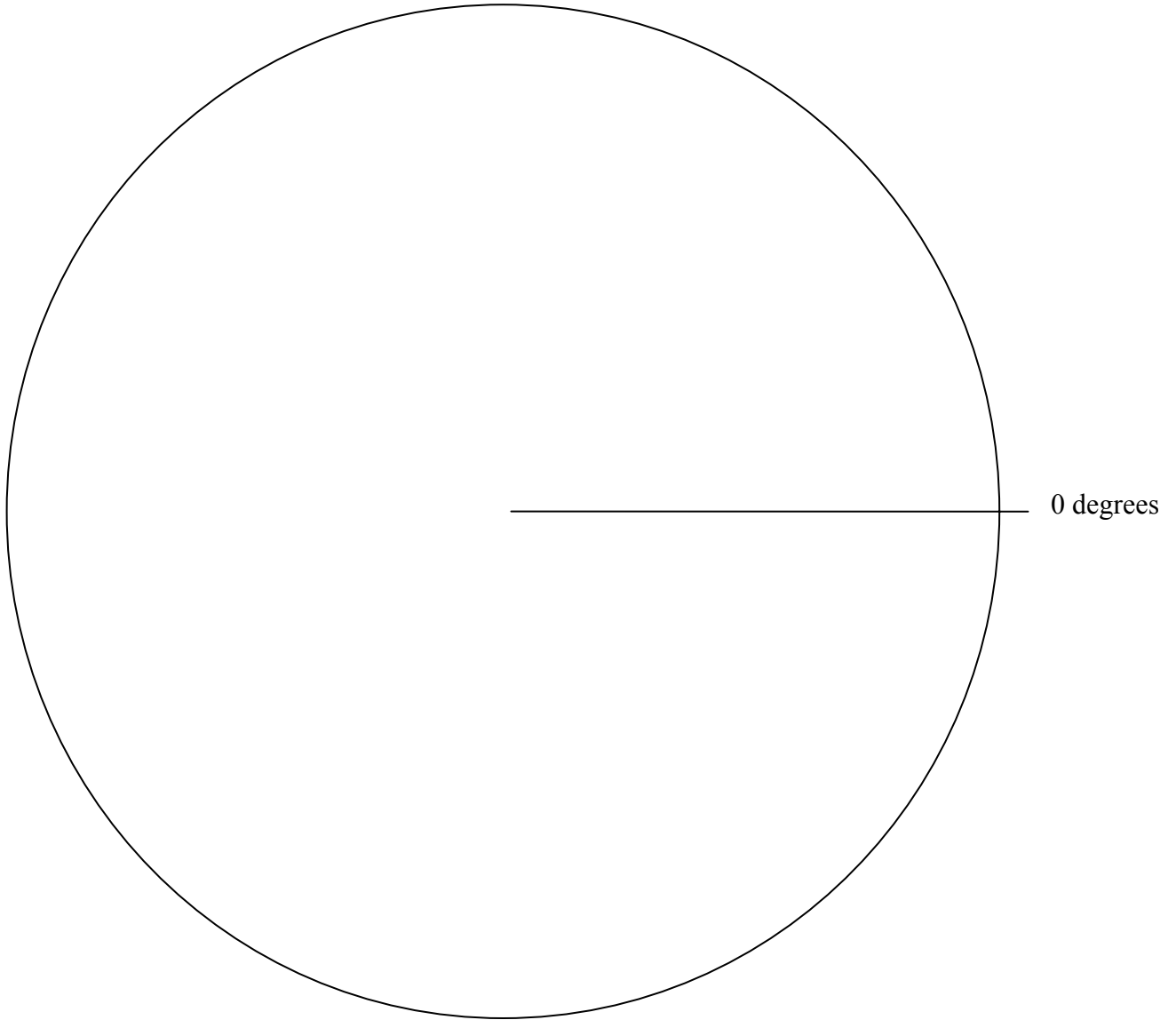


# Hexagons and Parallelograms

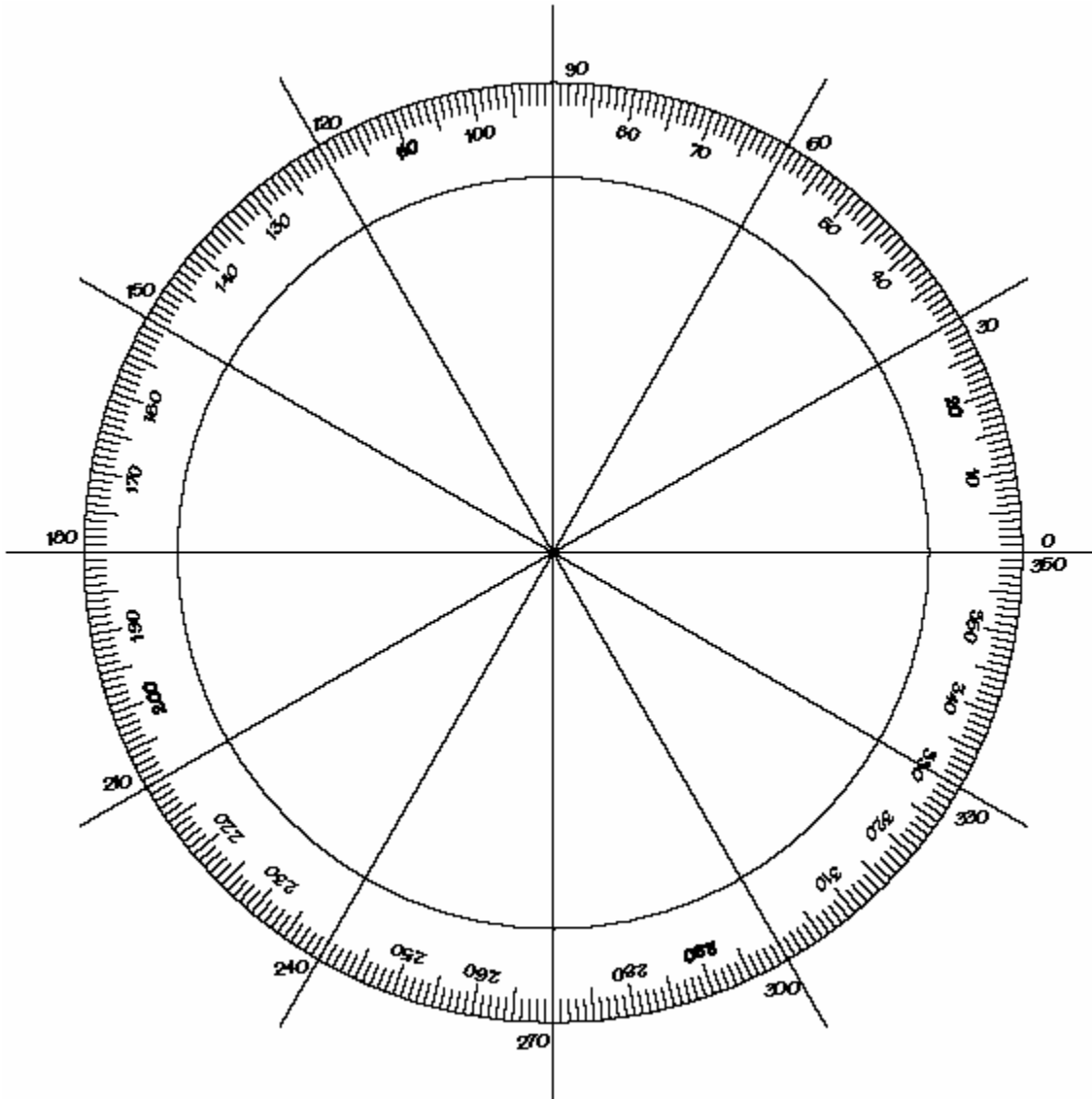


# Protractor Circle

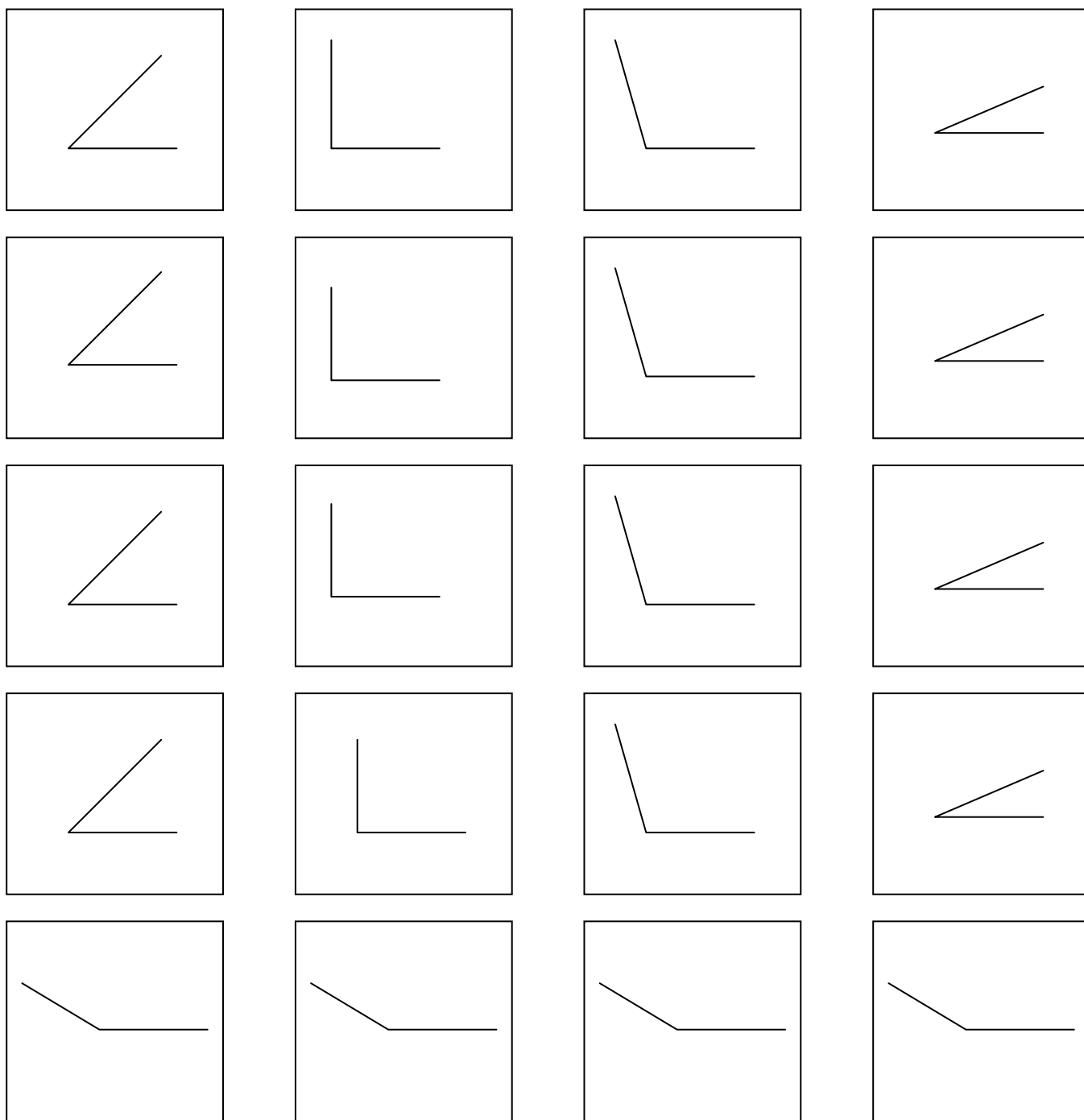
3



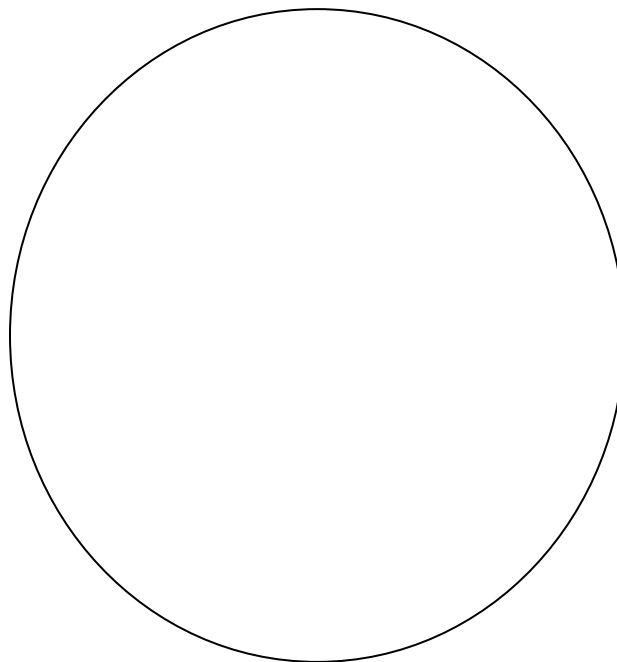
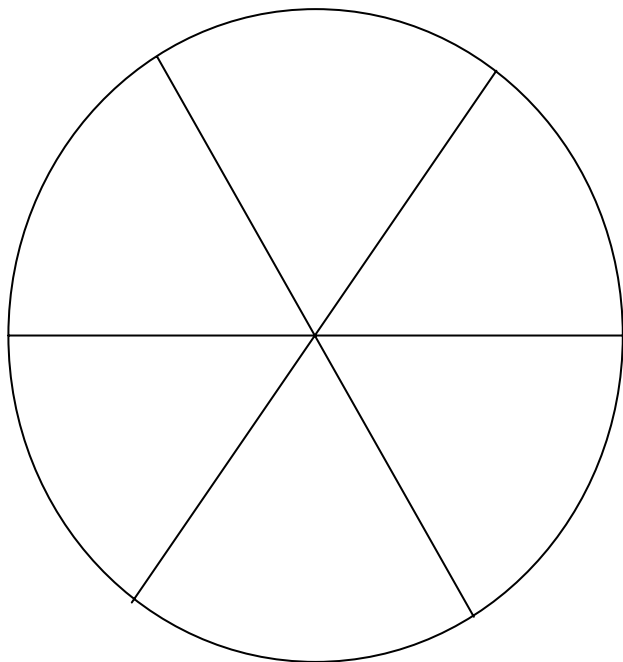
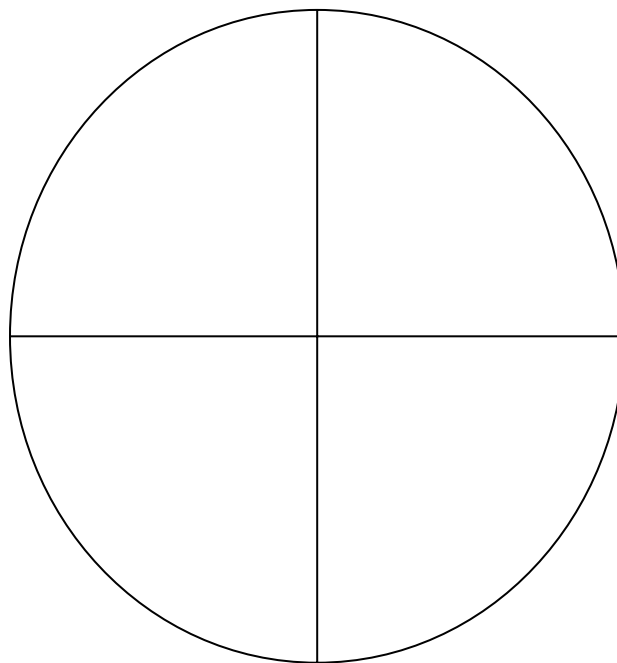
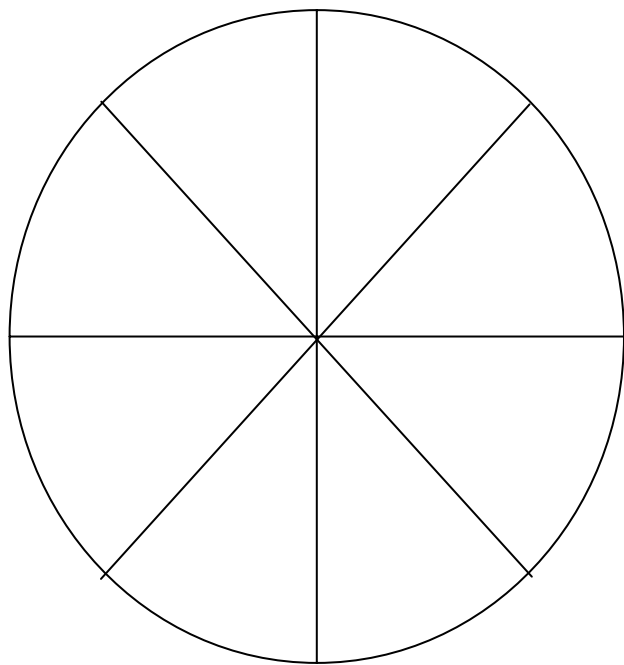
# Final Protractor



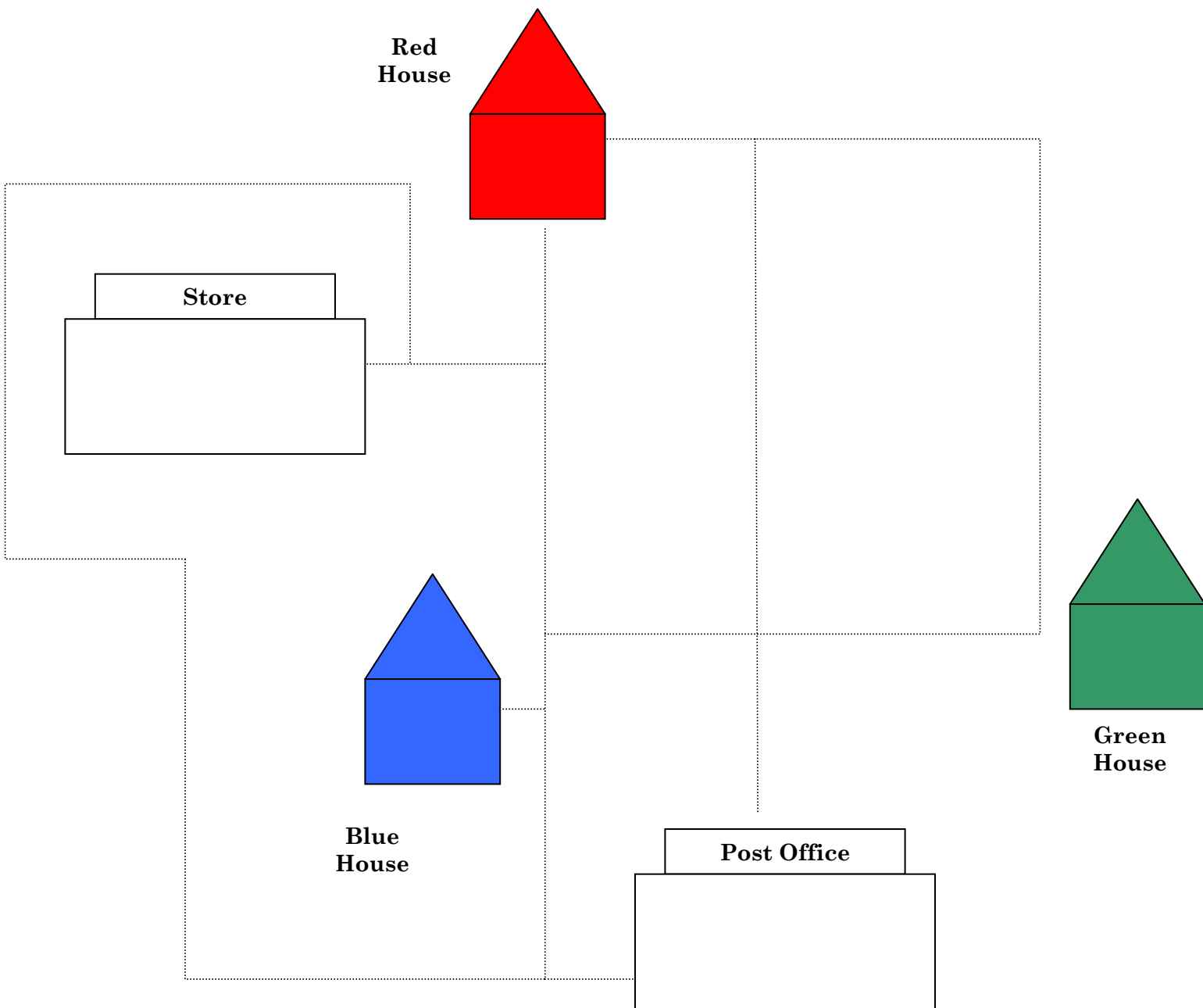
# Angle Cards



# Complete the Circle



# Neighborhood Map





## Distance Cards

**Move  
Forward 2  
Feet**

**Move  
Forward 1  
Foot**

**Move  
Forward  
1.5 Feet**

**Move  
Forward 3  
Feet**

**Move  
Forward 6  
Inches**

**Move  
Forward 4  
Feet**

**Move  
Forward 1  
Foot**

**Move  
Forward 5  
Feet**

**Move Back  
2 Feet**

**Move Back  
1 Foot**

**Move Back  
1/2 Foot**

**Move Back  
6 inches**

**Move Back  
2 Feet**

**Move Back  
1 Foot**

**Move Back  
3 Feet**

**Move Back  
1.5 Feet**



## **Length Hunt**

**Find items to match the following descriptions:**

**An object that is 2 inches long:**

**An object that is 5 feet from the door:**

**An object that has one side that has two sides of different lengths:**

**Length of one side:**

**Length of another side:**

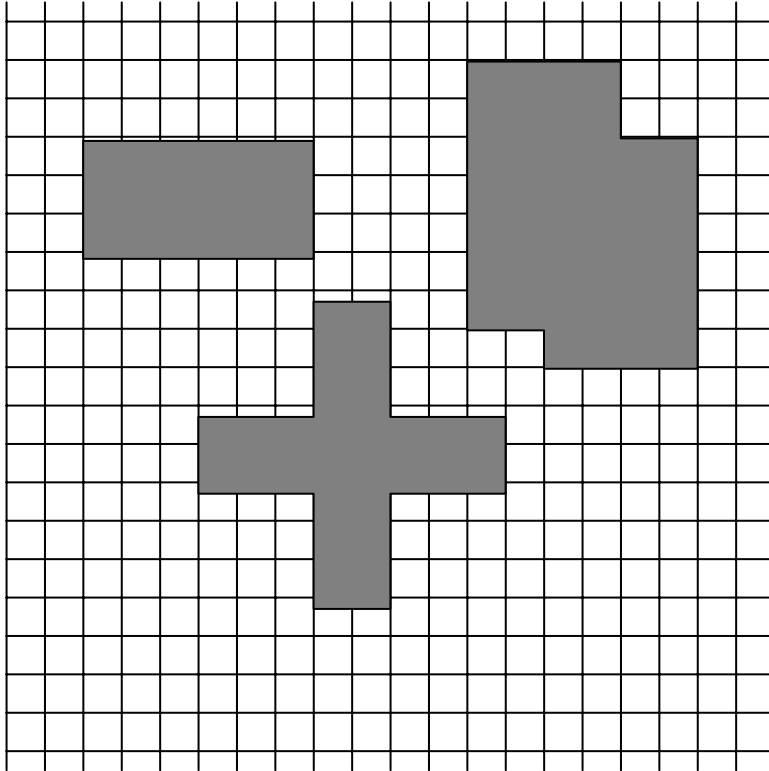
**Measure your nose:**

**An object the same length as your nose:**

**An object that is about 3 feet long:**

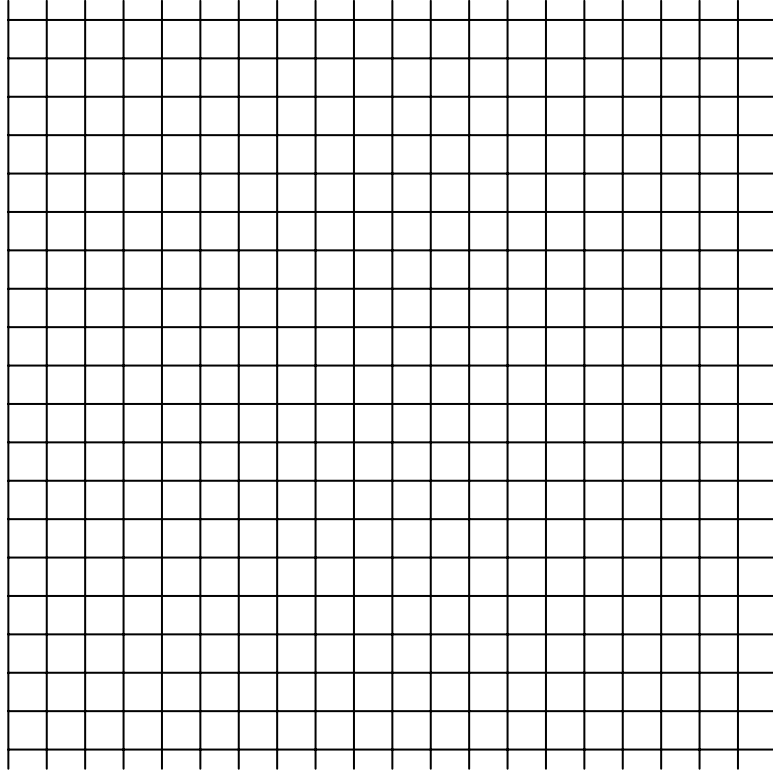
**An object that is found 7 feet from your 3-foot long object from the previous question:**

## Perimeters



# Grid Paper

*11*

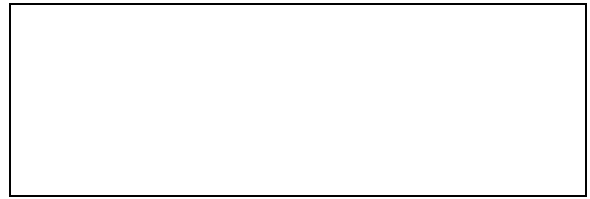


## Rectangles

Rectangle #1



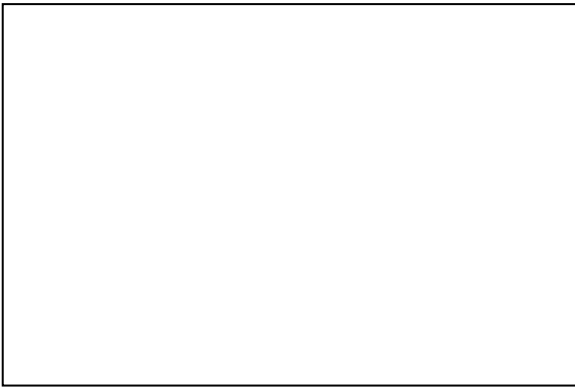
Rectangle #2



Rectangle #3



Rectangle #4



Rectangle #5

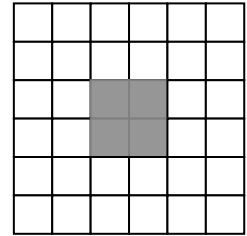
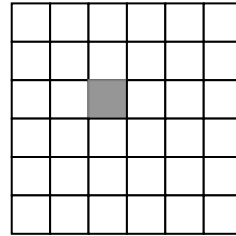


Rectangle	Side 1	Side 2	Side 3	Side 4	Perimeter
#1					
#2					
#3					
#4					
#5					

## Perimeter Cards

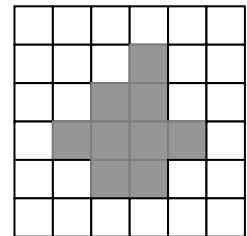
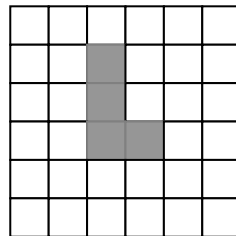
Perimeter =  
  
4 Units

Perimeter =  
  
8 Units



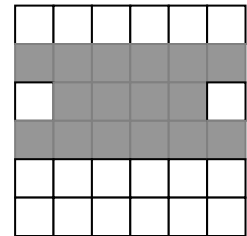
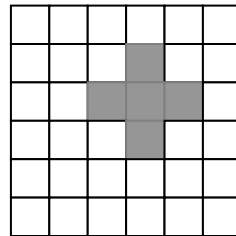
Perimeter =  
  
10 Units

Perimeter =  
  
16 Units



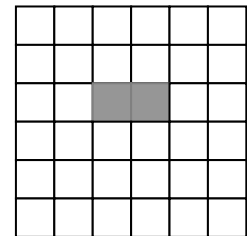
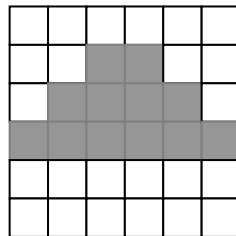
Perimeter =  
  
12 Units

Perimeter =  
  
22 Units



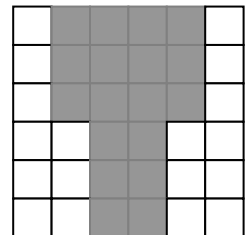
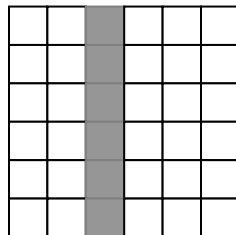
Perimeter =  
  
18 Units

Perimeter =  
  
6 Units



Perimeter =  
  
14 Units

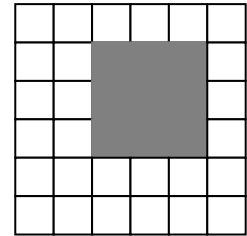
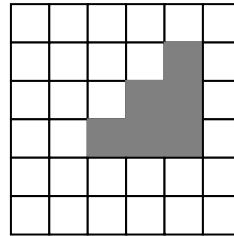
Perimeter =  
  
20 Units



## Area Cards

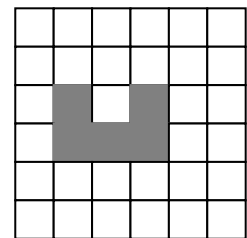
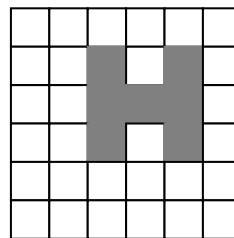
Area =  
6 Square  
Units

Area =  
9 Square  
Units



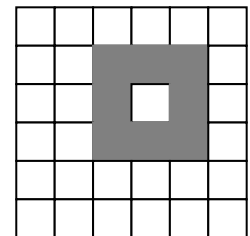
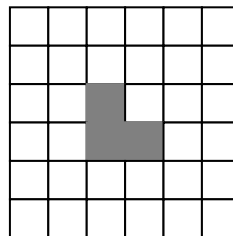
Area =  
7 Square  
Units

Area =  
5 Square  
Units



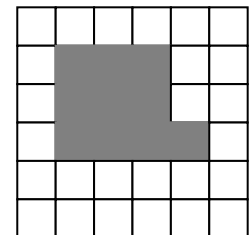
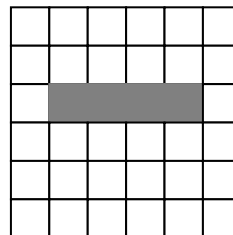
Area =  
3 Square  
Units

Area =  
8 Square  
Units



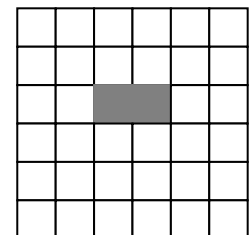
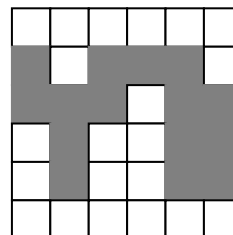
Area =  
4 Square  
Units

Area =  
10 Square  
Units

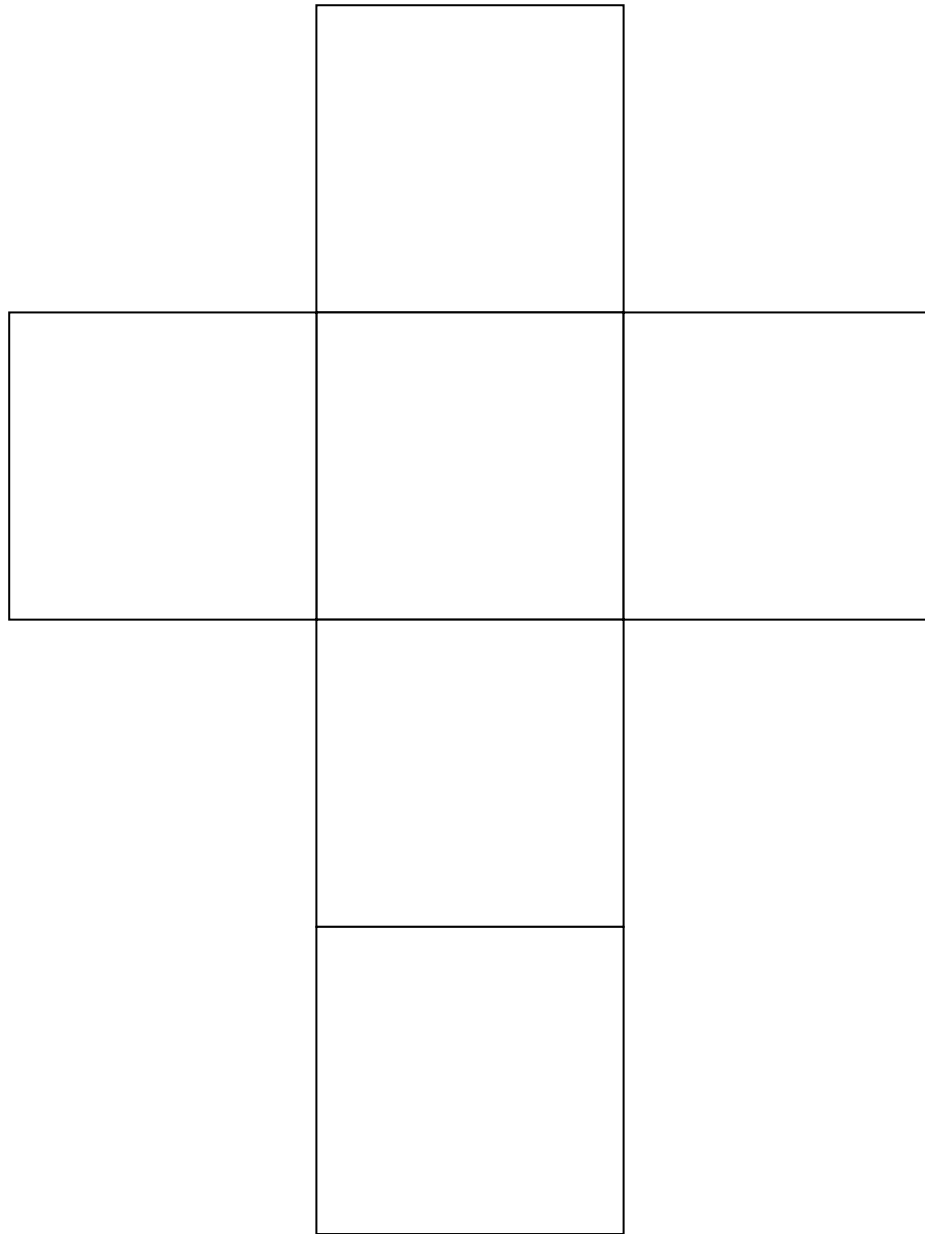


Area =  
15 Square  
Units

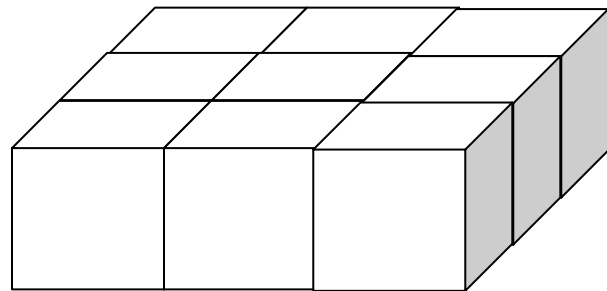
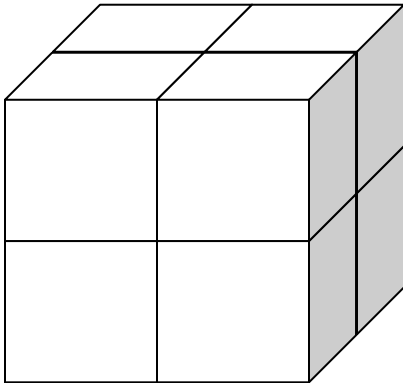
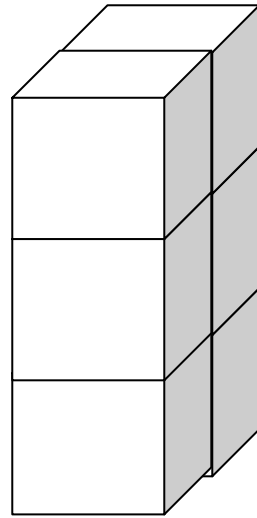
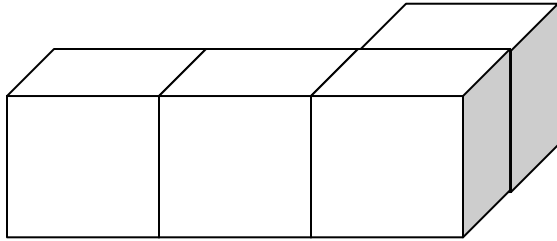
Area =  
2 Square  
Units



# Cube Net



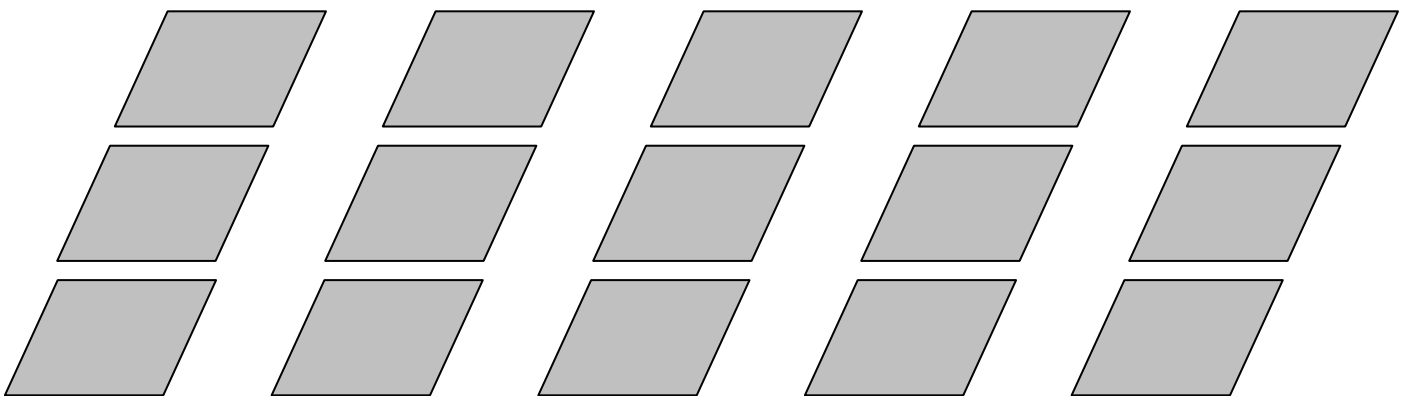
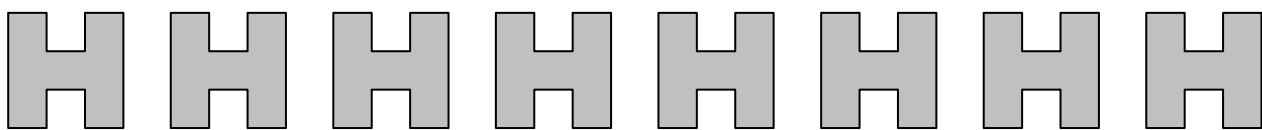
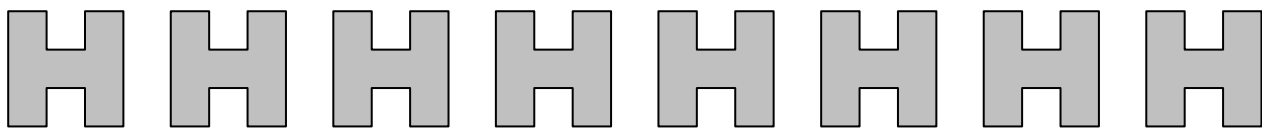
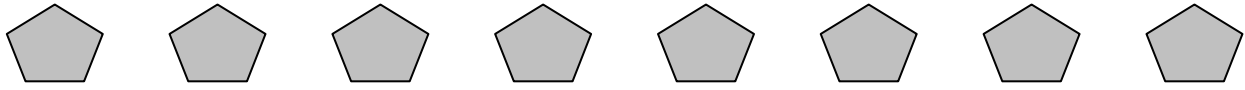
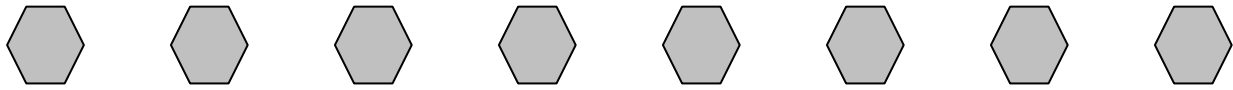
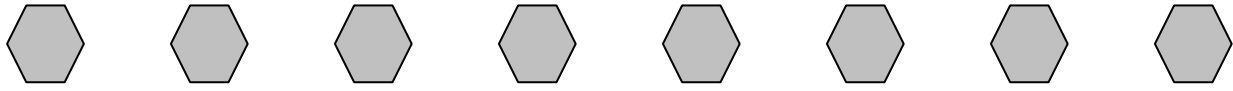
# Cube Packages



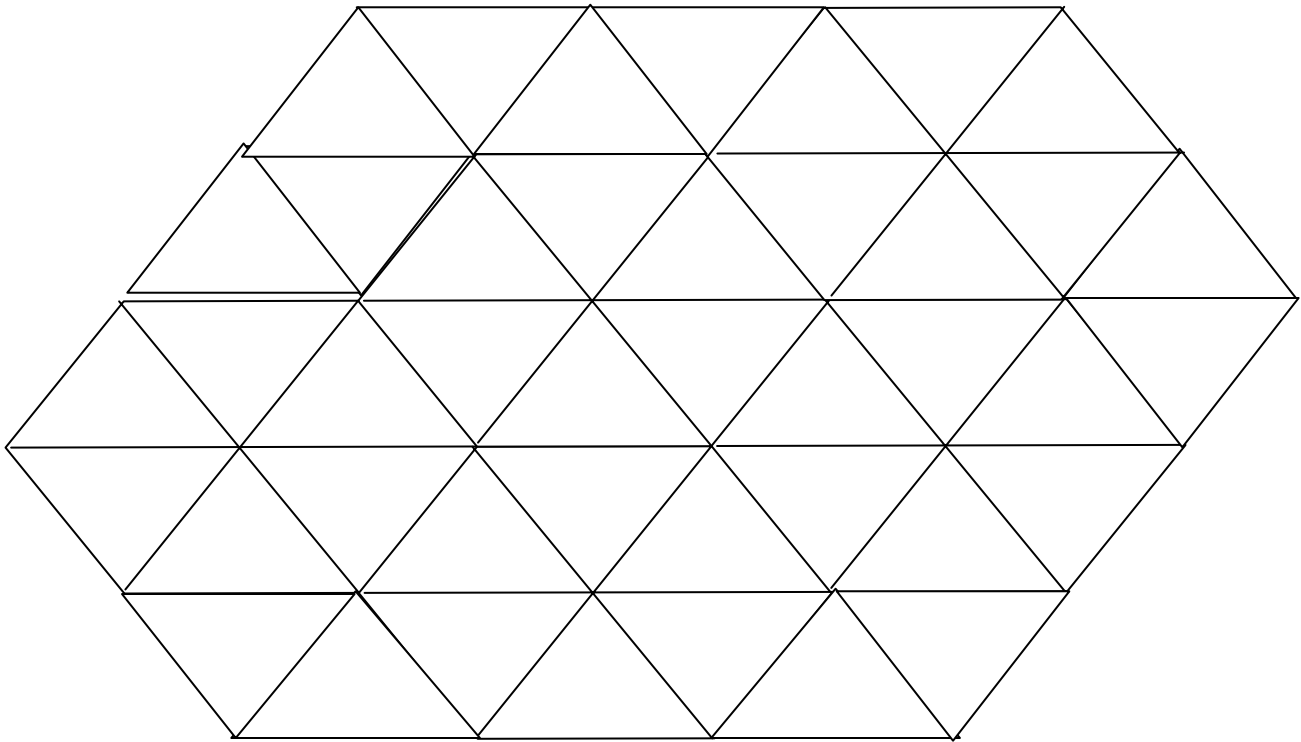
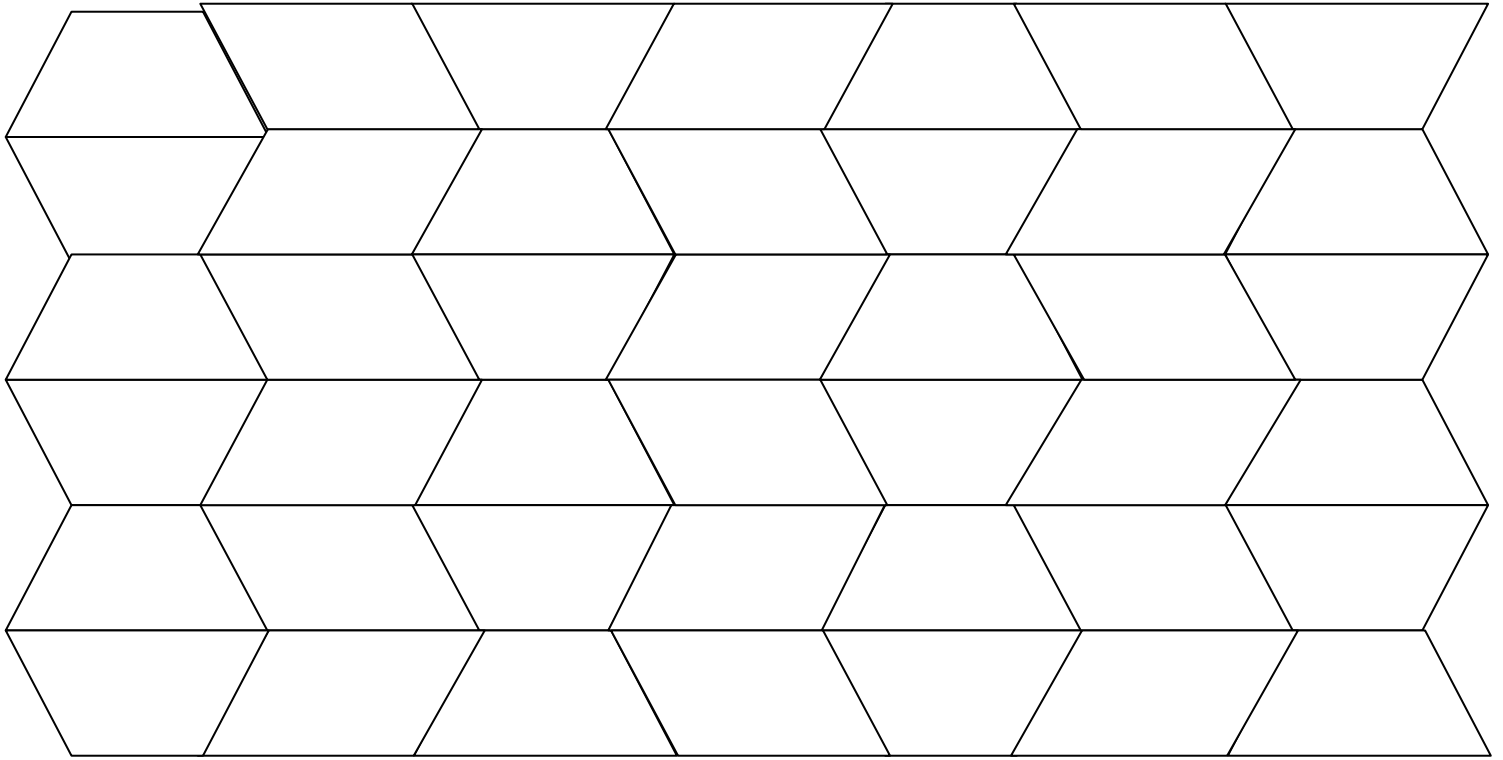
# Triangles



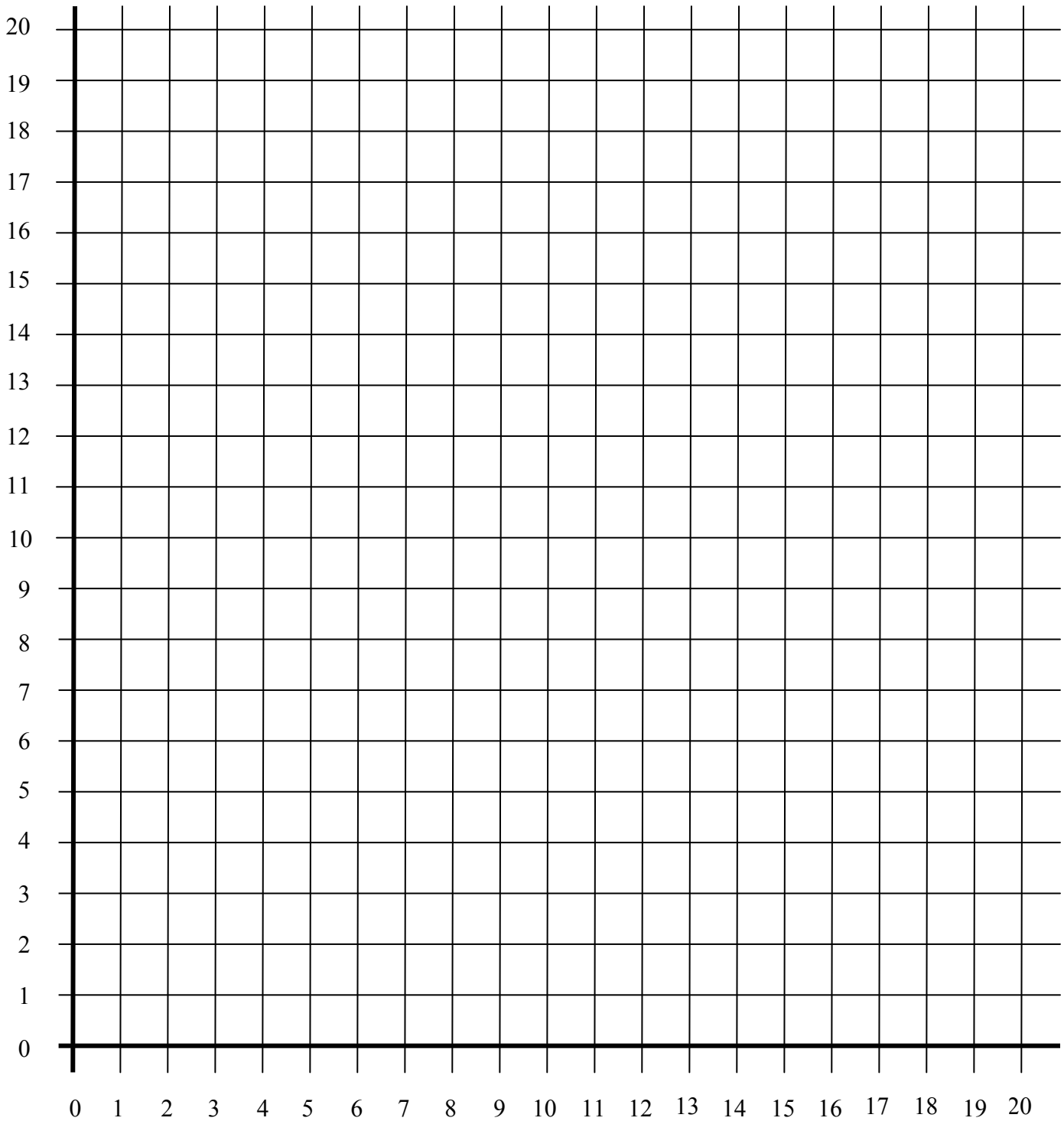
# Tessellation Pieces



# Tessellations



# Coordinate System



# Shapes in a Coordinate System

