

*Grades*  
3-5

# Number and Operation

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# Explorations on the Hundreds Chart

## Overview

### Mathematical Focus

- ▶ Number patterns
- ▶ Factors
- ▶ Multiples
- ▶ Primes and composites
- ▶ Arithmetic fact practice
- ▶ Relationships between operations

Students use the Hundreds Chart to explore number patterns and to look for patterns of multiples and factors. In playing Connect Four Multiples and the Factors Game, students practice multiplication and division facts while identifying patterns of factors and multiples. Students use their knowledge of multiples, factors, prime numbers, composite numbers, and patterns to solve mystery pattern puzzles.

### Preparation and Materials

**Before the session, gather the following materials:**

- ▶ Student Page 1: Hundreds Chart, several copies
- ▶ Dice
- ▶ Crayons or markers, 3 different colors (optional)
- ▶ Calculator (optional)

*Use of calculators:* Invite students to use calculators to explore factors and multiples and to check their answers.

# Activity

## Patterns on the Hundreds Chart

### 1. Make observations about the Hundreds Chart.

Students' observations may include the following:

- ♦ The smallest number is 0.
- ♦ The largest number is 99.
- ♦ All the numbers in the column under the 1 have a 1 in the ones place, all the numbers in the column under the 2 have a 2 in the twos place, . . . (etc.)
- ♦ Every other column has even numbers (or odd).

### Teaching Tip

Encourage students to use correct mathematical language, such as *ones place*, *tens place*, *ones digit*, *tens digit*, and *multiples*.

### 2. Look for number patterns on the Hundreds Chart.

Students' observations may include the following:

- ♦ In the first column the numbers are multiples of 10.
- ♦ The multiples of 2 make vertical stripes.
- ♦ The multiples of 5 are in two columns.
- ♦ The multiples of 11 make a diagonal line.
- ♦ The multiples of 9 make a diagonal line.
- ♦ The multiples of 3 make diagonal stripes.

### 3. Figure out what the mystery pattern is.

Choose a pattern from the list and keep it a secret. Circle all the numbers that fit your pattern. Challenge students to guess the rule. Encourage students to guess the pattern in as few guesses as possible. Take turns being the pattern guesser. You may also create your own mystery patterns and you may give students patterns to shade in on the Hundreds chart.

- ♦ Numbers with a ones digit of 5
- ♦ Numbers with the tens digit one more than the ones digit (e.g., 10, 21, 32)
- ♦ Multiples of 3
- ♦ Multiples of 11

- ♦ Numbers with 7 in the tens place
- ♦ Numbers with digits that add up to 10 (e.g., 91, 82, 73)
- ♦ Numbers with digits that add up to 9 (e.g., 81, 72, 63)
- ♦ Multiples of 10
- ♦ Odd numbers
- ♦ Multiples of 3 and 5 (e.g., 15, 30, 45)
- ♦ Multiples of 3 and 4 (e.g., 12, 24, 36)

## Multiples on the Hundreds Chart

### 1. Circle multiples of 3 and put squares around multiples of 5.

Give students another Hundreds Chart to mark multiples of 3 and 5. Ask: *Which numbers are multiples of both 3 and 5?* [Numbers with a circle and a square are multiples of 3 and 5 (e.g., 15, 30, 45, etc.).]

### 2. Find multiples of other pairs of numbers.

Have students find multiples of 3 and 4, 2 and 3, 4 and 5, 4 and 6, and 3 and 7.

### Connect Four Multiples

**Goal:** To cross off four numbers in a line.

**Players:** 2 or more

**Materials:** Hundreds Chart, two different-colored markers, die

**Instructions:** The first player rolls the die and crosses off a multiple of the number rolled. Then the second player takes a turn, crossing off a multiple of the number rolled, using a different-colored marker. The first player to cross off four numbers in a row (vertically, horizontally, or diagonally) wins.

## Factors on the Hundreds Chart

### 1. List the factors of several numbers.

Review the term *factor* with students. Have them list all factors for the following: 8, 9, 10, 12, 15, 16, 20, and 24.

### 2. Explore why certain numbers have an odd number of factors.

Ask students which numbers have an odd number of factors, and have them explain why those numbers have an odd number of factors.

## Teaching Tip

The square numbers (9, 16, 25, 36, etc.) have an odd number of factors because the factors can be arranged in pairs. For example, each factor of 12 can be matched with another factor of 12 such that the two factors multiplied equals 12: 1 and 12, 2 and 6, 3 and 4. But, if you pair up the factors of 16 in this way—1 and 16, 2 and 8—the factor 4 doesn't have a partner because 4 times itself is 16. This results in an odd number of factors.

### The Factor Game

**Goal:** To have the highest number of points

**Players:** 2 or more

**Materials:** Hundreds Chart, two different-colored markers or crayons

**Instructions:** Players alternate turns being the Number Chooser and the Factor Finder. When a player is the Number Chooser, he or she crosses off any number on the Hundreds Chart and adds that number to his or her current score. When a player is the Factor Finder, he or she crosses off any factors of the number that the Number Chooser chose, adding each factor to his or her score. Play continues, with a new Number Chooser on each turn, until all the numbers on the Hundreds Chart are crossed off, at which point the player with the highest total score wins. Once a number is crossed off it may not be used again (as the beginning number or as a factor).

#### Variations:

- ▶ Use a 0–36 Chart, then a 0–50 Chart, and finally a Hundreds Chart.
- ▶ For more than 2 players, the next player in line to be Number Chooser on each turn gets to be the Factor Finder for that turn.

### 3. Discuss strategies for playing the Factors Game.

Ask students:

- ♦ *What are the best numbers to choose to get a higher score?*
- ♦ *What numbers do you pick to keep other players' scores low?*
- ♦ *Would a calculator be useful while playing this game?*

## Teaching Tip

Students may notice that some numbers have no factors other than 1. These are called *prime numbers*. Prime numbers include 2, 3, 5, 7, 11, and 13. *Composite numbers* have more than two factors.

# Twenty Questions

## Overview

### Mathematical Focus

- ▶ Relative magnitude of numbers
- ▶ Order of numbers
- ▶ Names of numbers to 1,000,000

Students guess a mystery number by asking questions that can be answered with “yes” or “no.” The goal is to guess the mystery number in 20 questions or less. As students get better at playing the game, they try to guess the mystery number in 10 questions or less, then in the fewest number of questions. This activity helps students develop a sense of the relative magnitude and order of numbers. It also gives them practice naming numbers to one million. In playing the game, students develop basic logical thinking skills as they devise strategies for eliminating as many potential answers as possible.

### Preparation and Materials

**Before the session, gather the following materials:**

- ▶ Student Page 1: Hundreds Chart, several copies
- ▶ Student Page 2: 0–50 Number Line, several copies

# Activity

## Twenty Questions on a Number Line

### 1. Ask questions to determine what the mystery number on a number line is.

Select a mystery number between 0 and 50. Ask students to guess the mystery number by asking questions that can be answered with “yes” or “no.” Make a chart to record their questions and answers. As their questions are answered, ask students to cross out numbers on the 0-50 number line that can be eliminated as possible answers. Remember, the goal is to find the mystery number in 20 questions or less.



| Question                               | Answer | What #'s can we cross out?                             |
|--|--------|--|
| Is the mystery number smaller than 20? | No     | 0 - 19   |
| Is it greater than 40?                 | No     | 41 - 50  |
| Is it a multiple of 3?                 | Yes    | 20, 22, 23, 25, 26, 28, 29, 31, 32, 34, 35, 37, 38, 40 |
| Is it an even number?                  | Yes    | 21, 27, 33, 39   |
| Is it a multiple of 5?                 | Yes    | 24, 36   |
| Answer                                 | 30     |  |

### 2. Review your questions.

Once the mystery number has been found, give students an opportunity to describe the strategies they used when asking questions.

### 3. Play the game again, trying to guess the number using less questions (10 questions, or as few as possible).

### Teaching Tip

At first, students may guess individual numbers, e.g., “Is it 39?” “Is it 24?” etc. If students appear to be using a random guessing strategy, help them develop strategies that will give them better information with each guess. Look at specific questions they posed and ask these questions:

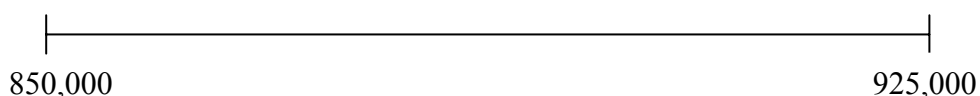
What kind of information did this question give you?

Which questions enabled you to eliminate the most numbers?

Students may begin to notice that questions that narrow the range of possible numbers will be the most useful. For example, if the answer to *Is it greater than 25?* is yes, the range narrows to 25–50, eliminating half the potential answers.

#### 4. Play Twenty Questions with bigger number ranges.

Increase the range of possible mystery numbers to 0–100; 0–1,000; or 850,000–925,000. As you increase the range of numbers you are playing with, it is no longer practical to draw out a detailed number line. Suggest that students label the two end numbers. After each question, they can change the end numbers to reflect the narrowed range.



## Twenty Questions on a Hundreds Chart

### 1. Play Twenty Questions without asking any “greater than” or “less than” questions, using the Hundreds Chart.

*Is it a multiple of 3?* is a legal question, but *Is it greater than 50?* is not a legal question. Record all questions and answers. Use Student Page 1, rather than a number line to keep track of possibilities that may be eliminated.

- ♦ Is the mystery number even? (Yes) *Cross out columns of odd numbers on the Hundreds Chart.*
- ♦ Is it a multiple of 10? (No) *Cross out multiples of 10.*
- ♦ Is it divisible by 3? (Yes) *Cross out remaining numbers that are not divisible by 3, or circle numbers that are divisible by 3.*

*Teaching Tip*

The type of questions used in this version of the game assumes that students have some facility with multiples. If this is not the case with your students, you may want to work with an activity that focuses on factors and multiples, such as Activity 5: Number Book.

### 2. Take turns picking numbers and being the guesser.

When you are the guesser, model questioning strategies for finding the mystery number. After students answer your questions, think aloud about how to use the information to eliminate possible answers.

### **3. Discuss strategies for playing Twenty Questions.**

Ask:

- ♦ Which kinds of questions helped eliminate the most possibilities?
- ♦ Why was it easy to cross out odd numbers?
- ♦ What multiples have a “straight up” pattern?
- ♦ What multiples have a diagonal pattern?
- ♦ Did any multiples get crossed out more than once? What multiples land on 10, 12, 36, etc.?

# Number Line Addition and Subtraction

# 3

## Overview

### Mathematical Focus

- Computation of whole numbers
- Properties of operations
- Number relationships

Students explore addition, subtraction, multiplication, and division on a number line, using the idea of jumping frogs. They investigate the inverse relationship between addition and subtraction, and they discover repeating patterns involving multiples.

### Preparation and Materials

**Before the session, gather the following materials:**

- Student Page 2: 0–50 Number Line, 2–3 copies

In this activity, the number line provides a concrete image for exploring computation; however, as students gain experience solving these problems, encourage them to rely less on physical jumps on the number line and more on arithmetic.

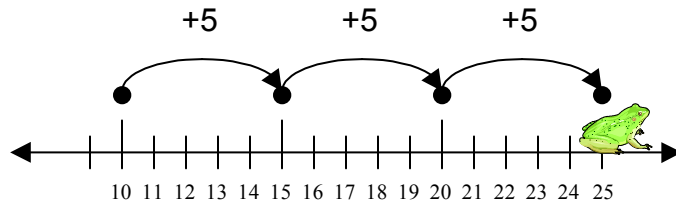
The problems presented in this activity do not involve negative numbers. However, the problem format can easily be used as a template for creating new problems with negative numbers.

# Activity

## Missing Starts and Stops

### 1. Learn about frog jumps on a number line.

Explain that a frog has the ability to jump forward (to the right) or backward (to the left) on a number line. Using Student Page 2 to demonstrate how a frog makes “plus 5” (+5) jumps. Start at 10 and show three jumps. The frog will land on 15, 20, and 25, consecutively.



Introduce a notation for presenting jumping frog problems. Write:

$$\underline{10} + \boxed{3} \times \textcircled{+5} = 25$$

Read: A frog starts at 10, makes three +5 jumps, and lands on 25. The start number for a jumping frog is given first (underlined), followed by the number of jumps, then the size of the jump, e.g., +5. The number the frog lands on is last.

### 2. Figure out where a frog started or landed given a number sentence.

Ask students to draw the jumps on the 0–50 Number Line for each problem. As students become comfortable solving the problems, challenge them to *predict* where the frog will land before drawing jumps on the number line.

$$\underline{13} + \boxed{3} \times \textcircled{+5} = ?$$

$$\underline{8} + \boxed{3} \times \textcircled{+5} = ?$$

$$\underline{11} + \boxed{3} \times \textcircled{+5} = ?$$

## Teaching Tip

Students may begin to notice that if the frog always makes three +5 jumps, the frog's total trip will be 15 no matter where it starts. Ask students how this knowledge could help them solve problems without drawing jumps on the number line.

Call students' attention to the number of jumps and the size of jumps in the two problems, below. These problems illustrate the property of *commutativity*.

$$\underline{20} + \boxed{3} \times \textcircled{+5} = ?$$

$$\underline{20} + \boxed{5} \times \textcircled{+3} = ?$$

Problems in which the missing information is the start number give students a chance to explore the inverse relationship between addition and subtraction.

$$\underline{6} + \boxed{2} \times \textcircled{+4} = ?$$

$$\underline{20} + \boxed{2} \times \textcircled{+4} = ?$$

$$\underline{?} + \boxed{2} \times \textcircled{+4} = 23$$

$$\underline{?} + \boxed{2} \times \textcircled{+4} = 18$$

Also explore with students “minus” jumps on a number line.

$$18 + \boxed{3} \times \textcircled{-3} = ?$$

start jumps size lands

$$20 + \boxed{2} \times \textcircled{+4} = ?$$

start jumps size lands

$$? + \boxed{3} \times \textcircled{-3} = 15$$

start jumps size lands

$$? + \boxed{3} \times \textcircled{-3} = 0$$

start jumps size lands

### 3. Solve more problems using any strategy you choose.

Make up additional problems for students to solve in which either the start number or the end number is missing. Invite students to draw their own number lines, if they wish.

## Teaching Tip

Observe the strategies students use for solving problems. *Do they use physical jumps on a detailed number line to find an answer? Do they draw a quick sketch of a partial number line? Do they use arithmetic? Do they use a “working backward” strategy for problems in which the start number is missing?* With each problem, encourage students to explain their thinking. Ask questions such as: *How does the type of frog affect the end number?*

### 4. Create your own problems.

Invite students to make up their own problems and solve them.

## Missing Jumps

### 1. Find the number of jumps or the size of the jumps that a frog made.

Sketch a number line. Put a 35 and a 50 on the left and right ends of the number line. Present the following problem:

$$\underline{35} + \boxed{?} \times \textcircled{+5} = 50$$

Read: A frog starts at 35. It makes a few +5 jumps and ends up at 50. Is that possible? If so, how many +5 jumps did the frog make?

$$\underline{35} + \boxed{8} \times \textcircled{?} = 91$$

Read: A frog starts at 35. It makes eight jumps and ends up at 91. Is that possible? If so, what size were the frog’s jumps?

Make up similar problems for students to solve. For each problem, give the number at which the frog starts, the number at which the frog ends, and *either* the number of jumps or the type of jump the frog makes. Challenge students to determine if the scenario is possible. If it is, ask them to supply the missing number. Decide whether fractional jumps are legal.

**2. Discuss your strategies for solving problems with missing jumps or jump sizes.**

Have students describe their strategies for solving these problems.

*Teaching Tip*

One strategy is to determine whether the difference between the start number and the end number is divisible by either the number of jumps or the size of the jump. If it is not, some would say the problem cannot be solved. On the other hand, some students may invent jump sizes that involve fractions, such as a +3 1/2 jump. Encourage creative solution strategies and give students an opportunity to check their answers with a calculator.

*Extension*

In the problems below, a frog starts at a given number and ends at a given number. Challenge students to determine different ways the frog could have jumped from the start number to the end number. Of course, one solution to all of the problems is to have the frog jump from the start number to the end number in a single bound! Ask the students if there are any problems for which this is the *only* solution?

Read: A frog starts at 15 and ends up at 39. Find one or more ways it could have gotten there.

$$\underline{15} + \boxed{?} \times \textcircled{?} = 39$$

$$\underline{85} + \boxed{?} \times \textcircled{?} = 58$$

$$\underline{31} + \boxed{?} \times \textcircled{?} = 42$$

$$\underline{100} + \boxed{?} \times \textcircled{?} = 40$$

$$\underline{53} + \boxed{?} \times \textcircled{?} = 66$$

# Windows



## Overview

### Mathematical Focus

- ▶ Computation of whole numbers
- ▶ Place value
- ▶ Regrouping
- ▶ The commutative property

Students put digits in window panes to make addition, subtraction, and multiplication problems. The goal can be to make the largest value, the smallest value, or a number close to a target value. Students apply and extend their knowledge of place value, regrouping, basic facts, addition, subtraction, and multiplication, and develop and explain computational strategies.

### Preparation and Materials

**Before the session, gather the following materials:**

- ▶ Student Page 3: 0–9 Number

Cut the 0-9 Number cards out ahead of time.

*Use of calculators:* Invite students to use calculators to check their answers and explore alternative arrangements of numbers. Calculators should also be used for multiplication and division computations involving numbers beyond two digits.

# Activity

## Windows Addition

### Window Numbers

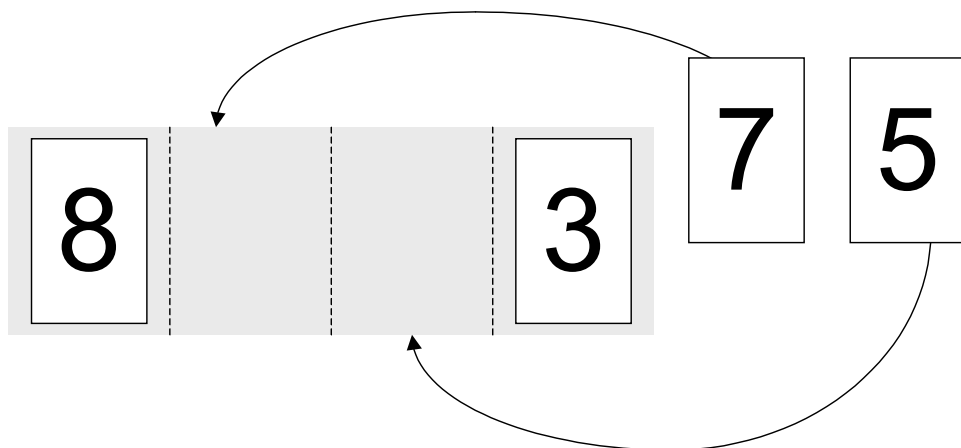
**Goal:** To create the largest number by placing digits in windows.

**Players:** 1 or more

**Materials:** Digit cards

**Instructions:** Begin by having each player draw a  $1 \times 4$  window. Shuffle the digit cards and place them face down. Turn over the first card.

Players write the digit in any pane of their  $1 \times 4$  windows. Once a digit has been written in a pane, it cannot be erased or moved. When everyone has written the first digit in a pane, turn over a second card and have the players write that digit in a pane. Continue until four cards have been turned over. Compare numbers to see who made the largest number. Ask students if the digits can be rearranged to make an even larger number.



**Variations:** Play again with the goal being to create the *smallest* four-digit number.

### Teaching Tip

Make sure your students understand the meaning of *digit*, i.e., the numbers 0 to 9. Examples: The number 34 has two digits—the digit 3 in the tens column and the digit 4 in the ones column. The number 34,216 has five digits (3, 4, 2, 1, and 6).

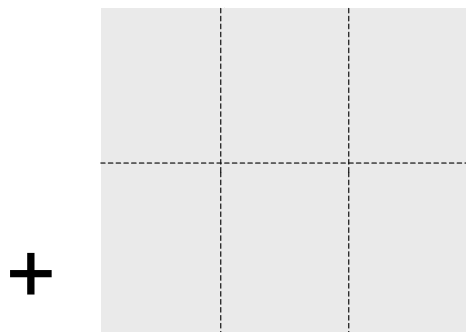
## Windows Addition

**Goal:** To create an addition problem that has the largest sum by placing digits in rows of windowpanes.

**Players:** 1 or more

**Materials:** Digit cards

**Instructions:** Each player draws this window:



As digit cards are turned over, each player writes each new digit in one of the panes of the window. Remember: Players must write each digit in a pane as soon as the card is turned over; they cannot erase or move it. When the windows are full, calculate the sums. Compare sums to see who made the largest sum. Try rearranging digits to come up with larger sums. Devise and discuss strategies for creating the largest sum.

*Teaching Tip*

Observe whether students use place value and regrouping as they play variations of the Windows game. Do they anticipate regrouping from the ones and tens columns to make a 10 or a 100? Do they understand why the hundreds column is the most important for getting close to a goal?

### Variations:

- ▶ If students have difficulty with the activity play the Windows Addition with all of the digits given at the beginning of the game. Players arrange the digits in their windows to meet the goal.
- ▶ The Windows games can also be played with smaller windows, such as a  $2 \times 2$  window rather than a  $2 \times 3$  window.
- ▶ Once students are comfortable making the largest sum, try changing the goal to making the smallest sum or to making a target number, such as 500 or 750.

## 1. Discuss what strategies you use when playing Windows Addition.

Ask questions like these:

- ♦ *When the goal is to make the largest sum, what digits do you want to place in the hundreds column? [large digits] What digits do you want to place in the ones column? [small digits] If a 7 came up as the first digit, where would you place it? Why?*
- ♦ *Is there more than one way to get the largest sum? [Yes, you can switch the top and bottom addends (commutative property). You can also move digits within a place value column.]*
- ♦ *If we know the largest sum, how can we quickly find the smallest sum? [Switch the digits in the hundreds column with those in the ones column]*
- ♦ *How are your strategies for making a target number different from your strategies for making the largest sum or smallest sum?*
- ♦ *What is the largest possible sum of two three-digit addends? [1,839] What is the smallest possible sum of two three-digit addends? [159]*

## Windows Subtraction

### 1. Discuss strategies for making the largest, smallest, or a target difference by filling in windows.

Ask questions like these about using windows to find a difference:

- ♦ *What do you need to do to make the largest difference?*
- ♦ *What do you need to do to make the smallest difference?*

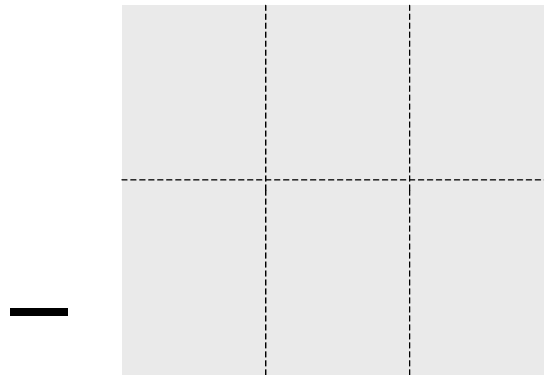
### Windows Subtraction

**Goal:** Create the largest, smallest, or a target difference between the numbers created by putting digits in the windowpanes.

**Players:** 1 or more

**Materials:** Digit cards

**Instructions:** All players should draw window panes looking like those shown below.



Draw cards from the digit card pile one at a time, and ask each player to write the digit in one of the panes. When all the panes are filled, players subtract the numbers and compare to see who has found the largest difference (or the smallest difference, or some target difference such as 20). After each game, ask if the digits could be rearranged to come closer to the target.

*Teaching  
Tip*

Students can get into negative numbers playing Windows Subtraction. If you wish to avoid negative numbers, let the goal be to get as close as possible to a positive target number.

## 2. Describe how Windows Subtraction differs from Windows Addition.

Students should see that in the subtraction game the result depends on which row you put your digits in, as well as their placement from left to right.

# Windows Multiplication and Division

## Windows Multiplication

**Goal:** To create the largest product (or the smallest product or a target product) by placing digits in the window panes of a multiplication problem.

**Players:** 1 or more

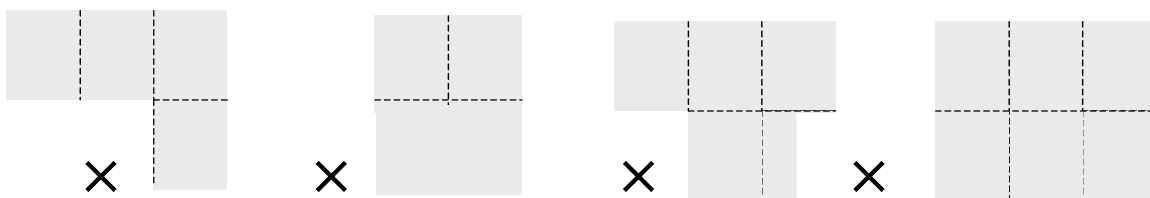
**Materials:** Digit cards

**Instructions:** Use windows and the operation of multiplication to create the largest number, smallest number, or a target number.

Ask students to think about the strategies they might use. Ask:

- ♦ *If the operation is multiplication, the frame is  $3 \times 1$ , and the goal is to make the largest number, what will you do?*
- ♦ *If the operation is multiplication, the frame is  $2 \times 2$ , and the goal is to make the smallest number, what will you need to do?*

Try using the different frames presented below.



## 2. Explain strategies for solving these problems.

Remember to share your strategies for thinking about the problems, as well as asking students to share their own.

### Windows Division

**Goal:** To create the largest number, the smallest number, or a target number by placing digits in windows to create a division problem.

**Players:** 1 or more

**Materials:** Digit cards

**Instructions:** Use windows and the operations of division to make the largest number, the smallest number, or a target number. Before playing each variation, ask students to think about what strategy they will use to reach the goal. Try using the different frames presented below.

# Number Book

## Overview

### Mathematical Focus

- Computation of whole numbers
- Number facts
- Factors and multiples
- Properties of operations (commutativity)

Students use arrays to investigate factors and multiples. In creating a Number Book for all of the single-digit multiplication facts, students develop meaning for multiplication as they build and describe arrays to show how a product is related to its factors.

### Preparation and Materials

**Before the session, gather these materials:**

- Graph paper, 20 sheets
- Student Page 4: Multiplication Table, 1 copy (for reference)

The thought of creating a Number Book for 81 number facts may seem daunting at first. Consider introducing the activity in one mentoring session and then letting students work on the book over the course of several sessions.

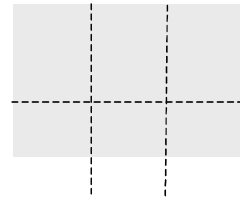
# Activity

1. **Shade different ways of representing the number six by making rectangles on graph paper.**

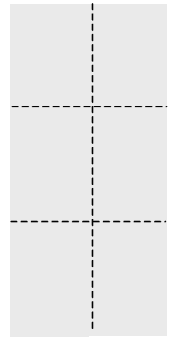
Each representation should be a rectangular array, like these:



$$1 \times 6 = 6$$



$$2 \times 3 = 6$$



$$3 \times 2 = 6$$

2. **Determine whether a  $2 \times 3$  array is the same as a  $3 \times 2$  array (or a  $1 \times 6$  as a  $6 \times 1$ ).**

Demonstrate how the  $2 \times 3$  array looks like the  $3 \times 2$  array and vice versa if the paper is rotated. This illustrates the property of *commutativity*:  $2 \times 3 = 3 \times 2$ . Explain that, for this activity, if an array can be rotated to look like another array, it will be considered the same. Only one picture is needed, but both number facts should be written.

3. **Create a Number Book.**

Challenge students to create a rectangular array for all of the single-digit multiplication facts ( $1 \times 1$ ,  $1 \times 2$ ,  $1 \times 3$ , . . . ;  $2 \times 1$ ,  $2 \times 2$ ,  $2 \times 3$ , . . . ; etc.). Assist students in organizing and labeling the number representations for their books. Students may want to use Student Page 4 to keep track of their work. Number facts can be checked off on the chart after a Number Book representation has been created.

4. **Make observations about your Number Book.**

When the book is complete, ask students to look back at the pictures and make observations. As students investigate the questions below, encourage them to use examples from their Number Books or draw new arrays to support their thinking.

- ♦ *Which rectangles have a side with two squares?* List them in order, starting with the smallest. *What do you notice?* [All of the rectangles that have a side with two squares are multiples of 2]
- ♦ *Which rectangles have a side with three squares?*
- ♦ *Which rectangles have a side with four squares?*
- ♦ *Which rectangles are also squares?* [These are called square numbers]
- ♦ Some square arrays can be divided evenly into groups of four. *Which ones?* [even squares] *What does that tell you about those numbers?* [They are multiples of 4, or they are divisible by 4]
- ♦ Sometimes multiplication is referred to as repeated addition; for example,  $3 \times 6$  can be thought of as  $6 + 6 + 6$ . Pick four different arrays and show how the array represents repeated addition.
- ♦ Look through your Number Book. *Which numbers have only one rectangle (or two rectangles if commutative pairs are considered, e.g.,  $1 \times 7$  and  $7 \times 1$ )?* These are called prime numbers. Numbers that have two or more rectangles are called composite numbers. “One” is a special number; it is neither prime nor composite.

## Extension

Have students explore arrays beyond single-digit multiplication facts. Use the suggestions below as starting points for different number explorations:

- ♦ Use rectangular arrays to represent the number 60. *How many different representations can you make?*
- ♦ *Can a multiple of 20 always be divided evenly into groups of four?* Explain your thinking. Give examples and draw arrays to support your answer.
- ♦ Whole numbers larger than 1 that are not prime numbers are called *composite numbers*. Each composite number can be written as the product of two or more primes. Pick four composite numbers and use arrays to show how each number can be written as the product of two or more primes.

# Using Fours

## Overview

### Mathematical Focus

- ▶ Computation
- ▶ Order of operations
- ▶ Arithmetic properties

Students try to make the numbers from 0 to 20 using only the digit 4, but using the least 4's possible in each expression. For example:

| Value | Expression             |
|-------|------------------------|
| 0     | $4 - 4$                |
| 1     | $4 \div 4$             |
| 2     | $(4 + 4) \div 4$       |
| 3     | $4 - (4 \div 4)$       |
| 4     | 4                      |
| 5     | $4 + (4 \div 4)$       |
| 6     | $[(4 + 4) \div 4] + 4$ |
| ...   |                        |
| 20    | $4 \times 4 + 4$       |

This activity gives students practice with addition, subtraction, multiplication, and division facts as they write and evaluate arithmetic expressions such as  $(44 - 4) \div 4 + 4 [= 14]$ . Students also use their knowledge of order of operations, properties of addition and multiplication, and the special cases of 0 and 1. Fewest Fours is a fun, puzzle-like challenge that can be tackled all at once or in several consecutive sessions.

### Preparation and Materials

- ▶ Student Page 5: Number Table
- ▶ Calculator

# Activity

1. **Create some numbers using only the digit 4, parentheses, and the following signs: +, −, ÷, and x.**

Ask: *How can we make the number 16 using only the digit 4?*

- ♦ One way to make 16 is  $4 + 4 + 4 + 4$ .
- ♦ Another way is  $(4 + 4)(4 + 4) \div 4$ . In this second solution, the use of parentheses is important. The calculations inside the parentheses are done first. So,  $4 + 4$  is 8.  $4 + 4$  is 8 again.  $8 \times 8$  is 64.  $64 \div 4$  is 16.

Ask students to make 0, 1, and 2. They should record their solutions on their Number Tables. Encourage students to review their solutions, looking for the solution with the fewest fours. [The expression with the fewest fours for 0 is  $4 - 4$  and for 1 is  $4 \div 4$ . One way to make 2 is  $(4 \div 4) + (4 \div 4)$ . A shorter way is  $(4 + 4) \div 4$ .]

## Teaching Tip

Before moving on, be sure students understand the use of parentheses to indicate the order that calculations are to be done.

Using the division fact  $4 \div 4 = 1$  is important for this activity. The 1 that results can be added to or subtracted from other numbers. For example:

One solution for 3 is  $4 - (4 \div 4)$ .

One solution for 5 is  $4 + (4 \div 4)$ .

One solution for 7 is  $(4 + 4) - (4 \div 4)$ .

One solution for 9 is  $(4 + 4) + (4 \div 4)$ .

## Extension

2. **Fill in the Number Table with solutions for each value from 0 to 20.**

Spend some time reviewing the solutions. Encourage students to find a shorter solution for each value, if possible.

- ♦ **Make 0 to 20 with Four 4s:** Students try to make all the numbers from 0 to 20, using exactly four 4s.
- ♦ **Make Larger Values:** Students use 4s to make numbers greater than 20, such as the whole numbers through 100.

# Recognizing and Comparing Fractions

## Overview

### Mathematical Focus

The use of fraction strips as concrete models for the following:

- ▶ Representing fractions
- ▶ Identifying fractions
- ▶ Comparing fraction sizes
- ▶ Identifying equivalent fractions

Students construct fraction strips and engage in a series of activities and games that give them practice (1) identifying and representing fractions as numerals and as fraction strip pictures, (2) determining whether two fractions are equivalent, and (3) comparing relative sizes of fractions.

### Preparation and Materials

- ▶ Student Page 6: Fraction Strip Chart (for reference only)
- ▶ Index cards, approximately 60
- ▶ Student Page 7: Fraction Strip Cards Template, 10 copies
- ▶ Tape or glue

Make a set of Fraction Numeral Cards: Write the following fractions on 28 index cards: twelfths ( $1/12$  to  $12/12$ ), sixths ( $1/6$  to  $6/6$ ), fourths ( $1/4$  to  $4/4$ ), thirds ( $1/3$  to  $3/3$ ), halves ( $1/2$  to  $2/2$ ), and 1. Students will make Fraction Strip Cards to match these Fraction Numeral Cards.

- ▶ These activities use all the Fraction Numeral Cards. For a simpler set of activities, use only the halves, thirds, sixths, and 1.
- ▶ Save Fraction Numeral/Strip Cards for use in Activities 8 and 9.

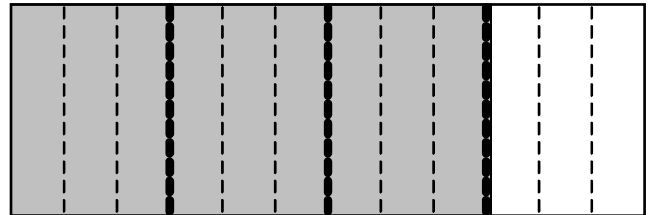
# Activity

## Identifying, Representing, and Comparing Fractions

1. **Select a few fraction cards and shade fraction strip cards to match them.**

Show students the set of Fraction Numeral Cards you prepared and give them the Fraction Strip Card Template. Tell students to first divide a strip on the template into the correct number of sections to represent the denominator of the fraction by drawing dark lines for the sixths, thirds, or halves. Next, they should color or shade the number of spaces that represent the numerator of the fraction.

$$\frac{3}{4}$$



2. **Shade a Fraction Strip to go with each of the Fraction Numeral Cards.**

The pictures can then be cut out and taped or glued to blank index cards to create a set of Fraction Strip Cards. Make sure that students do not write the fraction numeral on the Fraction Strip Card. When students have finished, there should be 28 Fraction Numeral Cards and 28 Fraction Strip Cards.

### Teaching Tip

As your students shade in the number of spaces representing the numerator of a fraction, watch to see how they are doing it. Ask students if they can color in the same fraction (say  $3/12$ ) in more than one way—i.e., any three shaded spaces on a strip divided into 12 sections represents  $3/12$ . Ask students to compare two fractions they have shaded, showing, for example, which is larger. Help them see that fraction strips are easiest to compare if students are consistent in the way they color in the numerator portions. Eventually you might want to establish some conventions, for example:

Shade fractions from left to right, so that it is easy to compare them. Color each type of fraction with a different-colored marker or crayon, but make sure that fractions with the same denominator are all colored with the same color.

- 3. Pick pairs of Fraction Numeral Cards and compare their sizes, using the Fraction Strip Cards as needed.**

As students pick cards, ask questions such as: *Is one bigger than the other, or do they represent the same value? How do you know?* If students cannot tell which fraction is larger, ask them to find the fraction strip that goes with each card and compare the strips to see whether the fractions have the same value or if one is greater.

- 4. Order a set of 5 or 6 Fraction Numeral Cards from smallest to largest.**

If two cards represent the same value, they can be placed next to the cards they are equivalent to. As a way of checking their arrangement of the cards, students can line up the matching Fraction Strip Cards underneath the corresponding numeral cards.

## Fraction Games

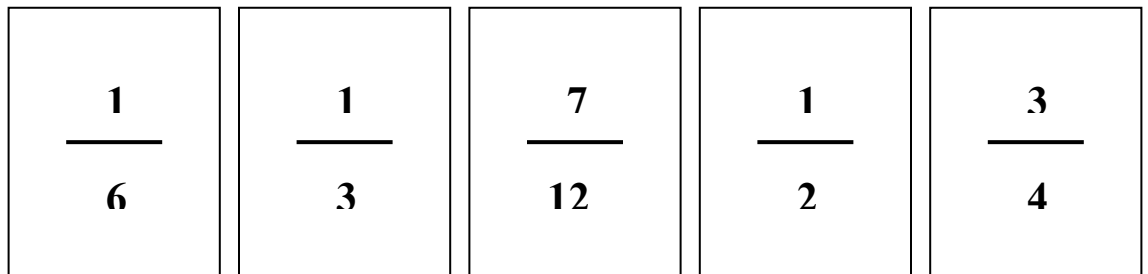
### Fracto!

**Goal:** To order five Fraction Numeral Cards from smallest to largest.

**Players:** 2 or more

**Materials:** Fraction Numeral Cards

**Instructions:** Shuffle the Fraction Numeral Cards, then deal five out to each player. When dealing the cards out, lay them out in a row, face-up, in front of each player. Players should leave the cards in whatever order they are dealt. Extra cards are placed in a facedown pile in the middle. Players alternate turns. On a turn, a player picks the top card from the stack and decides whether to discard it, or to replace one of his or her cards with it and discard the replaced card. Players should work towards having five cards that are ordered from lowest to highest (with the lowest fraction being on the left and the highest fraction on the right). When one player has five fractions in order, he or she calls “Fracto!” and wins.



**These cards are NOT in the proper order because 7/12 is larger than 1/2. So the player must replace one of these cards with a card that makes the sequence correct.**

### Variations:

- ▶ Play with Fraction Strip Cards instead of Fraction Numeral Cards for students who are struggling.
- ▶ Allow players to choose whether to take the top card in the face-up discard pile or the top card in the face-down draw pile, rather than making each player choose from the draw pile.

Teaching  
Tip

In this game—and any math game—you should model the strategies you would like the student to learn. If the student appears curious about how you decide what to do, you might ask if the student can think of a reason for a move you made.

The following strategies might apply to the column of cards shown above. If the player draws  $\frac{5}{12}$ , a card between  $\frac{1}{3}$  and  $\frac{1}{2}$ , he or she could use that card to replace the  $\frac{7}{12}$  and call “Fracto!” If the player draws a card between  $\frac{7}{12}$  and  $\frac{3}{4}$ , he or she could use that card to replace the  $\frac{1}{2}$ , and call “Fracto!”

Advanced strategy: If the player draws a card greater than  $\frac{3}{4}$ , that card should replace the  $\frac{3}{4}$  to allow a wider range next time (there are more fractions between  $\frac{7}{12}$  and a card greater than  $\frac{3}{4}$  than between  $\frac{7}{12}$  and  $\frac{3}{4}$  so it is more likely that a card that can replace the  $\frac{1}{2}$  will be drawn next turn if the  $\frac{3}{4}$  is replaced by a larger fraction). If the player draws a card between  $\frac{1}{3}$  and  $\frac{1}{6}$  ( $\frac{1}{4}$  or  $\frac{3}{12}$ ), that card should replace the  $\frac{1}{3}$  to allow a wider range of choices next time (there are more fractions between  $\frac{1}{4}$  and  $\frac{1}{2}$  than between  $\frac{1}{3}$  and  $\frac{1}{2}$  so there is a higher likelihood of finding a card to replace the  $\frac{7}{12}$  with next turn).

### Fraction Concentration

**Goal:** To gather the most pairs of equivalent fraction numeral cards.

**Players:** 2 or more

**Materials:** Fraction Numeral Cards

**Instructions:** Start with a shuffled set of Fraction Numeral Cards. Place all cards face down on a table (or the floor) in a rectangular array, in random order. The first player turns over two cards. If the cards show equivalent fractions, the player keeps both cards and takes another turn. If the cards do not match, the player turns them both face down, and the second player takes a turn. The game ends when there are no more cards left on the table. The player with the most cards wins.

### Variations:

- ▶ Play with Fraction Strip Cards instead of Fraction Numeral Cards for an easier version of the game (or at least have the Fraction Strip Cards available while playing for comparisons).

## Fraction Tic-Tac-Toe

**Goal:** To cross off 3 fractions in a row on the playing board.

**Players:** 2 players

**Materials:** Draw a  $3 \times 3$  playing board. Write one of the fraction numerals from the set of Fraction Numeral Cards in each square on the board. Start with a deck of Fraction Numeral Cards and different-colored markers for each player to use to cross off fractions.

**Instructions:** Players take turns drawing a card from the deck. If the card equals or is equivalent to one of the fractions on the board, the player covers that fraction with a marker. Players take turns until one player has three (or four) in a row or the board is full. If the board is full, the player with the largest number of markers wins.

**Variations:**

- ▶ Use a  $4 \times 4$  playing board instead of a  $3 \times 3$  playing board

## Extension

### Matching Fractions on a Number Line

*Materials*

- ▶ A number line divided into twelfths—marked but not labeled except for the ends, which are labeled 0 and 1; the spaces on the number line need to be long enough and wide enough so that index cards can be placed at each mark along the number line without overlapping
- ▶ Fraction Numeral Cards and Fraction Strip Cards

*To Play*

- ▶ Show students the number line. Ask them to find and mark the locations of  $\frac{1}{2}$  and  $\frac{2}{2}$  on the number line; then ask them to do the same with  $\frac{1}{3}$ ,  $\frac{2}{3}$ ,  $\frac{3}{3}$ ,  $\frac{1}{6}$ ,  $\frac{2}{6}$ , . . .  $\frac{6}{6}$ , and so forth until every tick mark on the number line has one or more numerals under it.
- ▶ Ask students to place each Fraction Numeral Card where it goes on the number line.
- ▶ Cover each Fraction Numeral Card with the corresponding Fraction Strip Card. Ask students if they can explain why the picture on the Fraction Strip Card represents the same fraction as the location on the number line between 0 and 1.

# Adding and Subtracting Fractions

## Overview

### Mathematical Focus

- Addition and subtraction of fractions

Students use fraction strips as a model to represent, understand, and practice adding and subtracting fractions.

### Preparation and Materials

**Before the session, gather the following materials:**

- Student Page 8: Fraction Segments (1), 1 copy
- Student Page 9: Fraction Segments (2), 1 copy
- Student Page 10: Fraction Segment Mat, several copies
- Fraction Numeral Cards, 1 set (see Preparation and Materials, Activity 7)
- Fraction Strip Cards, 1 set (see Preparation and Materials, Activity 7)

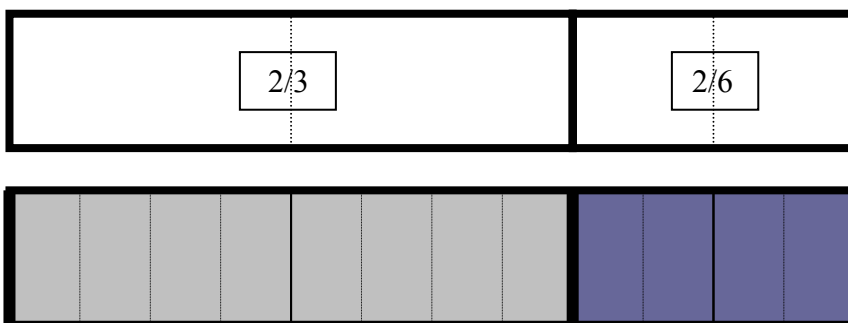
The following activities are written to use all the Fraction Numeral and Fraction Strip Cards. For a simpler set of activities, use a subset that includes only halves, thirds, sixths, and 1.

## Adding Fractions

1. **Explore ways to use fraction segments to fill one whole strip on the fraction segment mat.**

Ask: *How many pairs of fraction segments can you combine to make one whole (a full fraction strip on the fraction segment mat)?* Students should write down each sum as an equation equal to 1 (for example,  $\frac{2}{3} + \frac{2}{6} = 1$ ). Ask students to sketch their sums in two colors on the Fraction Strip Mat. Have plenty of copies of the Fraction Strip Mat available.

Ask: *Are there any more pairs you could use to add up to one whole?*  
 For advanced students, ask: *Can you combine three or more cards to make one whole?* [Example:  $\frac{1}{4} + \frac{3}{12} + \frac{1}{2} = 1$ ]



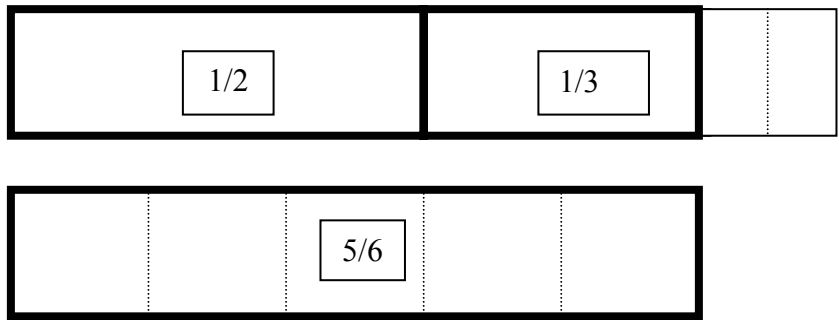
2. **Explore combinations of fractions that add to 1 by adding Fraction Numeral Cards.**

Students may use the fraction segments to help them if they are not sure if a particular pair of fractions adds up to 1.

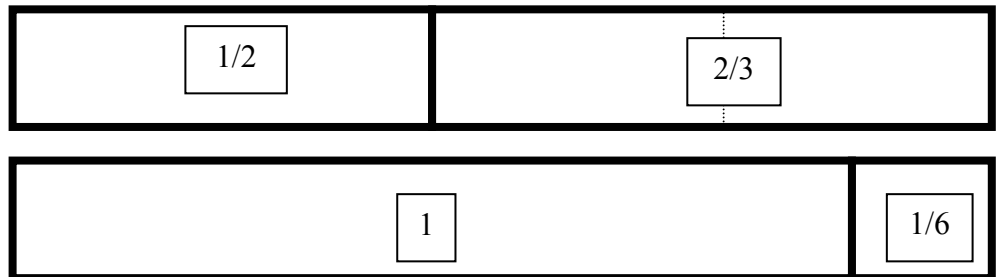
3. **Explore combinations of fraction segments that do not add to one**

Students pick any two fraction segments and “add” them by laying them next to each other on a Fraction Segment Mat. They find a third segment that is the same length as the combined strips.

For example, if students put  $\frac{1}{2}$  and  $\frac{1}{3}$  on the Fraction Segment Mat, they will need to find another Fraction Segment that is the same length as these two segments added together (such as  $\frac{5}{6}$  or  $\frac{10}{12}$ ).



If the total is more than 1, students will need to find a unit fraction strip and another fraction strip so that the total shaded part equals the combined shaded parts of the two original fraction strips.



*Teaching Tip*

This might be a good time to remind your students about *equivalent fractions*. Tell them that another way to add fractions is to find equivalent fractions that use the same denominator as both fractions to be added. Students should put the equivalent fractions next to the fractions being added, and count the total shaded portion. For the two examples above, both halves and thirds can be represented as sixths— $1/2 + 1/3 = 5/6$  is the same as  $3/6 + 2/6 = 5/6$ , and  $1/2 + 2/3 = 7/6$  is the same as  $3/6 + 4/6 = 7/6$ . This illustrates the method students are usually taught in school for adding unlike fractions using “common denominators.”

If students come up with an answer such as  $4/6$ , for which there is an equivalent fraction with smaller numbers in the numerator and denominator ( $2/3$ ), ask them to try to come up with that equivalent fraction. Explain that this is called *reducing to lowest terms*.

## Subtracting Fractions

1. Choose a fraction strip and determine what amount is left when that fraction is subtracted from 1.

[Hint: The strip you would add to the first strip to make 1 is the amount left.] Have students draw the result on a Fraction Strip Mat and write an equation to represent what they found.

**2. Choose any two fraction strips that are not equivalent, then find a third fraction strip that represents the difference between the first two.**

Have students draw the result on a Fraction Strip Mat and write an equation to represent what they found.

### **Fraction Differences**

**Goal:** To have the most cards when there are no cards left in deck.

**Players:** 2 or more

**Materials:** Fraction Strip Cards

**Instructions:** The first player takes two cards from the top of the deck. If the cards are equivalent, the player sets them aside. If one card is greater than the other, the player looks for a card in the deck that represents the difference between the two cards. If the player can find one, he or she keeps all three cards. If not, the player returns the cards to the deck, and it's the next player's turn. When there are no cards left, the player with the most cards wins.

**Variations:**

- ▶ Play with Fraction Numeral Cards instead of Fraction Strip Cards

### **Fraction Tic-Tac-Toe**

**Goal:** To mark three squares in a row on the playing board.

**Players:** 2

**Materials:** Fraction Numeral Cards, draw a 3 x 3 playing board with  $10/12$ ,  $3/4$ ,  $4/6$ ,  $2/3$ ,  $1/2$ ,  $4/12$ ,  $2/2$ ,  $5/6$ , and 1 written in the 9 boxes of the playing board, markers of two different colors

**Instructions:** The first player draws two Fraction Numeral Cards. If their sum equals one of the fractions on the board, the player covers that sum with a marker. Players take turns until one player has three in a row or the board is full. If the board is full, the player with the largest number of markers wins.

**Variations:**

- ▶ To play Subtraction Tic-Tac-Toe, write the following fractions in the squares of a 3 x 3 board:  $1/2$ ,  $1/3$ ,  $1/4$ ,  $1/6$ ,  $1/12$ ,  $2/12$ ,  $2/4$ ,  $2/6$ ,  $3/12$ . Players check if the difference between the cards they draw equals one of the fractions on the board rather than the sum. If the cards are equal, the player loses his or her turn.

## Adding Up

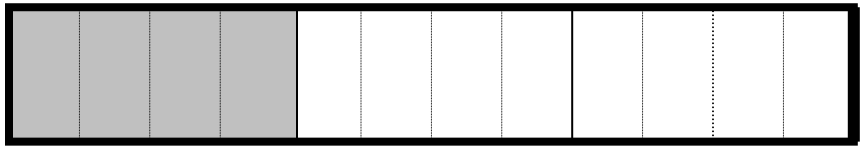
**Goal:** To reach a previously agreed-upon total by adding fractions

**Players:** 2 or more

**Materials:** Fraction Numeral Cards, several copies of the Fraction Segment Mat, and different-colored markers for each player

**Instructions:** Select a goal for a total greater than 2 to be reached exactly (3, 4, etc.). Spread out all Fraction Numeral Cards face up on the table, in a rectangular array. Players take turns choosing a card from the table, coloring the value of that card on the Fraction Segment Mat in such a way that the amount for each successive card is added to the previous total. The player must also call out the total. If the total is larger than one unit, continue by sketching in extra elements on the next fraction segment unit. Once a card is played, it may not be used again in that round. The player who places the Fraction Numeral Card that results in the goal being met wins the game.

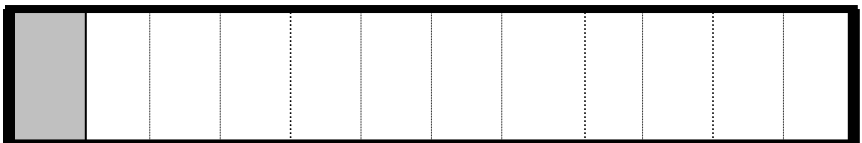
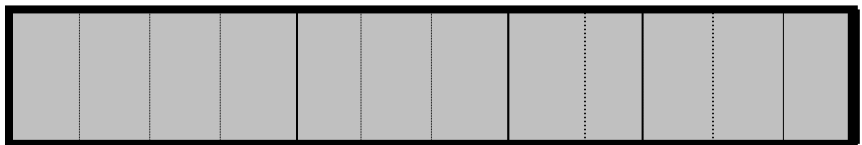
Play 1:  $\frac{1}{3}$



Play 2:  $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$



Play 3:  $\frac{1}{3} + \frac{1}{4} + \frac{3}{6} = 1\frac{1}{12}$



Each new value is added to the next (e.g., if Player One puts down  $\frac{1}{3}$  and shades in  $\frac{4}{12}$ , Player One writes  $\frac{1}{3}$  under the shaded portion. Player Two puts down  $\frac{1}{4}$ , shades  $\frac{3}{12}$  to reach the  $\frac{7}{12}$  mark, and writes  $\frac{7}{12}$  under the mark. Player One then adds  $\frac{3}{6}$  and shades in  $\frac{6}{12}$  to reach  $1\frac{1}{12}$ ).

**Variations:**

- ▶ Make the goal to avoid placing the last card. The player who is forced to place a card that reaches or exceeds the agreed-upon goal loses the game. If a player can make a legal move—one that reaches or is less than the agreed-upon goal—the player must make a play.
- ▶ Instead of displaying all the cards on the table, choose cards one at a time from the deck. This makes the game easier by depending more on chance and less on strategy.
- ▶ Play the game without using a Fraction Segment Mat. Players keep a running total, writing down the sum after each play.
- ▶ Play the game by subtracting. Start with an agreed-upon number of fraction strips laid out end to end. Trade fraction strips in order to remove the fraction played, always leaving a length of strips that correctly represents the amount left.

**Frac-Jac**

**Goal:** To have a set of cards that add up to an exact total of 2, or to have the highest total less than 2.

**Players:** 2 or more

**Materials:** Fraction Numeral Cards or Fraction Strip Cards, shuffled.

**Instructions:** Select one player to be dealer. The dealer deals two cards face down and one card face up to each player. Players look at their own cards and determine whether they want to be dealt an additional card or to pass (i.e., stick with the cards they have). Keep playing rounds until all players stop requesting cards. Then all players turn over their face-down cards. All players whose total is larger than 2 are out. All players who have an exact total of 2 win. If no player has an exact total of 2, the player(s) with the total closest to 2 win(s).

# Quick Ideas



## Overview

### Mathematical Focus

A collection of quick ideas for number exploration is presented below. Some ideas are problems, some are story starters, and some are games. Each number exploration can be used individually—for example, as a 10-minute warm-up at the beginning of a session—or several explorations can be used together as the activity for an entire session. The activities provide opportunities for students to explore different ways to make and represent numbers, use computational tools and strategies to solve problems, explore fractional concepts, and investigate large numbers.

# Activity

## Here's the Answer!

Write down a number. That number is “the answer.” Create several problems for which that number is the answer. Calculations can focus on addition, subtraction, multiplication, or division. Example: The answer is 15. Possible problems for which 15 is the answer:  $3 \times 5$ ;  $10 + 5$ ;  $30/2$ . Also create story problems with that number as the answer. For example: *A school bus holds 60 children. Four children can sit in each row of seats. How many rows of seats are there?*

## Arithmetic Stories

Create and illustrate a story for  $17 \times 3$  or for  $17 \div 3$ . Write additional problems focusing on addition, subtraction, multiplication, and/or division. Problems may also include fractions and decimals. Make up a story to go with each problem, and illustrate each story.

## Calculator Calculations

What is the greatest three-digit number you can multiply by 7 and get a product less than 3,000? Use a calculator to help you solve the problem. Record all of the numbers you try.

Fill in the blanks with different numbers: What is the greatest three-digit number you can multiply by \_\_\_\_\_ and get a product less than \_\_\_\_\_? Record all of the numbers you try. Explain your thinking and share any interesting discoveries you make.

## Decisions, Decisions

Which would you rather have: 5 cents for 30 days, or 10 cents for 20 days? Use a picture or diagram to represent the problem. Explain your thinking.

## Order Up!

At the local deli, you can order your sandwich on rye bread, white bread, or wheat bread. You can have a turkey, ham, salami, or veggie sandwich. How many different sandwiches can you order? List them.

Use the same problem given above, but now you can also have lettuce, tomato, and/or cheese on your sandwich. How many different sandwiches can you order?

### **Gauss**

Once there was a mathematics teacher who asked his students to add all the numbers from 0 to 100. He hoped to keep his students busy for a while with this activity. One bright young student named Gauss solved the problem in two minutes. How did he do it?

### **Cookies**

Would you rather have  $\frac{3}{8}$  of a double-chocolate chocolate-chip cookie, or  $\frac{1}{2}$ ? Draw a picture to show each portion.

### **Beware of One**

Use two number cubes for this game. Roll the number cubes and add the two numbers. The goal is to get to 100. Roll as many times as you want, but if a 1 comes up on one of the cubes, you lose your count for that turn. If a 1 comes up on both cubes, your total goes back to 0. And any time you roll a 1, you lose your turn.

### **Tic-Tac-Add to 15**

Draw a Tic-Tac-Toe board ( $3 \times 3$ ). The object is to get a row, column, or diagonal to add to 15. One player uses the numbers 1, 3, 5, 7, and 9, and the other player uses the numbers 2, 4, 6, and 8. Each number may be used only once. Players take turns writing numbers in the boxes. A player may block an opponent by putting a number in a square to make it impossible for that row to add to 15.

### **Largest Fraction Game**

This game, played with Fraction Numeral Cards, is similar to the card game “War.” All cards are split between 2 players. During each round, players place the top card from their piles face up on the table. If the cards are not equal, the player of the higher cards takes both cards. If the cards are equal, each player places four more cards on the table (3 face down and 1 face up). The player of the higher card wins all cards on the table. When a player has no cards to play, he or she uses the cards won so far to continue playing. A player wins when his or her opponent runs out of cards.

# 0–99 Chart



|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |
| 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |

# 0–50 Number Line



# 0–9 Number Cards

3

|   |   |   |   |   |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|

|   |   |   |   |   |
|---|---|---|---|---|
| 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|

# Multiplication Table



| x  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
|----|----|----|----|----|----|----|----|----|----|-----|
| 1  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 2  | 2  | 4  | 6  | 8  | 10 | 12 | 14 | 16 | 18 | 20  |
| 3  | 3  | 6  | 9  | 12 | 15 | 18 | 21 | 24 | 27 | 30  |
| 4  | 4  | 8  | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40  |
| 5  | 5  | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50  |
| 6  | 6  | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60  |
| 7  | 7  | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70  |
| 8  | 8  | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80  |
| 9  | 9  | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90  |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

# Number Table



| Value | Expression |
|-------|------------|
| 0     |            |
| 1     |            |
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| 3     |            |
| 4     |            |
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| 13    |            |
| 14    |            |
| 15    |            |
| 16    |            |
| 17    |            |
| 18    |            |
| 19    |            |
| 20    |            |

# Fraction Strip Chart



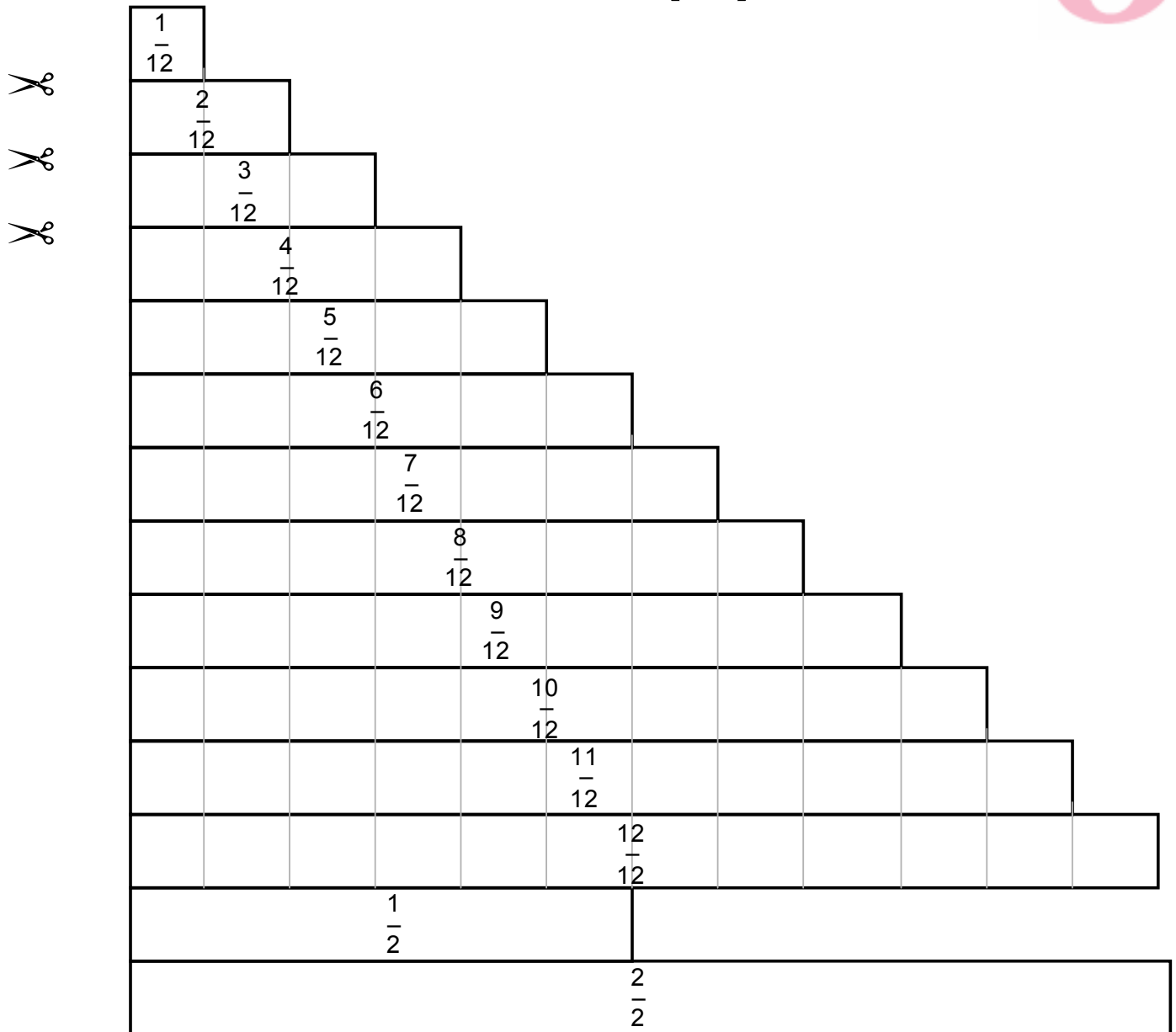
|                |                |                |                |                |                |                |                |                |                |                |                |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $1$            |                |                |                |                |                |                |                |                |                |                |                |
| $\frac{1}{2}$  |                |                |                |                |                | $\frac{1}{2}$  |                |                |                |                |                |
| $\frac{1}{3}$  |                |                |                | $\frac{1}{3}$  |                |                |                | $\frac{1}{3}$  |                |                |                |
| $\frac{1}{4}$  |                |                | $\frac{1}{4}$  |                |                | $\frac{1}{4}$  |                |                | $\frac{1}{4}$  |                |                |
| $\frac{1}{6}$  |                | $\frac{1}{6}$  |                | $\frac{1}{6}$  |                | $\frac{1}{6}$  |                | $\frac{1}{6}$  |                | $\frac{1}{6}$  |                |
| $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ |

# Fraction Strip Cards Template

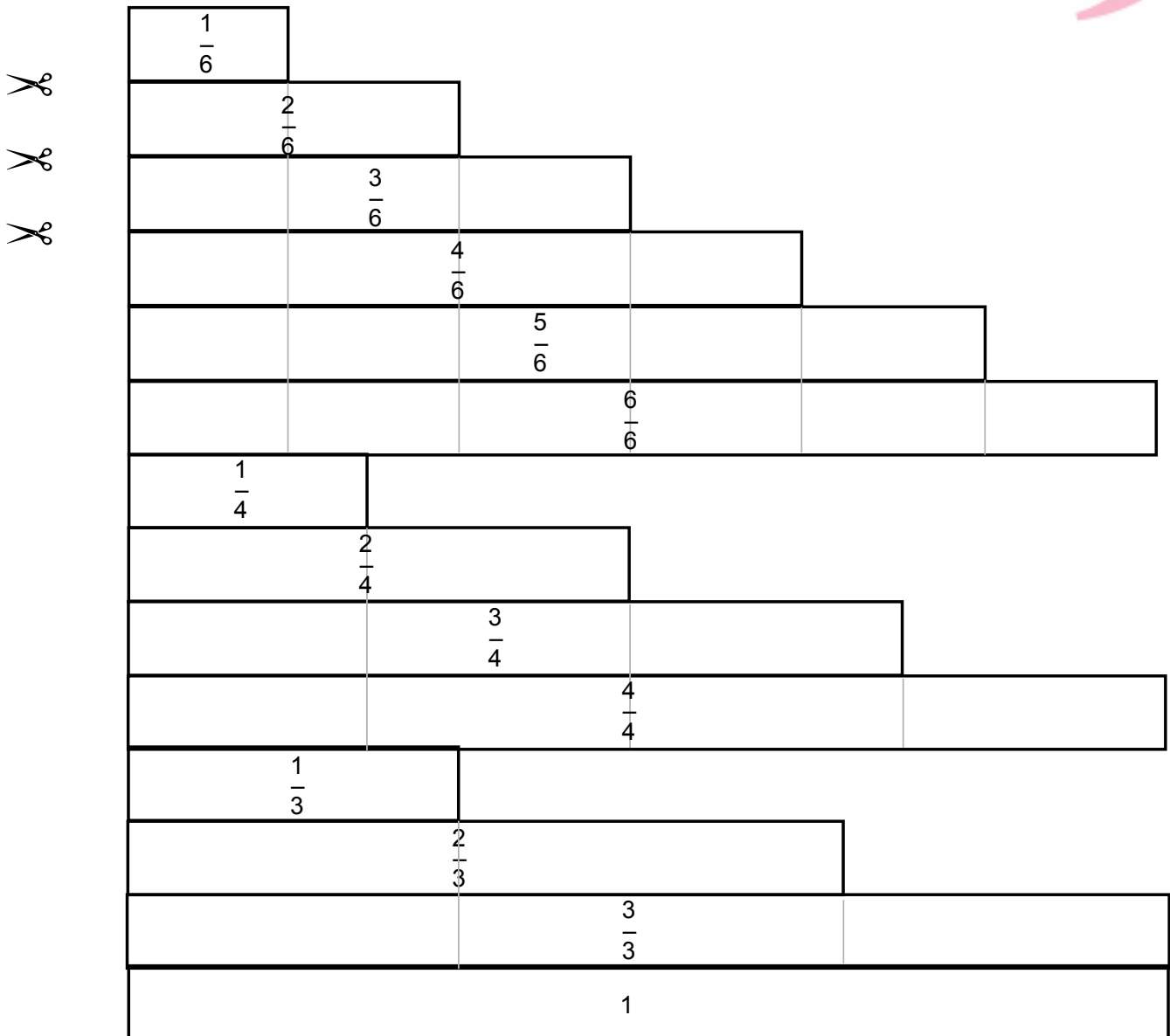
7

The image shows a template for fraction strip cards. It consists of four rectangular boxes arranged in a 2x2 grid. Each box contains a vertical strip with ten horizontal dashed lines, intended for writing fractions.

# Fraction Strips (1)



# Fraction Strips (2)



# Fraction Strip Mat

10

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