Grades 3–5

Patterns and Functions

TABLE OF CONTENTS

Patterns and Rules

1

Overview

Mathematical Focus

- Recognize, describe, and extend growing and shrinking patterns
- Use visual and numeric patterns to make and verify predictions
- Use rules to describe visual and numeric patterns

Students explore and build growing and shrinking patterns with pattern blocks. They make data tables to record the number of blocks in term of the pattern. As students analyze the emerging pattern, they are challenged to describe a *rule* for adding blocks. As students describe and then numerically record the picture patterns they create, they begin to bridge from the pictorial level to the symbolic level. In the second part of the activity, students explore and extend the patterns they worked with in Part 1. Using the rule from each pattern and the constant feature of a calculator, students predict and generate additional terms in the sequence.

Preparation and Materials

Before the session, gather the following materials:

- Student Page 1
- Student Page 2
- Student Page 3

Cut out 4-5 sets of shape pieces from Student Page 1 ahead of time. Shapes made out of heavy paper will be easier for students to manipulate.

- Commercially available Pattern Blocks may be used for this activity instead of the shape pieces on Student Page 1. If pattern blocks are available, use them.
- For additional work with growing and shrinking patterns, see Activity 4 in the Number and Operations, Grades K-2 unit.

Activity

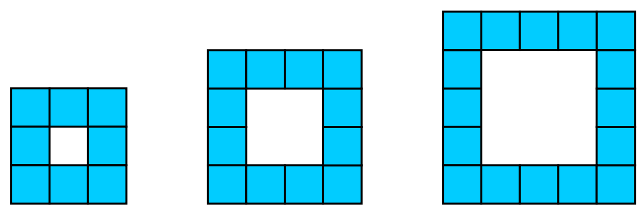
Part 1: Growing and Shrinking Patterns

1. **Explore a growing pattern made from squares.**

   Begin by using pattern blocks, paper manipulatives or graph paper to build or sketch the pattern pictured below. Present the following story: *A frame-maker builds different size square frames. The smallest frame he makes is a Size 1 frame, the next size up is a Size 2 frame, etc.*

   Do students see a pattern in the way the frames increase in size? Ask: *How many squares are added to any given size frame to make the next size frame?* [4] *Can you predict what a Size 4 frame will look like? How many squares will be used?*

   Pattern 1



   Size 1 Size 2 Size 3

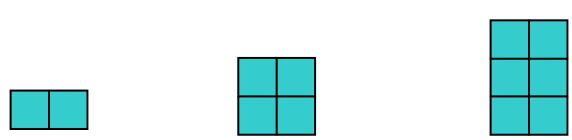
   Explain that this is a growing pattern. A growing pattern is a sequence of shapes or numbers that increases in a regular way. While the frame-maker calls the first frame a *Size 1 frame*, the second a *Size 2 frame*, etc., we call the first frame in the pattern *Term 1*, the second frame *Term 2*, and so forth.

   There is a rule for adding squares to a frame. Ask students if they know the rule. Explain that the number of blocks added to each term of the pattern is the *rule*. The rule for this pattern is +4. Have students use the rule to build the 4^{th} and 5^{th} terms of the pattern.

2. **Explore a second growing pattern made from squares.**

   Using pattern blocks, build and explore. Use shape pieces or pattern blocks to explore a different pattern.

   Pattern 2

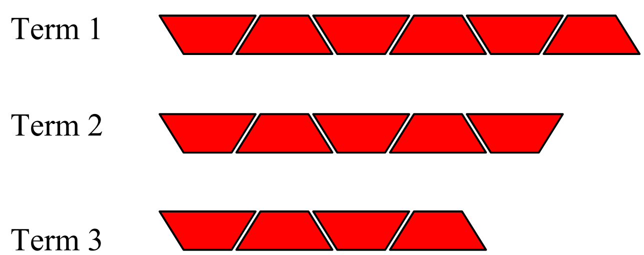


Ask students to describe Pattern 2. Ask: *How does the pattern change from one term to the next? Predict the fourth term.*

Pattern 2 is also a growing pattern. The rule for this pattern is +2. Have students build the fourth term.

3. Investigate shrinking patterns.

Pattern 3



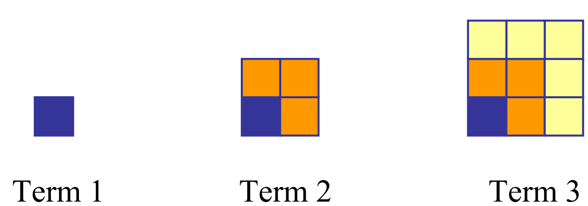
Ask students to describe Pattern 3. Ask: *How is this pattern different from the others? Predict the fourth term. What is the rule for this pattern?* [-1]

Patterns that decrease in a regular way can be called *shrinking* patterns.

4. Identify "same-step" patterns and "changing step" patterns.

Explain that the growing and shrinking patterns that have been explored so far are SAME-STEP patterns. The patterns grow or shrink by the same amount. Not all patterns change by the same amount from term to term. Have students consider Pattern 4, below:

Pattern 4



Ask: *Is this a growing or shrinking pattern?* [growing] *How does it grow? How many blocks are added to the first term to get the second, the second to get the third, etc.*? *Does the pattern grow by the same amount each time?* [no] *How many blocks need to be added to the third term to get the fourth?*

Explain that this is an example of a CHANGING-STEP pattern–a pattern that grows by step amounts which increase or decrease in a regular way.

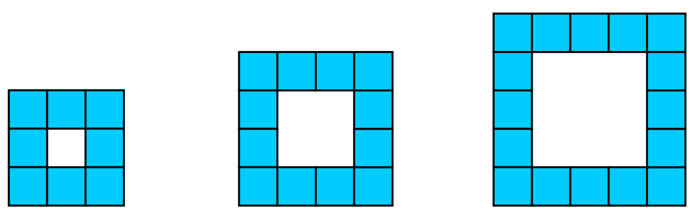
5. Build patterns and describe how each pattern is generated.

Give students an opportunity to build different growing and shrinking patterns. Have students determine if the pattern is a growing or shrinking pattern; describe how the pattern is growing or shrinking; and give the rule.

Part 2: Describing visual patterns numerically

1. Use a data table to record information about the pattern.

Return to Pattern 1 from Part 1. Give students a copy of Student Page 2. Let students have an opportunity to sketch a term of the pattern. Explain that the data table provides a way to record information about patterns. Have students use the table to record information. Record the information in the table.



Term 1 Term 2 Term 3

Term	Total
1	8
2	12
3	16
4	
5	

2. **Analyze patterns in the table and use them to make predictions.**

   Ask: *Do you need to have enough blocks to determine each term, or is there another way? How can you determine the tenth term without blocks?*

   Have students determine how to answer these questions. Point out that there is an increase with each term. Students may recognize that if you add 1 to any term number and multiply that by 4, you get the total number of squares for that term.

3. **Use the constant feature on a calculator to investigate growing and shrinking patterns.**

   Use the calculator to explore growing and shrinking patterns by repeatedly adding or subtracting a *constant*. Press the + key, then 4 (the amount the pattern grows by), then the = key, then enter 8 as the starting number (the total number of blocks in the first term). Press the = key once, which adds 4 to the starting number. The total is 12, which agrees with the second term total in the data table. Continue to press the = key. Record each new term in the table. Ask students to predict what each new term will be before they press the = key.

4. **Represent and analyze patterns.**

   Give students a copy of Student Page 3 and several copies of Student Page 2. Students are asked to represent and describe each pattern on Student Page 3. This can be done either verbally or in writing. Students determine a rule for the pattern and use the rule to build the next term of the pattern. Students create a Pattern Data Table and use the constant function on the calculator to complete the table.

5. **Use patterns to make predictions.**

   Have students predict the 12^{th} term for each of the patterns and check their predictions with the calculator.

Extension

Number Pattern Challenges

Ask students to solve these calculator challenges. Challenges involve addition, subtraction, multiplication, and division.

- Build growing or shrinking patterns that have the number 20 as the fourth term.
- Start at 100. Build as many shrinking patterns as possible that end exactly at zero.

- Start at any number and choose a constant. Predict the first four outputs and check your predictions using the calculator.

Function Machines

2

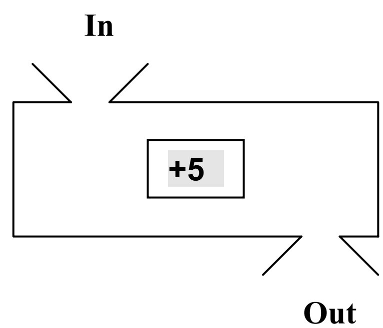
Overview

Mathematical Focus

- Represent patterns in tables
- Use input/output rules to describe functions

Function machines are a way to investigate and describe relationships between quantities that vary in a predictable way. A function machine is a device with a slot in which an input number can be placed, a rule for that input number, and a slot for the resulting output. Below is a [+5] machine, which adds five to any input and outputs the result. Students are the number machine operators. Put a 2 into the machine, the student adds 5 to the input and shows the output of 7. Put a 3 into the machine, the student adds five to produce an output of 8. The series of inputs and outputs is recorded in an input/output table.

Rule = +5	
Input	Output
2	7
3	8
4	9
5	10



Preparation and Materials

Before the session, gather the following materials:

- Student Page 4
- Counters

For younger students, make a 3-D function machine out of a cardboard box. Label the “IN” slot and the “OUT” slot. Make sure objects can be easily manipulated while they are in the box, adding to or taking away from them as indicated by the rule. Have students be the machine operator so that when, for example, a 2 comes in the machine, they add 5 and send out 7.

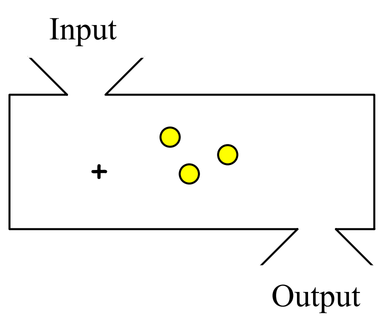
Activity

Part 1: Introduce Function Machines

1. Learn about function machines.

Explain that a function machine, or number rule machine as they are sometimes called, has an input slot and an output slot. When you put something in the input slot, the function machine uses a *rule* to produce a new number, which comes out the output slot.

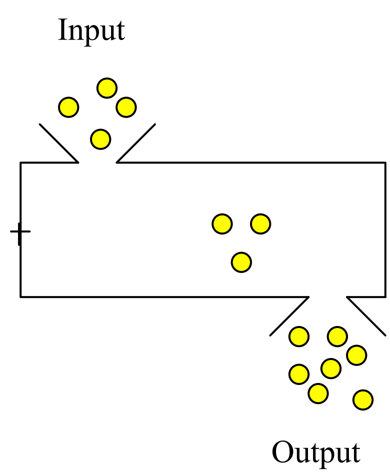
Draw a +3 function machine like the one below. Explain that if you put something into the input slot of this machine, the machine will add three to it (the rule) and the result will come out the output slot.



A "plus three" function machine.

2. Use a +3 machine to explore several examples.

Put four counters above the "in" slot. Ask: *What will happen if I put these four counters in the function machine?* Give students an opportunity to show with counters what will happen. Answer: The machine will add three more to the four and seven counters will come out.



Try a few more examples. Have students manipulate the counters to show inputs and outputs.

- *Put two counters in. How many come out?* [5]
- *Put 5 counters in. How many come out?* [8]
- *Put 10 counters in. How many come out?* [13]

Change the question:

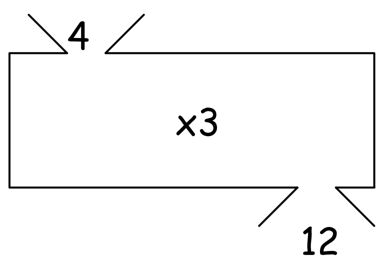
- *If 9 counters come out, how many went in? [6] How do you know?*

3. Record inputs and outputs for a +3 machine in an input/output table.

Have students track function machine data by creating an input/output table. Give students a copy of Student Page 4. Have students record some inputs and outputs for the +3 function machine.

4. Investigate a x3 machine and record inputs and outputs in an input/output table.

Make a [x 3] machine. Ask: *What will happen if you put 4 in a x3 machine?* [12 will come out]. *What will happen if you put a 10 in the machine?* Explain that the [x 3] machine makes a total of 3 copies of the input. Have students determine the outputs for several different inputs and record this information in an input/output table for the [x 3] machine.



5. Determine what happens to the output when fractions or decimals are used as inputs.

For older students who are comfortable working with fractions and decimals, try using fraction and decimal inputs. Have students predict what the output will be before solving the problem. Students may want to check their answers using a calculator.

Note: Using function machines may be a good way for students to get computational practice with fractions and decimals.

6. **Explore a variety of function rules.**

   Have students explore some of the function rules below. Have extra copies of Student Page 4 available for students to record input/output data.

   +5

   +8

   x4

   ÷2

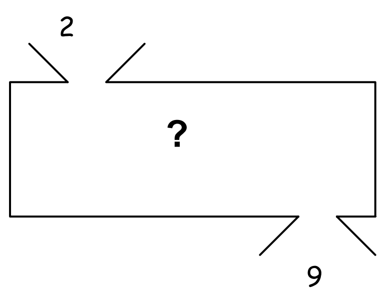
7. **Draw diagrams of function machines and record inputs and outputs.**

   Give students an opportunity to think creatively about different kinds of function machines and how they would work. Remember: a function machine takes an input, uses a rule, and produces an output. Have students draw pictures of their machines and record several inputs and outputs using either pictures or input/output tables.

Part 2: Mystery Machines

1. **Learn about Mystery Machines.**

   Students give the mystery function machine an input, the machine uses a rule (which is a mystery) and produces an output. Begin with a [+ 7] mystery machine. Remember: Do not tell students the rule. Have students give you inputs. For each input, add 7 to their input and give the output. Record each input and output in an input/output table. Continue until students are able to solve the mystery by determining the pattern in the input/output table and stating the mystery rule.



Mystery rule: +7

Rule: mystery	
Input	**Output**
2	9
5	12

2. **Solve a second mystery machine.**

   Try a mystery machine with a rule of [– 3]. Have students give inputs and you record outputs. Record each input and output in an input/output table. Continue until students are able to solve the mystery.

3. **Using data from an input/output table, determine the mystery rule.**

   On a piece of paper, or a copy of Student Page 4, write the input/output table shown below. Give students time to figure out the function rule using the table. Encourage students to explain their strategy for determining the rule. Once students have suggested a rule, ask: *Does your rule work for all of the input/output pairs in the table? Check it.*

Rule: mystery	
Input	**Output**
20	10
14	7
24	12
6	3

[The rule for this mystery machine is: ÷2]

4. **Try a few more examples.**

   Have students use the data in the tables to determine the mystery rules. Encourage students to tell how they solved each mystery. Ask questions such as: *What strategies do you use to solve the mystery? How can you be sure you have the rule?*

Rule: mystery	
Input	**Output**
3	21

[The rule for this mystery machine is: x7]

Rule: mystery	
Input	**Output**
2	1/2
100	25
60	15

28	7

[The rule for this mystery machine is: ÷4]

5. Create your own mystery machines.

When students can solve mystery machines let them create their own mystery machines: *Think of a rule, but keep it a secret. Using the rule, create an input/output table. Give the input/output table to another person to solve.*

Teaching Tip

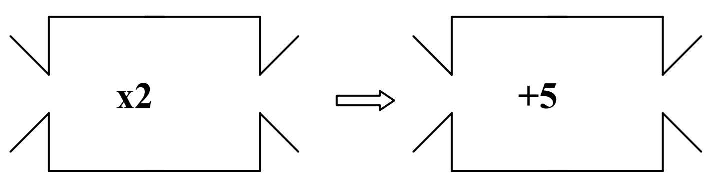
Have students determine which numbers are helpful in solving mystery machines. For example, students may say that using 0 as an input number helps determine whether the machine is an addition machine or a multiplication machine (also, subtraction or division). After students guess the rule for a machine, challenge them to test their rule by giving them outputs and seeing if they can figure out the inputs.

For students who need computation practice with fractions and decimals, encourage them to use fractions and decimals as inputs.

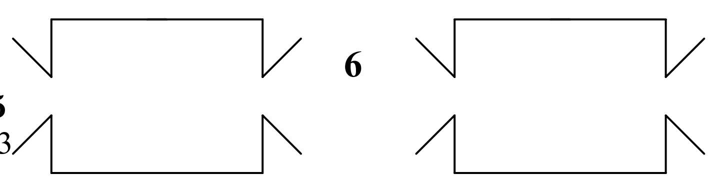
Part 3: Composite Machines

1. Introduce composite machines.

Draw the composite function machine pictured below. Ask students: *What do you think will happen if I attach a x2 machine and a +5 machine together? Try using 3 as an input.*



Students may say that the output of the x2 machine (in this case, 6) is the input for the second machine, resulting in a final output of 11. Explain that attaching two machines in this way makes what is sometimes called a composite function machine. A composite function machine uses the output of one machine as the input for the next machine.



3 x2 ⇒ +5 11

(3 x 2) (6 + 5)

Work through a few more examples. For older students try fractional and/or decimal inputs. Have students use a function record sheet to record input/output data.

Possible inputs: 5, 8, ½, 11, .1, ¼, 100

Rule: x2, +5	
Input	**Output**
3	11
5	15
8	21
1/2	6

2. **Look for patterns in the data.**

   Ask students to examine the input/output data in their table and look for patterns. Ask questions such as: *What happens when you use an even number as an input? An odd number? Do you think this will be true for all even and odd inputs? Explain your thinking.*

3. **Work backwards, using outputs to find inputs.**

   Using the same function record sheet, write 31 in the output column. Ask: *If the second output for the machine we have been working with (x2, +5) is 31, what would the input be?* [13] *Explain how you found the answer.*

   By running the machines backwards, students can determine that 26 must have gone into the second machine (31-5 = 26), and therefore must be the output of the first machine. Running the first machine backwards means dividing by 2 rather than multiplying by 2: 26 ÷ 2 = 13. Give a few more outputs and challenge students to find the corresponding inputs.

Teaching Tip

Working backwards may be difficult for some students. Another strategy is to test some numbers and try to get closer to the target output. For example, using the machine above (x2, +5) and the target output of 31, students might try different possible input numbers:

If the input is 5, the output would be 15. That's too small. If the input is 10, the output would be 25. That's too small too. If the input is 20, the output would be 45. That's too big. The input must be between

10 and 20. Students can continue in this way to narrow the range of possible inputs.

4. **Make your own composite machines.**

Give students an opportunity to create and explore their own composite machines. For each machine, have students take several different inputs and determine the outputs. Also have students take several outputs and work backwards to determine the inputs. Here are a few composite machines students can try:

[+ 2, x 4]

[x 10, - 10]

[x 4, x 1/2]

Additional challenges:

Can you make a machine that will always give you an even output?

Can you make a machine that will always give you an output that is divisible by 5?

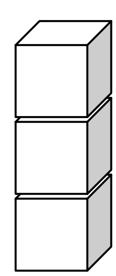
Teaching Tip

Encourage students to look for patterns in their input/output data. Systematic selection and recording of input numbers (e.g. 1, 2, 3, …) may make patterns in the data easier to see.

Extension

Surface Area of Cubes

Stack 3 cubes on top of each other. Ask: *What is the surface area of this tower of cubes if each side has a surface area of 1 square unit?* [14] *As more cubes are added to the stack, how does the surface area change?*



Ask students to use additional cubes to explore the question. Have them make a table to show the number of cubes (height of cube stack) and the surface area of different-sized stacks. Ask students how this can be thought of in terms of a mystery machine. Using data from their table, have students predict the surface area of a tower with 50 cubes. Students may realize that there are four faces on each cube, so you need to include 4 times the number of cubes, and then add on the top and bottom faces. So, surface area of these towers equals the number of cubes times four, plus two.

Number of cubes	Surface area (square units)
1	6
2	10
3	14
4	18
5	22

Patterns on a Lattice 3

Overview

Mathematical Focus

- Recognize, describe, and extend a variety of patterns on a number lattice
- Analyze the correspondence between the visual patterns formed by shaded numbers on a lattice and the counting pattern.

A number lattice is an ordered table of numbers. Lattices are named by the number of columns. For example, a 10-lattice has ten columns of numbers. The hundreds chart, pictured below, is a 10-lattice. On a 10-lattice, each number is 10 more or less than the number below or above it. One can think of the lattice as a number line that has been cut into segments of ten, with the segments then stacked on top of each other. Like number lines, number lattices can be extended in either direction. In this activity, students investigate the relationships between spatial arrangements of numbers on a 10-lattice and their numerical relationships. By visually exploring and analyzing patterns on a lattice, students develop their understanding of number relationships, place value, and arithmetic operations.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59

60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Preparation and Materials

Before the session, gather the following materials:

- Student Page 5
- Student Page 6, several copies
- Student Page 7

If students are more familiar with the 1-100 number chart than the 0-99 chart, or a number chart that starts at the bottom and builds up rather than one that starts at the top and builds down, you may want to use that chart instead. Student Page 7 can be used to create the chart with which students are most familiar.

Several other activities in the MathPARTNERS curriculum include explorations on a hundreds chart: Grades K-2 Patterns and Functions: Activity 5; Grades K-2 Number and Operation: Activity 5; and Grades 3-5 Number and Operation: Activity 1.

Activity

Part 1: Build a Number Lattice

1. **Construct a complete 0-99 number lattice.**

   Give students a copy of Student Page 5 (or a similar chart created on Student Page 7—see note in Preparation and Materials section of this activity). Explain that a number lattice is a table of ordered numbers. The hundreds chart is one type of number lattice. Students may already be familiar with the hundreds chart.

   Some of the numbers are missing from the lattice on Student Page 5. Ask students to fill in the missing numbers to build the lattice. Skip around on the lattice. As you point to squares, ask:

   What number goes in the empty square that is…

   - *to the right of an existing number?*
   - *to the left of an existing number?*
   - *three squares to the right of a number?*
   - *four squares to the left of a number?*
   - *above a number?*
   - *below a number?*
   - *two cells above a number?*
   - *five cells below a number?*

2. **Develop and extend problem-solving strategies.**

   For each question above, ask students to explain their problem-solving strategies. Students may say that each number is ten more than the number above it and ten less than the number below it. If they know one number in a column, they should be able to count by tens to determine the remaining numbers in the column.

   Pick a number on the lattice and ask questions such as the following:

   - *What number is four squares up and two squares to the right of this number?*
   - *What number is three squares to the left and three squares down from this number?*

Part 2: Diagonal Patterns

1. Investigate patterns in the diagonal numbers on a 10-lattice.

Have several copies of Student Page 6 available. On one copy, use a colored marker to mark several numbers along a right main diagonal, starting at the number three.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Give students time to make observations about the numbers on the diagonal. If students have difficulty finding patterns, ask questions such as:

- *What is the difference between 3 and 14? Between 14 and 25?*
- *What is the difference between the digits in each number on this diagonal?*

Students may notice that the differences between the numbers are the same: each number is 11 more than the number diagonally above it. Students may also notice that the difference between the digits in any number on this diagonal is always three. Invite students to look for other number patterns for this diagonal.

Note: A right main diagonal is one that starts in the upper left corner and goes down to the right. A left main diagonal starts in the upper right corner and goes down to the left.

2. Explore the patterns in a different diagonal on the 10-lattice.

Mark another right main diagonal that starts at a different number, such as 2 or 4. Ask students if the same patterns apply to this diagonal as well. Have students predict whether the same pattern will apply to all diagonals, right or left. Give students an opportunity to mark other diagonals and observe the numeric patterns.

3. **Analyze the correspondence between visual patterns formed by shaded numbers on a lattice and the numeric pattern.**

   Give students a copy of Student Page 6. Ask them to mark the following diagonal: 8, 17, 26 and look for numeric patterns in the visual pattern. Next, have students mark the same visual pattern in another part of the lattice, for example: 28, 37, 46, and check to see if the numeric patterns are the same. Ask students to explain why the numeric patterns are the same or different.

4. **Explore, analyze, and compare additional diagonal patterns on the 10-lattice.**

   Give students an opportunity to explore additional diagonal patterns and record their observations. As students describe numeric patterns, challenge them to explain why the pattern makes sense. As students compare different visual patterns, challenge them to explain why the underlying numeric patterns are the same or different.

Extension

Exploring Rules on a Lattice

Give students a copy of the 10-Lattice and present the following challenges:

- Describe a rule for finding the sum of any three horizontally adjacent numbers; any three vertically adjacent numbers; any three diagonally adjacent numbers. Explain why the rule works.
- Describe a rule for finding the sum of any 2x2 array of numbers; 3x3 array of numbers. How could you change the rule to find the sum of any 6x6 array?

Exploring Negative Numbers on a Lattice

For students who have trouble understanding the relationship of negative numbers, you may want to try Part 1 of this activity on a 10-lattice that includes negative numbers. Begin the activity by constructing the 10-lattice from a number line that includes negative numbers. Cut a –50 to 50 number line into equal segments and arrange them as a lattice. Explore the questions from Part 1 on the new lattice.

Paths

4

Overview

Mathematical Focus

- Use arrow paths as visual models to represent and solve equations
- Explore properties of equivalence, commutativity and inverse relationships

Students explore the relationship of numbers on a 10-lattice by creating arrow paths between them. An arrow path consists of a starting number, a set of arrows (←, ↑, →, ↓), and an ending number. An arrow path between numbers on a lattice indicates both the direction and magnitude of a movement from one number to another. Using the arrow symbol system, students express arrow paths as arrow equations, for example: 12 →→↑= 4. Students solve and create a variety of arrow equations, such as 5 ↓↑↓→ = ?

Preparation and Materials

Before the session, gather the following materials:

- Student Page 6 or Student Page 7
- Student Page 8

If students are more familiar with the 1-100 number chart than the 0-99 chart, or a number chart that starts at the bottom and builds up rather than one that starts at the top and builds down, you may want to use that chart instead. Student Page 7 can be used to create the chart with which students are most familiar.

Activity

Part 1: Follow the Path

1. **Introduce arrow paths as a symbolic way of representing mathematical relationships.**

   Give students a copy of Student Page 6. Have several extra copies available. Ask what they remember about the 10-lattice: *How are the numbers arranged? What is to the right of a number? To the left? Above? Below?*

   Explain that students will be using arrows to create paths on the 10-lattice. Write 14→. Ask: *What do you think this means?* Encourage students use the lattice to explain their thinking. If students do not suggest it on their own, explain that 14→ is another way of saying 14 and one to the right, or 15. Circle 14 on the lattice and draw a right arrow to 15.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14→	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

2. **Practice following arrow paths on the 10-lattice.**

   Give several more single arrow path examples. For each example, have students circle the starting number, draw the right, left, up and/or down arrows on the lattice, and then circle the end number. Eventually, students may be able to simply trace the arrow path on the lattice and determine where the path will end.

   - 14←
   - 21→→
   - 33↑

- 64→↑
- 45←←←

3. **Determine what happens when you get to the edge of the lattice.**

   Write: 29→. Ask students: *What will happen when you get to the edge of the lattice?* To help student think about the problem, suggest that they think of the lattice stretched out into a number line. Instead of rows of numbers, all of the numbers would be in a continuous line. Then ask what 29→ would look like on the number line. Students should agree that 29→ ends at 30.

4. **Use arrow paths to illustrate properties of equivalence, commutativity, and inverse relationships.**

   Introduce problems such as the following: 15↑→↓←.

   For this example, students should see that the successive arrows move around in a square.

   Write: 46↑↓↑↓↑↓.

   Ask students to first predict the ending number before tracing the path on the lattice. Students may recognize that following this arrow path leads back to the starting number. They may suggest that each pair of up-down arrows cancels each other out.

Teaching Tip

Some students may need lots of practice before they understand that opposite arrows undo each other. Be sure to give students plenty of time and opportunities to discover this shortcut for themselves.

Write: 81→→↑←←↑↓.

Ask students to determine the end number. Have them explain their strategies for solving the problem. Student may recognize possible shortcuts, such as canceling as many pairs of opposite arrows as possible and then following the arrows that are left.

Part 2: Path Puzzles

1. **Explore different types of path puzzles.**

   Path puzzles are arrow path equations with missing information. To solve the puzzles, students must fill in either the starting number, the ending number, or missing arrows.

   Work through the examples below. For each puzzle, ask:

   - *What information is missing?*
   - *What will your strategy be for solving the puzzle?* Once students have figured out one way of solving the puzzle, ask them if they can think of a different way to approach the problem.
   - *Can you explain your thinking?*

   Missing End Number
   Write: 87 ↓↑↓↑→→ = ?

   Missing Arrows
   Write: 72 [   ] = 57

   Missing Start Number
   Write: [?] ←←↑ = 25

2. **Solve path puzzles.**

   Give students a copy of Student Page 8. Have them solve the path puzzles. When they are finished, discuss strategies for solving each type of puzzle.

3. **Invent new path puzzles.**

   Have students make up new path puzzles for others to solve.

Extension

Challenge students to solve the following path puzzles:

- 1, 056 ↓↓→ = ?
  [1,037]

- ? ↑→↓←↑ = 438
  [448]

- 962 [   ] =954
  [↓→→]

- [?] ↑↑→→↑↑→ = 2,735
  [2,692]

Equivalent Paths

5

Overview

Mathematical Focus

- Use arrow paths as visual models to represent and solve equations
- Use arrow paths to illustrate properties of equivalence, commutativity and inverse relationships

Students build on their experiences with arrow paths from Activity 4 by creating equivalent arrow paths. To make a path equivalent to a given path, the new path must begin at the same starting number and end at the same ending number as the given path. As students compare equivalent arrow paths, they learn that the order of arrows does not affect the ending number. This illustrates the commutative property. Students also learn that the ending number is not affected by adding or subtracting pairs of opposite arrows. For example, adding ↑↓ or ←→ to an arrow path will not affect the ending number. Experience working with these properties enables students to develop an understanding of algebraic concepts using a concrete visual model.

Preparation and Materials

Before the session, gather the following materials:

- Student Page 6
- Student Page 9
- Index cards or small pieces of paper

Activity 4: Paths introduces students to the idea of arrow paths on a number lattice. Students need this introduction before beginning Activity 5.

Activity

Part 1: Equivalent Paths

1. **Explore the idea of equivalent arrow paths.**

   Write each of the following arrow paths on a slip of paper or index card:

   ↑→↑
   ↑↑→
   →↑↑

   Give students a copy of Student Page 6. Ask students to:

   - Pick a start number on the lattice and circle it.
   - Choose one of the path cards and follow the path indicated on the card.
   - Circle the end number.
   - Using the same start number, follow the paths on the other two cards.

   Ask:

   - *What is similar about these three paths?*
   - *What is different?*
   - *If you chose a different start number and followed all three paths from that number, do you think the same thing would happen?*

   Explain that these three paths are equivalent paths. They have the same start number and the same end number but the arrows are in a different order.

2. **Investigate equivalent paths.**

   Write each of the following new arrow paths on index cards or slips of paper:

   ↑→↑↑↓
   ←↑↑→→
   →↑→↑←

Ask students to begin at their original circled start number and follow each of the paths on the index cards. Ask:

- *How are these paths similar to the first three paths you looked at?*
- *How are they different?*

Explain that the paths on all six cards are equivalent paths. They have the same start number and the same end number but a different number of arrows or a different order of arrows between start and end numbers.

Part 2: Creating Equivalent Paths

1. **Create several equivalent arrow paths.**

Give students a new copy of Student Page 6. Ask students to show an arrow path from 32 to 54. Have the student write the equation for the arrow path. Using a different color marker, ask students to show a different arrow path from 32 to 54. Write the equation for that arrow path below the first equation. Continue until students have created at least 4 different arrow paths. Possible arrow paths might include:

32↓↓→→ = 54

32 ↓→↓→ = 54

32 →→→↓↓← = 54

32 ↓↓↓↓→→↑↑ = 54

Ask:

- *How are these paths similar? How are they different?*
- *Which path uses the fewest arrows? Which path uses the most arrows?*

Have students compare the two paths that use the fewest arrows. Ask:

- *Can you make a path from 32 to 54 that uses fewer arrows?*
- *What do you notice about the order of the arrows?*

Point out that all of the paths from 32 to 54 are equivalent paths.

2. **Solve arrow path challenges.**

Give students a copy of Student Page 9 and several copies of Student Page 6. For each pair of numbers on Student Page 9, encourage students to find arrow paths with the same number of arrows and arrow paths with a different numbers of arrows. Ask students to put a star beside the path that uses the fewest arrows possible for getting from the start number to the end number.

3. **Share strategies for creating equivalent paths.**

   After students have had an opportunity to work on Student Page 9, have them share and demonstrate the strategies they used for creating equivalent paths.

4. **Investigate the strategy of changing the order of the arrows.**

   Students may mention that changing the order of the arrows will produce different but equivalent paths. To illustrate this strategy, take one of the paths that students created and write it on a piece of paper. Ask: *What will happen if I change the order of the arrows? Will the end number be the same? Why or why not?*

   Ask students to re-write the equation, changing the order of the arrows, and check their prediction. Students should see that changing the order of the arrows does not affect the ending number.

   Example:

   70 →→↑↑↑ = 42   70 ↑→↑→↑ = 42

5. **Explore the strategy of adding inverse arrows.**

   Choose an arrow path from the student page that uses the fewest arrows possible to get from the start number to the end number. Ask: *What will happen if we add this pair of arrows (↑↓) to the arrow equation?* Have students re-write the equation, adding the pair of arrows, and check to see if the end number changes [it does not]. Ask:

   - *What will happen if we add this pair of arrows → ← to the arrow equation?*
   - *Will the end number change or stay the same?*
   - *How do you know?*

   Students may begin to see that adding pairs of inverse arrows to an equation does not change the ending number. Ask students to test the idea with a few more examples.

Shape Holders

6

Overview

Mathematical Focus

- Represent the idea of a variable as an unknown quantity using symbols

Students explore addition, subtraction, multiplication, and division using shape holders. Shape holders, like letters in algebraic expressions, are used to represent numbers. Whatever number is used to fill one type of shape within an expression must be used to fill all instances of that shape in that expression. As students begin to think algebraically, it is easier to fill in a space rather than substitute for a letter. By using shape holders to represent unknown quantities in an expression, students begin to develop the algebraic concept of variable.

Preparation and Materials

Before the session, gather the following materials:

- Student Page 10

Activity

Part 1: Shape Holders

1. **Learn about shape holders.**

   Write the following expression:

   $$\square + \square + \square = 12$$

   Ask students what number can be put in the square holders to make the expression true. Explain that whatever number is used to fill in one type of shape must be used to fill in all of the other holders of the same shape in that expression. Encourage students to try some numbers to see if they can make it true. If students are unsure how to begin, model a strategy for trying numbers, for example:

   - *Let's try 1. Is that too big or too small?*
   - *What number should we try next?*

   Students should see that a 4 can be placed in each of the squares to make the expression true. Ask: *Is there any other number that could be put in the square to make the expression true?* [No]

2. **Combine two variables in the same equation.**

   Try another example with holders of different shapes.

   $$\triangle + \triangle + \diamond = 12$$

   Note: If two holders have different shapes, it is legal to use the same number for each holder; however, if two holders have the same shape, the same number must be used to fill those holders. So, in the example above, the triangles and the diamond could all hold fours, but the diamond could not hold a five while one triangle held a three and one held a four.

   Model a strategy for trying different numbers. Ask questions such as:

   - *If we put a 1 in both of the triangles, what will we need to put in the diamond?*

- *If we put a 2 in both of the triangles, what will we need to put in the diamond?*
- *Does the diamond need to be an even number or an odd number, or does it matter? Can you explain your thinking?*
- *Is it possible to have more than one solution?*
- *How could we organize the solutions to make sure we have them all?*

3. Create a data table to organize possible solutions to a shape holder equation.

One way of organizing solutions is to create a data table such as the one below.

△ + △ + ◇ = 12

1	1	10
2	2	8
3	3	6
4	4	4
5	5	2

4. Solve shape holder challenges.

Give students a copy of Student Page 10. Work through several of the problems on the student page together. Remind students of the rule*: If two holders have the same shape, the same number must be used in the holder.*

5. Look for multiple solutions to each challenge and share problem-solving strategies.

Once students have filled the holders to make the expression true, ask: *Is there a different way that you could fill the holders?* For most expressions, there will be more than one possible solution. Encourage students to find two or more possible solutions for each problem. For each problem, ask students to share their problem-solving strategies.

Example 1:

△ + □ + 7 = 14

Possible problem-solving strategy: Recall that 7 + 7 = 14. You already have one 7, so find two numbers that add to seven, and put those numbers in the triangle and square.

Have students organize their solutions in a data table.

△ + □ + 7 = 14

3	4
4	3
2	5

Example 2:

□ - △ = 6

One problem-solving strategy: Take any number greater than 6, and put that number in the square. Find the difference between 6 and the larger number, and put the difference in the triangle.

Possible solutions: 7 - 1 = 6, 8 – 2 = 6, 9 – 3 = 6

Example 3:

⬡ + △ + ⬡ + △ = 24

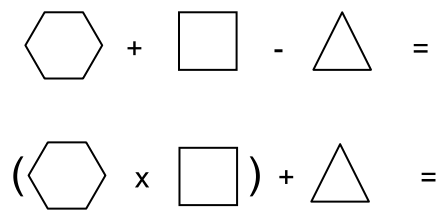
One problem-solving strategy: Find two even numbers that add to 24, such as 4 and 20. Divide each of those numbers by 2: 20 ÷ 2 = 10 and 4 ÷ 2 = 2. Put 10's in each hexagon and 2's in each triangle.

Possible solutions: 10 + 2 + 10 + 2 = 24, 4 + 8 + 4 + 8 = 24, 6 + 6 + 6 + 6 = 24

Part 2: Smallest Number/ Largest Number

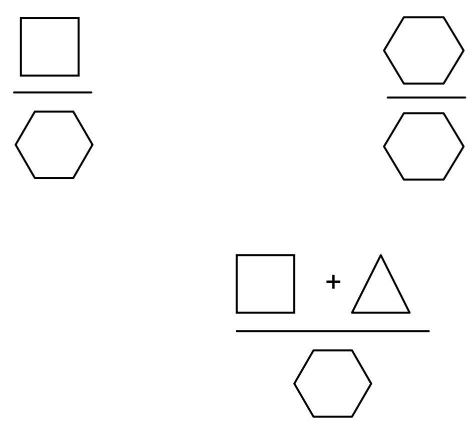
1. **Using a limited set of numbers, solve shape holder challenges to create the smallest/largest number possible.**

Put a different twist on the challenges from Part 1. Using only the numbers 2, 3, and 4, ask students to fill in the holders that follow to make the smallest possible number. Then make the largest possible number. For each challenge, encourage students to explain their strategy for making the largest or smallest number.



2. **Create the smallest/largest fraction possible using a limited set of numbers.**

   If students are familiar with fractions, try these additional problems. Again, using only the numbers 2, 3, and 4, ask students to fill in the holders to make the smallest possible number. Then ask the student to make the largest possible number. Ask questions such as: *To make a small fraction, what kind of number will you want to put in the denominator? the numerator? Does the same hold true for making the largest fraction possible?* As students think about the problems, it may be helpful to sketch a circle or a bar and divide it into pieces to solve the fraction problems. For each challenge, encourage students to explain their strategy.



Number Puzzles

7

Overview

Mathematical Focus

- Investigate mathematical relationships using equations
- Apply and adapt a variety of problem-solving strategies

Students explore patterns involving arithmetic operations as they investigate different number puzzles. Solving number puzzles develops students' ability to describe, analyze and generalize numerical patterns and relationships, thus fostering early algebraic thinking.

Preparation and Materials

Before the session, gather the following materials:

- Calculator, optional.

Activity

Part 1: Mystery Numbers

1. Solve mystery number challenges.

For each mystery, work through the clues step-by-step. Write each clue. Give students a chance to think about the clue and decide if it is possible to solve the mystery with the information they have, or if more information is needed. With each clue, encourage students to list numbers that could possibly be the mystery numbers and to eliminate numbers that cannot be the mystery numbers.

Teaching Tip

Teaching Tip: For younger students, you may want to work with mystery number clues that involve only addition and subtraction, while older students can be challenged with clues that use multiplication and division.

Mystery 1

- There are two numbers.
- Their sum is 10.
- *Ask: Do you know the numbers yet? Can you think of any solutions?*
- Their difference is 2.
- *What are the mystery numbers?* [4 and 6]

Mystery 2

- There are two numbers.
- Their difference is 6.
- *Ask: Do you know the numbers yet? Can you think of any solutions?*
- Their sum is 12.
- Strategy hint: List pairs of numbers whose sum is 12. *Which pair of numbers whose sum is 12 also has a difference of 6?*
- *What are the mystery numbers?* [9 and 3]

Mystery 3

- There are two numbers.
- Their difference is 3.
- Their sum is 27.
- Both numbers are between 10 and 20.
- *What are the numbers?* [15 and 12]

Mystery 4

- There are two numbers.
- Their sum is 11.
- Their product is 24.
- When I subtract one from the other, the answer is 5.
- *Ask: Do you know the numbers now? Did you need all the clues? Which clues could I have left out?*
- Strategy hint: List pairs of numbers whose sum is 11. *Which pair of numbers whose sum is 11 also have a product of 24?*
- *What are the mystery numbers?* [3 and 8]

Mystery 5

- There are four numbers.
- Their sum is 8.
- Their product is 12.
- *What are the mystery numbers?* [1, 2, 3, 2]

Mystery 6

- There are four consecutive numbers.
- Their sum is 18.
- The largest number is divisible by 6.
- *What are the mystery numbers?* [6, 5, 4, 3]

2. **Make your own mystery number challenges.**

   Challenge students to create their own mysteries for you and/or other students to solve. The process of creating problems sharpens the student's ability to analyze patterns and think algebraically. Help students create their first mystery by having them choose two numbers and then helping develop clues about those numbers. Encourage them to take the mystery home with them and try it with their family.

Part 2: Hidden Numbers

1. **Learn about hidden number challenges.**

   Hidden Numbers is a good calculator activity, but it can be done without the calculator as well. Have students think of a number from 1 to 10. Use the following sequence of steps with students. At each step, have students carry out the calculation.

   - Start with your number.
   - Add 5.
   - Add 4 more.
   - Subtract 9.
   - *What number do you have?*

   If students did the calculations correctly they will end up with the same number they started with.

2. **Write an equation to represent each step of the hidden number calculation.**

   As you read the sequence of steps again, have students try a different number. This time, ask students to write an equation for each step that represents the calculation to be carried out. For example, if the number 5 is chosen, the step-by-step calculations would be as follows:

   - Start with your number: 5
   - Add 5: $5 + 5 = 10$
   - Add 4 more: $10 + 4 = 14$
   - Subtract 9: $14 - 9 = 5$
   - What number do you have? 5

3. **Use shape holders to unravel the secret behind the hidden number challenges.**

   Ask students to try a few more numbers. Challenge them to explain why the sequence of steps always gives you the starting number.

   Have students use a shape holder (introduced in Activity 6) to represent the starting number. Ask them to write out the sequence of steps using the shape holder in place of a number:

   $\square + 5$

   $\square + 5 + 4$

$$\square + 5 + 4 - 9$$

Students should see that you are always adding 9 to the starting number (5 + 4) and then taking 9 away again, leaving you back where you started.

4. Solve another hidden number challenge.

Again, tell students to think of a number from 1 to 10 and write the number at the top of a piece of paper. Present the following sequence of steps to students. As each step is presented, have students write an equation for each step that represents the calculation to be carried out.

- Multiply your number by 10.
- Divide by 2.
- Divide by 5.
- What is your number?

Students should end up with the same number they started with. Challenge them to explain why the sequence of steps always gives the starting number.

5. Invent your own magic number challenges.

Ask students to create their own hidden number challenges. Help students to see that whatever is done to the starting number must be undone with an inverse operation. For example, if 4 is added to the starting number, then five is added to the starting number, a total of nine must be subtracted.

Extension

Hidden Number Challenge

Start with a number between 1 and 10.

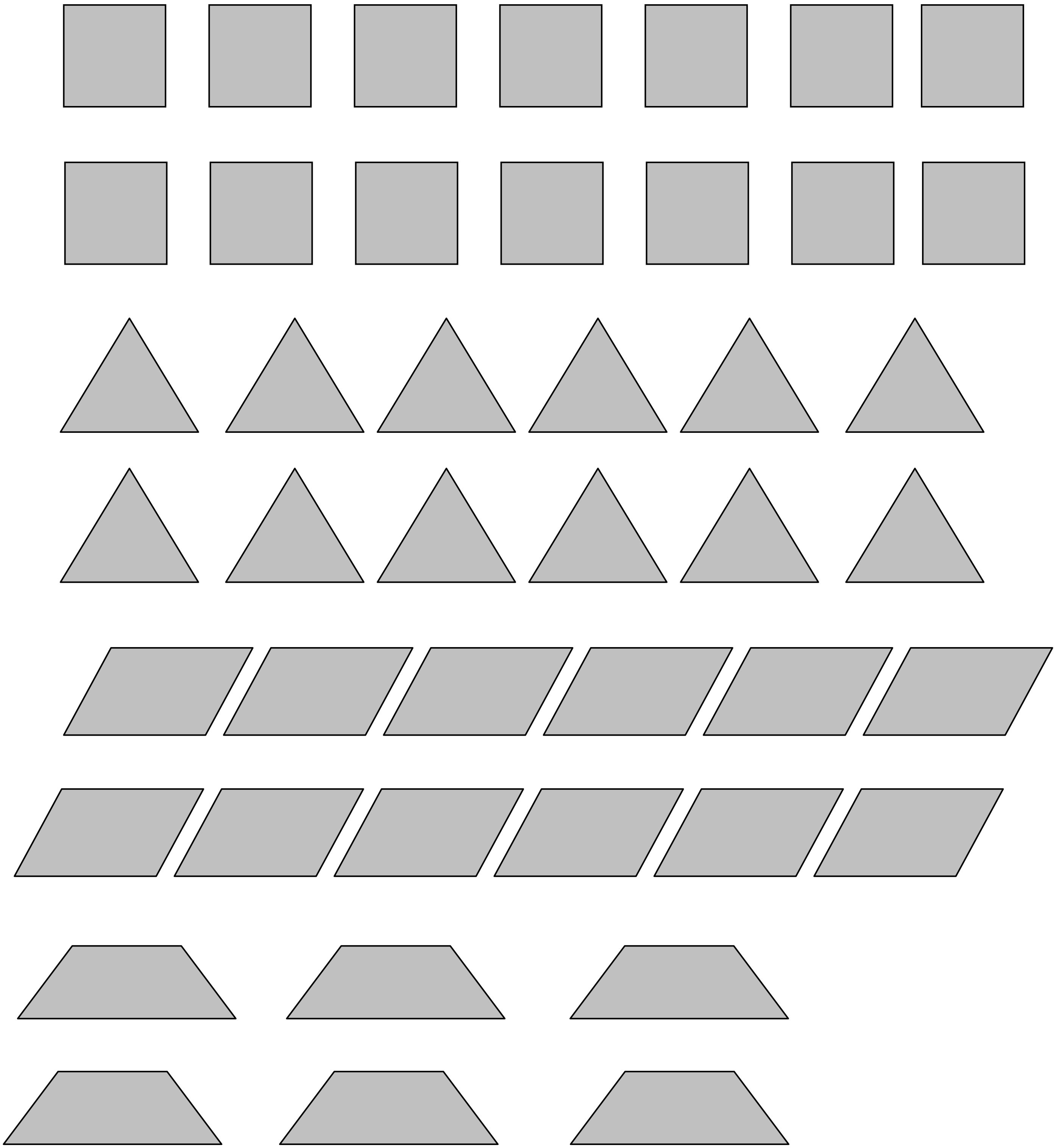
- Multiply by 10.
- Subtract 4.
- Divide by 2.
- Add 2.
- Divide by 5.

What is your number?

Invite students to analyze this hidden number challenge and explain why it always gives the original starting number.

1

Shape Pieces



Pattern Data Table

Name of pattern:

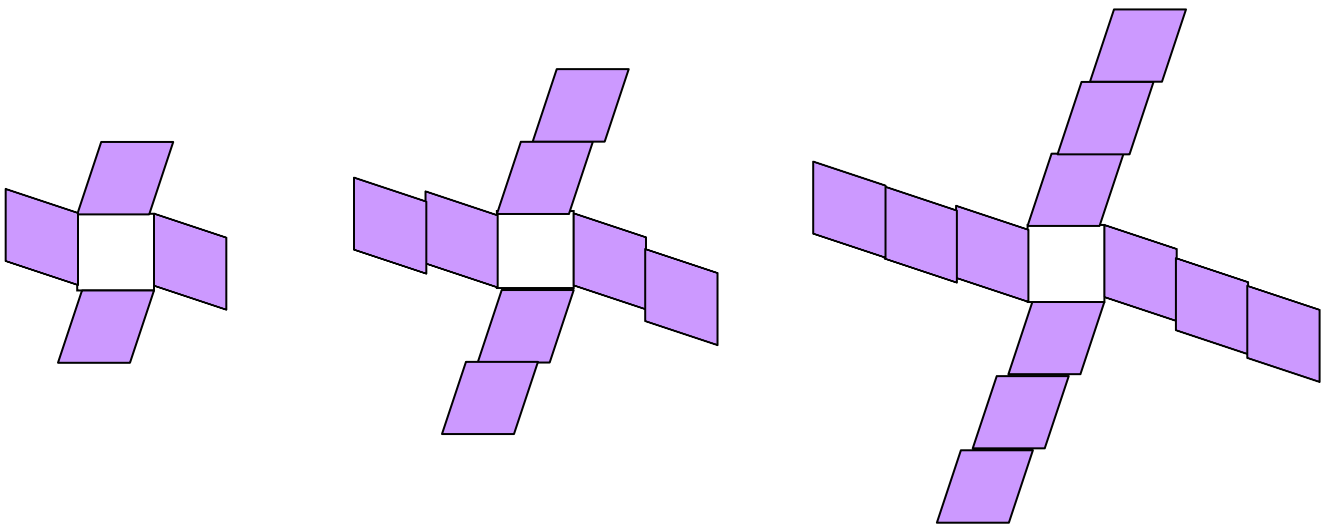
Description or sketch of pattern:

Pattern rule:

Term	Total

Build and Describe a Pattern

Pattern 1



Pattern 2: Party Plan

1 guest: 3 utensils [insert graphic of: knife, fork and spoon]
2 guests: ? utensils
3 guests: ? utensils

Pattern 3: Spiders

1 spider: 8 legs [insert graphic of a spider]
2 spiders: ? legs
3 spiders: ? legs

Pattern 4: Make-Your-Own Pattern

Function Machine Record Sheet

4

Rule:	
Input	**Output**

5

Build a 10-Lattice

0	1	2	3	4	5	6	7	8	9
10	11		13	14	15	16	17		
20	21	22		24	25	26	27	28	29
30		32	33		35	36	37	38	
	41	42	43	44		46	47	48	49
50	51		53		55			58	59
	61	62		64	65	66	67		69
70	71	72	73			76	77	78	79
80	81			84		86		88	
		92		94	95	96	97	98	99

10-Lattice (0-99 Chart)

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

7

Blank 100's Chart

8

Path Puzzles

37 ________ = 59

19 →→→↓ = □

□ ←←↑↑ = 25

88 ________ = 64

96 ↑↑→ → ← ↓ = □

□ →→→↑↓↑↑ = 54

□ ↓→ = 80

42 ________ = 12

49 ↑→→ = □

□↑↑↑→↓ = 57

9

Equivalent Paths

Find different arrow paths from each start number to the end number. Can you find paths with the same number of arrows? Can you find paths with different numbers of arrows?

Start	Equivalent Paths	End
70	→→↑↑↑	42
	↑→↑→↑	
	→→↑↓↑↑↑	
	↑→↑→↑→→←←	

Start	Equivalent Paths	End
65		31

Start	Equivalent Paths	End
18		39

Start	Equivalent Paths	End

24		57

10

Shape Holder Challenges

△ + □ + 7 = 14

□ - △ = 6

⬡ + △ + ⬡ + △ = 24

□ + □ - △ = 15

⬡ x □ = 18

△ - □ + △ = 40

⬡ + ⬡ + □ = 6 + □