Grades 6–8

Patterns and Functions

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Target Numbers

1

Mathematical Focus

Overview

- Order of operations
- Writing of numeric expressions
- Evaluation of numeric expressions

Target Numbers is a game with 56 cards, all marked differently from 1 to 25. The game can be played with one or more players. The object is to use five number cards to make a target number. Players may use addition, subtraction, multiplication, division, and parentheses in their solutions. This game provides practice with order of operations and writing and solving numeric expressions.

Prior to playing Target Numbers, the student solves a series of puzzle-like problems that involve performing basic operations, using parentheses to group calculations and the standard order of operations. For example:

Use the numbers 1, 3, 5, and 7 to complete the equation, using each number once:

$$(\square + \square - \square) \times \square = 1$$

Preparation and Materials

Before the session, gather the following materials:

- Four sets of equation puzzles, using the templates in Part 1, below
- A deck of cards from Student Page 1: Target Numbers Cards (using heavy paper or card stock), in the following amounts:
    - three each of the numbers 1–6
    - four each of the numbers 7–10
    - two each of the numbers 11–17
    - one each of the numbers 18–25

Once the Target Numbers game becomes familiar to students, it may be used as a quick warm-up activity during any of the mentoring sessions.

Activity

Part 1: Solve the Equation Puzzles

1. Solve a series of number sentence problems.

Create a series of number sentence problems for students to solve. Examples of four problem types are shown below.

If students are not familiar with the order of operations, spend some time reviewing parentheses, then multiplication and division, then addition and subtraction.

Type 1: Insert parentheses to make this equation true:

$$7 \times 9 \div 3 \times 7 = 3$$

Type 2: Use the numbers 1, 3, and 5 to make each of the following equations true, using each number only once:

$$\square - \square - \square = 1$$

$$(\square + \square) \div \square = 2$$

$$\square - (\square - \square) = 3$$

$$\square + \square - \square = 7$$

Type 3: Use the numbers 2, 4, and 6 (using each only once) and any operations— +, -, x, and ÷ —to make true equations of the following:

$$\square \quad \square \quad \square = 0$$

$$(\square \quad \square) \quad \square = 1$$

$$\square \quad \square \quad \square = 2$$

$$\square \quad (\square \quad \square) = 3$$

$$\square \quad \square \quad \square = 4$$

Type 4: Use the numbers 3, 5, 10, 12, and 15 (using each only once) and any operations— +, -, x, ÷, and ( ) —to make true equations of the following:

= 1

= 2

= 3

= 4

…

2. Play Target Numbers with the 1 through 10 cards.

Rules for Playing Target Numbers:

Place five cards (numbers up) in the center of the game table. Then turn up a sixth card, which will be the Target Card. Each player uses the numbers on the five cards to make the Target Card number. All five cards must be used, in any order, and each can be used only once. Players may use addition, subtraction, multiplication, division, and/or any combination of each. Parentheses may be used to group calculations and to indicate the order in which calculations are to be performed. Players must write out their solutions.

- *Sample hand:*
  Cards: 1, 3, 7, 1, 8
  Target Card: 1

  One possible solution:
  [(3 – 1) + 7] – (8 ÷ 1) = 1

The first player to reach a solution says "Target!" and then has 30 seconds to explain his or her solution. If the solution is correct, the player receives 1 point for that hand. If the player cannot explain the solution or the solution is not correct, the player receives a –1 for that hand.

Ten hands of Target Numbers equal one game. After 10 hands, the winner is the player with the most points.

3. Play Target Numbers with the 1 through 25 cards.

Rules for Target Numbers with the 1 through 25 cards are the same as those for Target Numbers with the 1 through 10 cards (see number 2, above) except that the cards 1 through 25 are used.

- *Sample hand:*
  Five cards: 2, 1, 2, 2, 3
  Target Card: 24

  One possible solution:
  [(2 x 2) x 2] x 3 x 1 = 24

- *Another sample hand, using parentheses to group calculations:*
  Cards: 24, 22, 23, 20, 21
  Target Card: 1

  One possible solution:
  (24 + 22) ÷ 23 + 20 – 21 = 1
  (Applying the order of operations, first do what is in

parentheses: Add 24 and 22. Then divide by 23. Then add 20. Then subtract 21.)

Extension

More Target Numbers

Play Target Numbers with exponents, square roots, negative numbers, and factorials.

For example, 4! = 4 x 3 x 2 x 1, or 24.

$\sqrt{16} = 4$

$2^4 = 16$

Four 4s

2

Overview

Mathematical Focus

- Computation
- Order of operations
- Arithmetic properties

Students try to make all the numbers from 0 to 100 by writing arithmetic expressions using exactly four 4s. They can use any of the operations and parentheses. For example: $4 \times 4 + 4 \div 4 = 17$ and $44 + 44 = 88$.

This activity gives students practice with addition, subtraction, multiplication, and division facts as they write and evaluate arithmetic expressions. Students also use their knowledge of order of operations, properties of addition and multiplication, and the special cases of 0 and 1. Four 4s is a fun, puzzle-like challenge that can be tackled all at once or in several consecutive sessions.

Preparation and Materials

Before the session, gather the following materials:

- Paper and pencil
- Student Page 2: 0–99 Chart
- Calculator (optional)

Start the session by reviewing the order of operations: parentheses first, then multiplication and division, then addition and subtraction.

If possible, have a four-function calculator available for students. At several points during the session, stop and discuss whether the calculator is a useful tool for this activity. Ask students when they think a calculator might be helpful.

Activity

Part 1: How Many Numbers Can You Make with Exactly Four 4s?

1. Make a few numbers together by using exactly four 4s.

Start with a copy of Student Page 2: 0–99 Chart. Explain the goal of the activity, which is to write a mathematical expression to make each of the values 0, 1, 2, 3, etc. through 100, using exactly four 4s. Students may use addition, subtraction, multiplication, and division. They may also use parentheses to group calculations. Try a few together. Ask: *How can we make the number 7 using four 4s?* [One way to make 7 is: $44 \div 4 - 4$. Another way is: $4 + 4 - (4 \div 4)$.]

2. Make as many of the numbers from 0 to 100 as you can using exactly four 4s.

Encourage students to use information about how to make some numbers to make other numbers. For example: $44 \div 4 + 4 = 15$ and $44 \div 4 \times 4 = 44$. Have students keep a record of all their solutions on the 0–99 Chart. With the above examples, students can cross off 7, 15, and 44 on the 100 chart.

Part 2: Making Numbers with Exactly Four of Some Number

1. Make numbers using exactly four 5s.

Ask: *Which numbers can you make with exactly four 5s? Can you make more numbers with four 4s or with four 5s?*

2. Check which numbers can be made in more than one way with exactly four 4s or 5s.

Look back at the students' solutions for the Four 4s. Ask: *Which numbers can be made in more than one way? Are these the same numbers that can be made in more than one way with four 5s?*

3. Choose a different digit and make numbers using exactly four of that digit.

Have students choose a different digit (not 1, 4, or 5). Ask: *Which numbers can you make with exactly four of that digit?*

Extension

How Many Numbers Can You Make with 1, 2, 3, and 4?

Now, change the problem again. Ask: *Which numbers can you make with the digits 1, 2, 3 and 4? Is this problem easier or more difficult than the Four 4s problem? Why?*

Function Machines

3

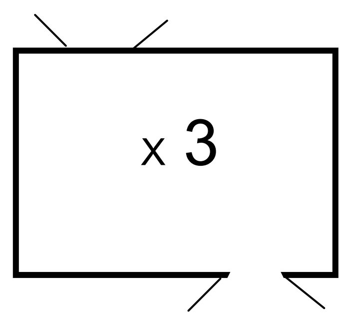
Overview

Mathematical Focus

- Functions as input/output rules (i.e. for every possible input number there is a rule that determines an output number)
- Concrete representation of a machine as a kind of function
- Equivalent functions
- Commutative property of addition and multiplication

Students explore the use of function machines to represent sequences of computations that can be used repeatedly with many numbers. (A function machine is an imaginary device with an input hopper in which an input number can be placed, and an output spout through which a result comes out.) The figure below shows a [x 3] machine, which multiplies its input number by three and outputs the result. Putting a 2 into the machine results in an output of 6. Putting a 5 into the machine results in an output of 15, and so forth. A series of inputs and outputs can be recorded in a table.

Input	Output
2	6
3	9
4	12
5	15



Preparation and Materials

Before the session, gather the following materials:

- Several copies of Student Page 3: One-Step Function Machine Record Sheet and Student Page 4: Two-Step Function Machine Record Sheet

- Calculator

Students should be able to use a calculator throughout this activity. When the same calculation is performed repeatedly, it is helpful to use the constant function on the calculator. Some calculators have a constant key. In this case, you will key in the calculation, for example: [+] [1] [1] (plus 11) and then use the constant key [CONS]. Some calculators do not have a constant key. In this case, key in the calculation—[+] [1] [1] —and then use the [=] key. Now, enter any input number and then use the [=] key. For example: [5] [=] will result in 16. [20] [=] will result in 31.

Throughout this activity, students may encounter negative numbers. If your students are not familiar with performing calculations with negative numbers, you can either spend some time teaching these concepts (see the unit Number and Operation, Grades 6–8), or set up the function machine problems so that the inputs and outputs are always positive numbers.

Activity

Part 1: One-Step Function Machines

1. Fill in input/output tables for one-step function machines.

Using copies of Student Page 3: One-Step Function Machine Record Sheet, make up a series of one-step machines. You may want to use some of the following rules:

[+ 11]	[+ 27]	[– 12]	[– 20]
[x 4]	[x 7]	[x 1.5]	[x 10]
[÷ 2]	[÷ 4]	[÷ 10]	[÷ 3]
[x .5]	[– .5]	[– 3.75]	[÷ 5]

For each function machine, students fill in an input/output table with 10 inputs and the corresponding outputs. It is helpful for students to have access to a calculator. (See Preparation and Materials, above.)

2. Determine the rule for one-step function machines by examining input/output pairs.

Using copies of the One-Step Function Machine Record Sheet, make up a series of input/output tables. For each table, keep the function rule a secret. Students will use the data in each table to determine a rule. You may want to use some of the following input/output pairs:

Input	Output
3	9
10	30
1	3
0	0
–5	–15
1.5	4.5
1/3	1
½	1.5
5	15
101	303

Input	Output
3	10
10	17
1	8
0	7
–5	2
1.5	8.5
1/3	7 1/3
½	7 ½
–1	6
100	107

Input	Output
2	24
0	0
–2	–24
5	60
10	120
100	1,200
½	6
3	36
¼	3
–1	–12

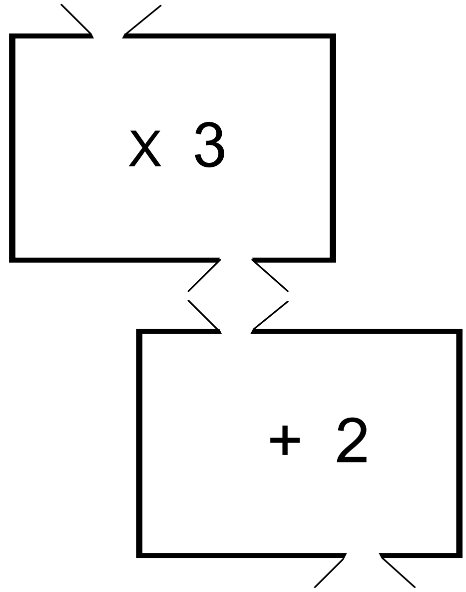
Input	Output
5	56
2	53
0	51
10	61
–5	46
–2	49
–10	41
½	51.5
100	151
–100	49

Input	Output
10	2.5
0	0
1	.25
3	.75
5	1.25
100	25
½	.125
4	1
–4	–1
12	3

Part 2: Two-Step Function Machines

1. Learn what a two-step function machine is.

Describe two-step function machines to students. A two-step function machine is constructed by linking the output of one machine to the input of another. The [x 3 + 2] machine shown in the figure multiplies its input by 3, adds 2 to that, and outputs the result. It corresponds to the equation **y** = 3**x** + 2, where **x** stands for the input and **y** stands for the output.



2. Fill in input/output tables for two-step function machines.

Using copies of Student Page 4: Two-Step Function Machine Record Sheet, make up a series of two-step machines. You may want to use some of the following rules:

[x 2 + 3]	[÷ 2 + 1]	[÷ 10 + 1]
[x 3 + 2]	[÷ 2 – 1]	[x 10 – 1]
[x 5 + 1]	[÷ 3 + 5]	[x 2 + 7]
[x 10 + 2]	[÷ 2 + 10]	[÷ 3 + 10]

For each function machine, students fill in an input/output table with 10 inputs and the corresponding outputs.

3. Determine the function rules for two-step function machines.

Using copies of the Two-Step Function Machine Record Sheet, make up a series of input/output tables. For each table, keep the function rule a secret. Students will use the data in each table to determine the two-step rule. You may want to use some of the following input/output pairs:

Input	Output
2	14
10	30
1	12
0	10
–5	0
1.5	13
1/3	10 2/3
½	11
5	20
101	212

Input	Output
2	3
0	–3
–2	–9
5	12
10	27
100	297
½	–1.5
3	6
¼	–9/4 or –2 1/4
–1	-6

Input	Output
5	3.5
2	2
0	1
10	6
–5	–1.5
–2	0
–10	–4
½	1 ¼
100	51
–100	–49

Input	Output
10	49
0	–1
1	4
3	14
5	24
100	499
½	1 ½
4	19
–4	–21
12	59

Students build a two-step function machine for each input/output table given.

Extension

Equivalent Machines

Here is a set of inputs and outputs for a mystery machine:

Mystery Machine	
Input	Output
3	10

10	17
1	8
0	7
–5	2
1.5	8.5
1/3	7 1/3

Your students might think it looks like a [+ 7] machine. Invite students to think about what other possibilities could be inside the mystery machine. [These possibilities include a [+ 3 + 4] machine, a [– 3 + 10] machine, and a [x 3 + 21 ÷ 3] machine.] Explain that these machines are called equivalent machines because they all have the same results and therefore the same input/output table.

The input/output table for another mystery machine is shown below. Ask students to examine the input/output table and then to try to build two equivalent two-step machines that fit this input/output table.

Mystery Machine	
Input	**Output**
3	10
10	24
1	6
0	4
-5	-6
1.5	7
1/3	4 2/3

Use a Spreadsheet

If you have access to a computer, show students how to build one- and two-step machines using a spreadsheet. Have students go back through this activity and see if they can build a spreadsheet for each machine.

Mystery Machines

4

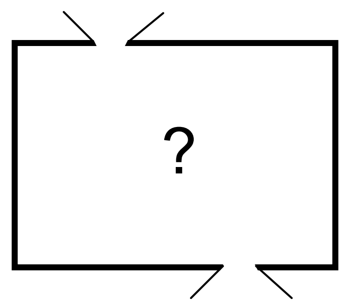
Overview

Mathematical Focus

- Functions as input/output rules (i.e. for every possible input number there is a rule that determines an output number)
- Concrete representations of a machine as a kind of function
- Input/output tables as a method to represent patterns

Just as with a function machine, a mystery machine receives an input and produces an output, but its component machines are hidden. The rule (or function) is discovered by giving the machine a series of test inputs, examining the outputs, and trying to determine the hidden rule. For example, this one-step machine gives some of the following input/output combinations:

Input	Output
7	23
2	8
16	50



Students continue testing the machine until they can accurately predict the output for any input and can draw a picture of a function machine that can produce the same outputs as the mystery machine.

Preparation and Materials

Before the session, gather the following materials:

- Several copies of Student Page 3: One-Step Function Machine Record Sheet and Student Page 4: Two-Step Function Machine Record Sheet

Students should be able to use a calculator throughout this activity. When the same calculation is performed repeatedly, it is helpful to use the constant function on the calculator. Some calculators have a constant key. In this case, you will key in the calculation, for example: [+] [1] [1] (plus 11) and then use the constant key [CONS]. Some calculators do not have a constant key. In this case, key in the calculation— [+] [1] [1] —and then use the [=] key. Now, enter any input number and then use the [=] key. For example: [5] [=] will result in 16. [20] [=] will result in 31.

Throughout this activity, students may encounter negative numbers. If your student is not familiar with performing calculations with negative numbers, you can either spend some time teaching these concepts (see the unit Number and Operation, Grades 6–8), or set up the function machine problems so that the inputs and outputs are always positive numbers.

Activity

1. Find the outputs for one-step function machines.

Using copies of Student Page 3: One-Step Function Machine Record Sheet, create some problems in which students have to find the outputs. You may want to fill in the inputs and the function rules using some of these examples:

Rule	Inputs
[x 12]	5, –5, 0, 1, –1, 10, –10, 2, –2, 12
[÷ 10]	0, 1, 5, 10, 20, 25, 30, 40, 50, 100
[– 15]	0, 1, 2, 3, 4, 5, –1, –2, –3, –4, –5

2. Find the outputs for two-step function machines.

Using copies of Student Page 4: Two-Step Function Machine Record Sheet, create some problems in which student have to find the outputs. You may want to fill in the inputs and the function rules using some of these examples:

Rule	Inputs
[x 3 + 1]	0, 2, 4, 6, 8, 10, 12, 20, 30, 50
[÷ 2 + 10]	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
[x 10 + 1]	0, 1, 2, 3, 4, 5, –1, –2, –3, –4, –5

3. Take turns creating and solving mystery machines.

Think of this as a two-person game with one person being the machine-maker and the other being the guesser.

The machine-maker starts by creating a one- or two-step function machine, keeping the function rule a secret. The machine-maker announces whether this is a one-step or two-step machine. The guesser gives an input number. The machine-maker calculates the output. The guesser records the inputs and corresponding outputs in a table.

Play continues until the guesser thinks he or she knows the rule. The guesser records a rule in a machine and tests it to see if it gives the correct outputs. When this happens, the mystery rule is known and players switch roles. (It is important to switch roles because there is a lot to learn from being the machine-maker as well as the guesser.)

Teaching Tip

The Mystery Machine activity provides an opportunity to develop and discuss strategies. Typically, students will start with random guessing. Next, they may use a systematic sequence of inputs, such as 1, 2, 3, 4, . . . or 10, 20, 30, 40, . . . After that, students may use 0 and 1 to learn something about the mystery rule.

Extension

Machines with Outputs Equal to the Inputs

- Build a two-step machine that inputs and outputs the same number.
- Build a four-step machine that inputs and outputs the same number.

Have students explain why each machine works.

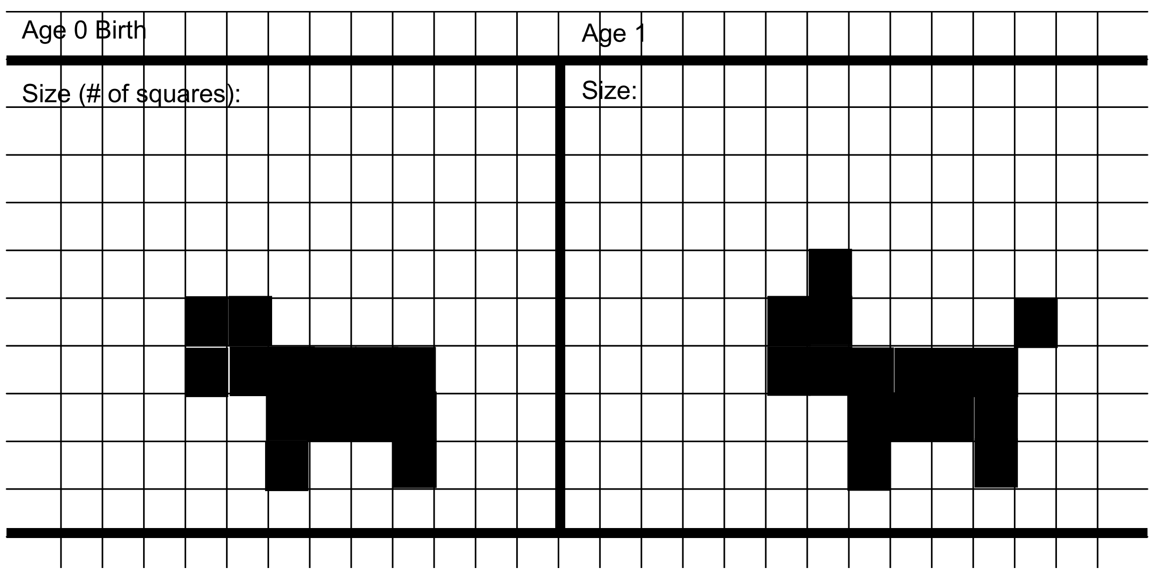
Growing Patterns

5

Overview

Mathematical Focus

- Visual growth patterns—identifying and describing them
- Rules as methods to describe visual and arithmetic patterns
- Input/output tables as a method to represent patterns



Growing Cats are imaginary creatures whose bodies grow by adding shapes in a patterned, linear way. Students study the Growing Cats at different ages and try to identify a pattern of growth. They use input/output tables to record the total number of tiles used to construct each Growing Cat. Students look for patterns in the visual as well as the numeric data. They write verbal and mathematical rules to represent the growth patterns for different types of Growing Creatures.

Preparation and Materials

Before the session, gather the following materials:

- Student Page 5: 1-cm Grid Paper
- Student Pages 6, 7, and 8: Growing Cat, Growing Baby, and Grow-Worm
- Several copies of Student Page 9: Input/Output Table Recording Sheet

One major learning goal of this activity is for students to identify a pattern of growth for each type of Growing Creature and to describe the rule verbally. If students seem ready, you can work with them to record their rule as a mathematical expression.

Activity

1. Examine a Growing Cat and describe how it is growing.

Tell students that Growing Creatures grow in patterned, linear ways. Different creatures have different parts that grow in regular ways, such as the arms, legs, neck, tail, feet, and so forth.

Start with Student Page 6: Growing Cat. Have students examine the cat and determine which parts of its body are growing [ears and tail]. Ask students to describe how each part is growing [by one square for each year].

2. Sketch a picture of the Growing Cat at age 3.

3. Create an input/output table for the Growing Cat.

Have students use Student Page 9: Input/Output Table Recording Sheet to create an input/output table for the Growing Cat.

Input (age of cat)	Output (size of cat)
0	
1	
2	
3	
4	
5	

4. Describe the growth pattern of the cat.

Ask: *Can you predict the size of the cat at age 4? Age 5? Age 10?*

Encourage students to look for patterns in the input/output table. Ask students to explain the growth pattern in words. Ask: *How does the cat grow from year to year? How is the amount the cat grows related to the age of the cat?*

Example:

- At birth, the cat's size is 17.
- At age 1, the cat's size is 20. [17 + 3 x 1 (year)]

  Size at age 1 = size when born plus three squares.

- At age 2, the cat's size is 23.

  Size at age 2 = size when born plus 6. [$17 + 3 \times 2$ (years)]
- Each year the cat grows three more squares. To find the size of the cat at any age, multiply the age of the cat by 3 and add the size at birth.
- If students are ready, you may want to show them how to translate the verbal rule into a mathematical expression, such as: Size = 17 + Age x 3 or $S = 3A + 17$.

5. Sketch other Growing Creatures and describe how they are growing.

Have students investigate Student Page 7: Growing Baby and Student Page 8: Grow-Worm.

6. Create an input/output table for each Growing Creature and write rules to describe their growth patterns.

Have students create an input/output table to represent the relationship between the age and size of the creature and then write a rule to describe the growth pattern. If possible, they may also translate the rule into a mathematical expression.

Extension

Make Your Own Growing Creature

Have students create their own creatures on Student Page 5: 1-cm Grid Paper or with square tiles. As the age of the creature increases by 1, their creatures should grow by adding a fixed number of tiles, which can be added to the arms, legs, neck, tail, feet, and so forth.

Toothpick Patterns

6

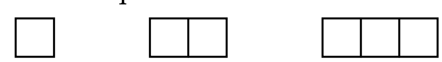
Overview

Mathematical Focus

- Patterns—identifying and describing them
- Rules for understanding arithmetic and visual patterns
- Input/output tables as a method to represent patterns

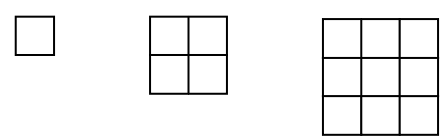
Students will answer the following questions:

- How many toothpicks are needed to construct the next rectangle in this sequence?



- How many small squares will be inside the next square in the sequence?

- How many toothpicks are needed to construct the next square in this sequence?



In the toothpick rectangles activity, students may begin by constructing the rectangle pattern with toothpicks or drawing the pattern on paper. Students continue drawing/constructing rectangles until they can predict the number of toothpicks needed for the next rectangle in the sequence. Students record the number of toothpicks needed for each of the rectangles they constructed in an input/output table. They look for patterns in the data and describe a rule for finding the number of toothpicks.

For the growing squares activity, students start by predicting the number of small squares that will be in the next growing square. Students construct squares and record the numeric data in a table. They soon discover an interesting number pattern!

In the third part of the activity, students use the visual and numeric patterns to write a rule for finding the number of toothpicks needed to construct the next growing square.

Preparation and Materials

Before the session, gather the following materials:

- 100 toothpicks

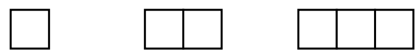
You may use toothpicks or popsicle sticks for this activity. Whatever material you choose to use, all the sticks must be the same length.

Activity

Part 1: Toothpick Rectangles

1. Construct the next two rectangles in a sequence.

Start by constructing the first three toothpick rectangles with toothpicks (or popsicle sticks, etc.).



Have students construct the next two rectangles.

2. Describe the pattern you see in the rectangle constructions.

For example, students may notice that for each new rectangle, three toothpicks are added—but for the first square there was one extra toothpick.

3. Create an input/output table to record the number of toothpicks needed to construct each rectangle.

Input (length of rectangle)	Output (number of toothpicks needed)
1	4
2	7
3	10
4	13
5	16
6	
7	

4. Look for patterns in the data.

Ask: *Can you predict the number of toothpicks needed to make a rectangle with a side that is 6 toothpicks long? A side that is 7 toothpicks long?*

If necessary, have students continue constructing rectangles until they notice a pattern and are able to describe it.

5. **Describe a rule for predicting the number of toothpicks needed to make a rectangle of any length.**

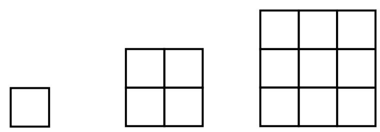
Students may have observed that the length of the rectangle multiplied by 3 plus 1 = the number of toothpicks needed to construct the rectangle.

If students are ready, you may want to introduce how to translate the verbal rule into a mathematical expression. For example: Toothpicks = Length x 3 + 1 or T = 3L + 1.

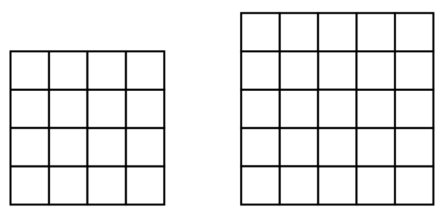
Part 2: Growing Squares

1. **Construct the next two squares in a sequence.**

Start by constructing the first three toothpick squares with toothpicks (or popsicle sticks, etc.).



Have students construct the next two squares.



2. **Create an input/output table to record the number of small squares inside each large square.**

Input (length of side of square)	Output (number of small squares inside)
1	1
2	4
3	9
4	16
5	25
6	
7	

Encourage students to look for patterns in the data. Ask: *Can you predict the number of small squares inside a large square with a side that is 6 toothpicks long? With a side that is 7 toothpicks long?*

If necessary, have students continue constructing squares until they notice a pattern and are able to describe it.

3. Describe a rule for predicting the number of small squares inside a large square with any length side.

Students may have observed that the length of the rectangle multiplied by itself = the number of small squares inside.

If students are ready, you may want to introduce how to translate the verbal rule into a mathematical expression. For example: Small squares = Length of side x Length of side or $S = L^2$.

Part 3: Toothpick Squares

1. Create an input/output table to record the number of toothpicks needed to construct each square from Part 2.

Input (length of side of square)	Output (number of toothpicks needed)
1	4
2	12
3	24
4	40
5	60
6	
7	

Encourage students to look for patterns in the data. Ask: *Can you predict the number of toothpicks needed to make a square with sides that are 6 toothpicks long? With side that are 7 toothpicks long?*

If necessary, have students continue constructing squares until they notice a pattern and are able to describe it.

2. Describe a rule for predicting the number of toothpicks needed to make a square of any length.

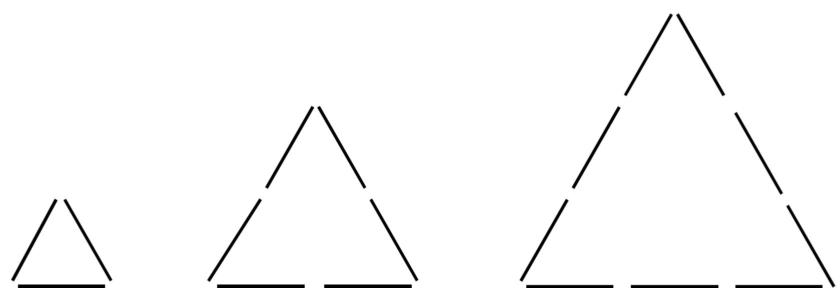
Some students will refer back to the toothpick constructions. These students may observe that the length of the square multiplied by 1 more than itself, then multiplied by 2 = the number of toothpicks needed [2n(n + 1)].

Other students will refer to the number patterns in the table. These students may notice that the outputs increase first by 8, then 12, then 16, then 20, etc.

Extension

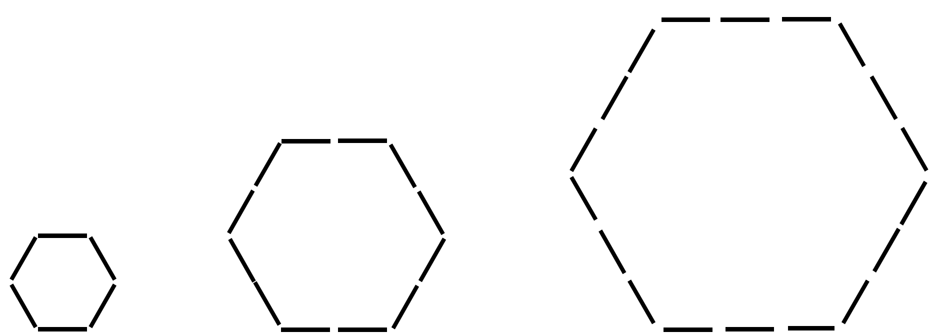
Toothpick Triangles

Have students construct the first five toothpick triangles. Then have students create an input/output table (input = length of side and output = number of toothpicks needed) and describe a general rule for finding the number of toothpicks needed to construct any size triangle.



Toothpick Hexagons

Have students construct the first five toothpick hexagons. Then have students create an input/output table (input = length of side and output = number of toothpicks needed) and describe a general rule for finding the number of toothpicks needed to construct any size hexagon.



Toothpick Polygons

Choose any polygon (e.g., pentagon, heptagon, octagon). Have students construct the first five toothpick polygons. Then have students create an input/output table (input = length of side and output = number of toothpicks needed) and describe a general rule for finding the number of toothpicks needed to construct any size polygon.

Balancing Equations

7

Overview

Mathematical Focus

- Solutions to single-variable equations
- Transformation of equations
- Concept of variable

Students are introduced to balance pictures as a representation of equations. Students learn to do the same thing to both sides of a balance picture to make a simpler, equivalent equation. Students represent balance pictures as equations and then solve the equations.

Preparation and Materials

Before the session, gather the following materials:

- Pan balance scale (optional)
- Pennies (optional)

If a pan balance scale is available, demonstrate how it is used. Place 10 pennies on each pan. The pans should balance. Remove five pennies from one pan. The balance should tip toward the heavier pan. Remove five pennies from the other pan so the pans balance. Now, add 12 pennies to one pan (for a total of 17 in one and 5 in the other). Ask: *How many pennies do we need to add to the other side to get the pans to balance?* [12] Be sure students understand that for the pans to balance you must always add or remove the same number of pennies from each side.

Activity

1. Learn about pan balance scales.

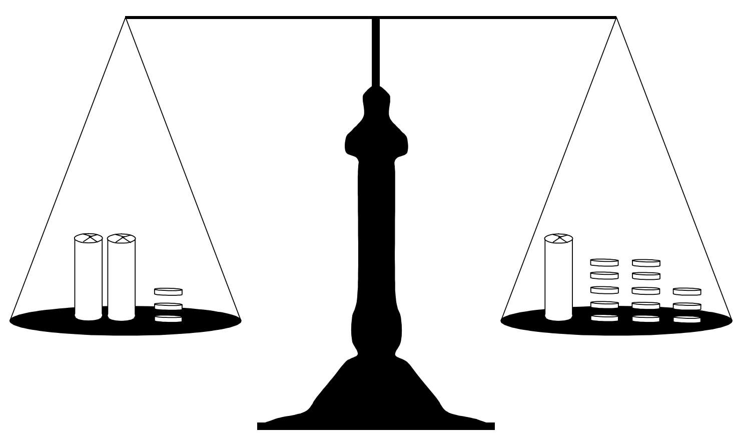
Explain the idea of a pan balance scales. (See Preparation and Materials, above.)

Tell students that rather than use the balance scale for this activity, they will draw balance pictures—that is, pictures that represent a balance scale.

2. Determine the number of pennies in a roll at a "strange bank."

Introduce the idea of balance pictures, using the following context of a "strange bank":

> *There is a very strange bank that does a very strange thing. The people who work there always wrap pennies in rolls of weightless paper, but each day they change the number of pennies in a roll—so every day the customers have to figure out how many pennies are in a roll! But the people who work there always give the customers some clues. On Monday, a customer went into the bank and was given this clue:*

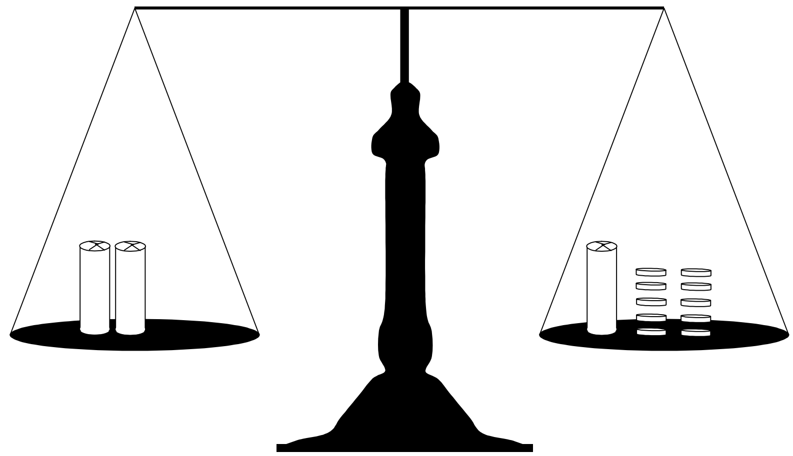


Explain that this is a balance picture showing that 2 rolls of pennies plus 3 loose pennies balances with 1 roll and 13 loose pennies.

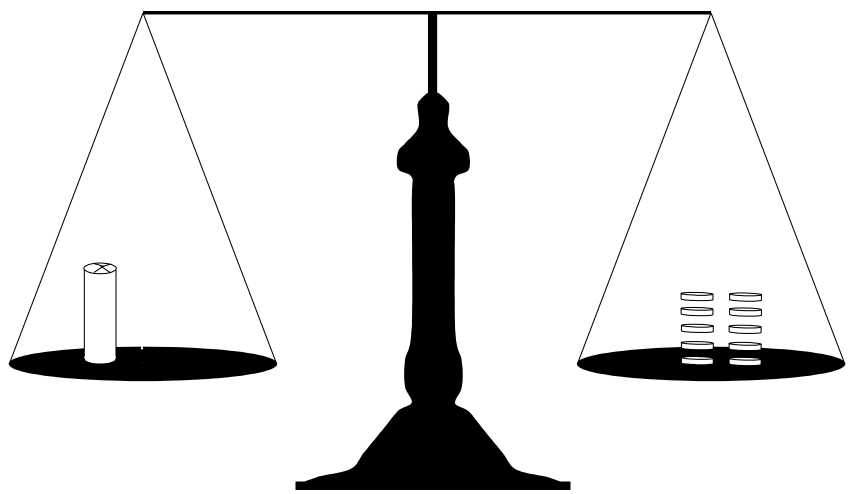
Ask: *How can we determine the number of pennies in a roll?*

Encourage students to develop a plan or strategy for finding the number of pennies in a roll. If necessary, give a prompt: *Let's remove some pennies from the pans to make the simplest balance picture we can.*

If you remove 3 loose pennies from each pan, the balance now looks like this:



Now, if we remove 1 roll from each pan, the balance looks like this:



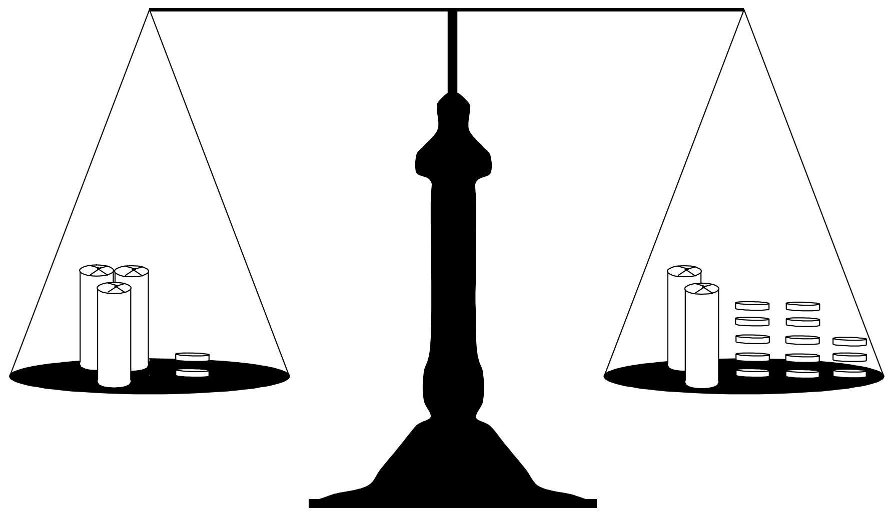
3. Solve another balance problem.

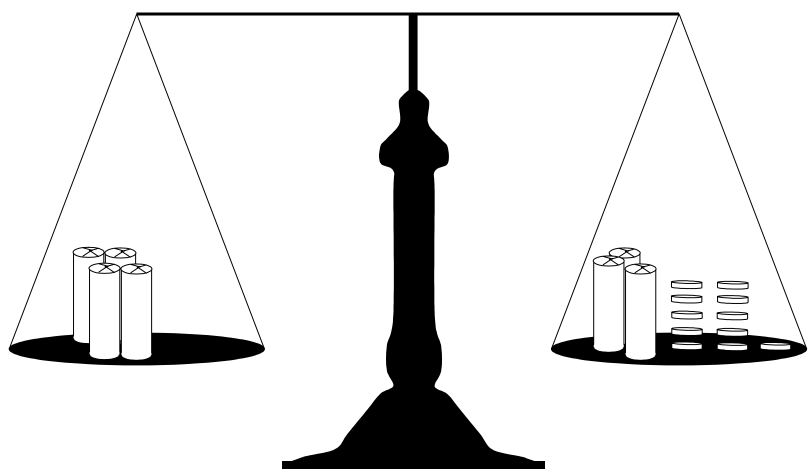
Present students with the following balance problem. Encourage students to develop a plan or strategy for finding the number of pennies in a roll. If necessary, give a prompt: *Let's remove some pennies from the pans to make the simplest balance picture we can.*

There are many possible answers. Here are a few:

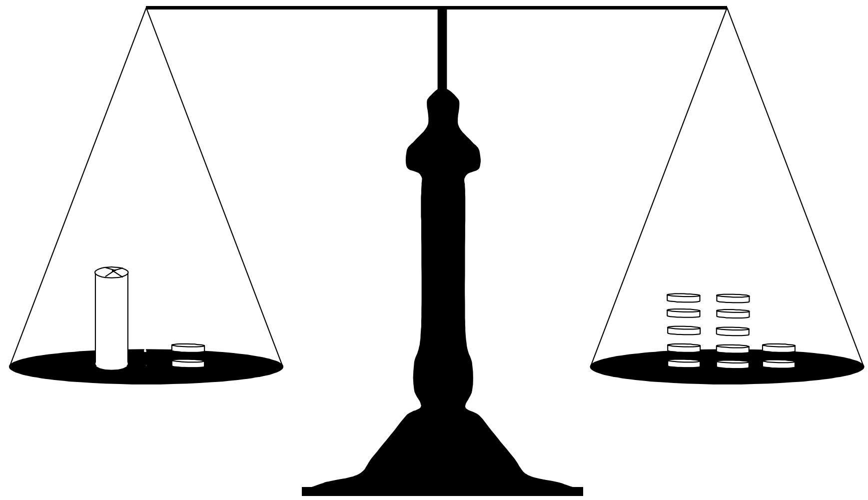
- Remove 2 loose pennies from each pan. The new picture would look like this:

- Remove 1 roll from each pan. The new picture would look like this:

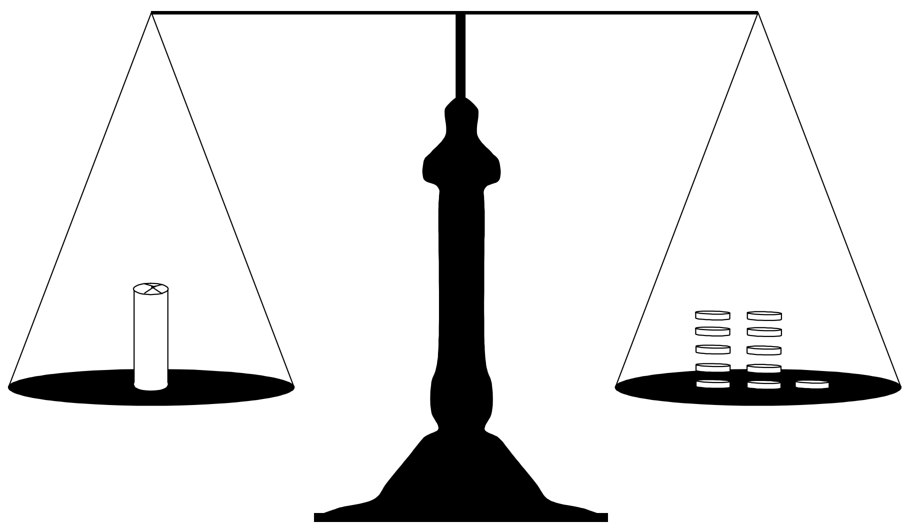




- Remove 3 rolls from each pan. The new picture would look like this:



Encourage students to continue to remove rolls and/or loose pennies until they have arrived at the simplest picture, which is:

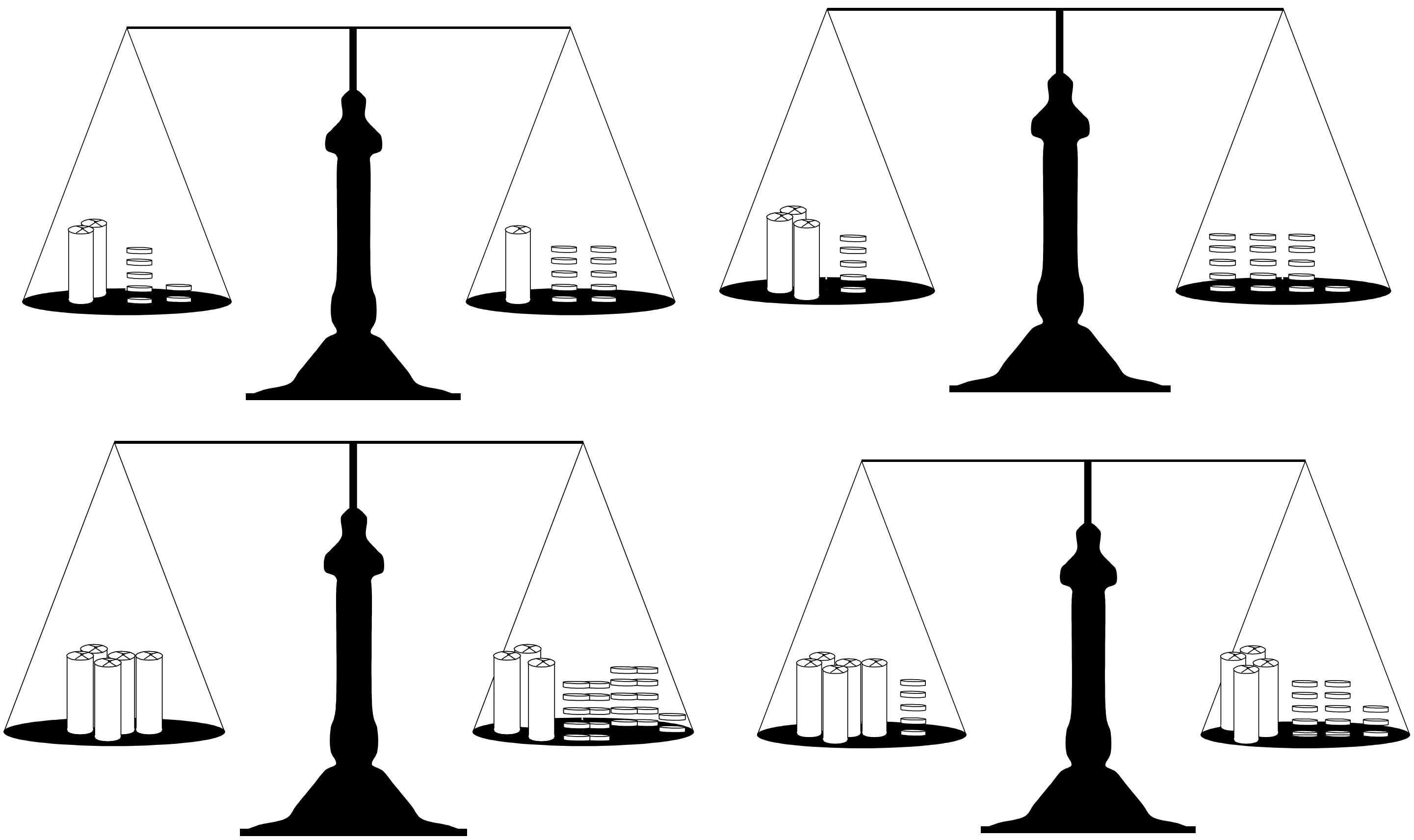


Introduce the method for recording the balance picture as an equation. For example, the balance picture from step # 5 above can be written as:

(4 x □) + 2 = (3 x □) +13

4. Record the equations that correspond to balance pictures.

Create a series of balance pictures and have students record the corresponding equation. You may want to use some of the pictures below.



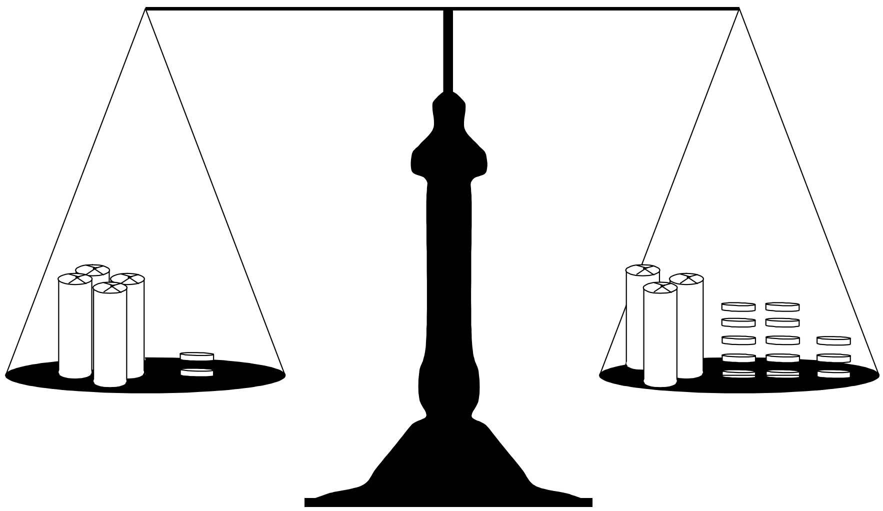
5. Draw the balance pictures that correspond to some equations.

Create some equations and have students draw the corresponding balance picture for each equation. You may want to use some of the equations below.

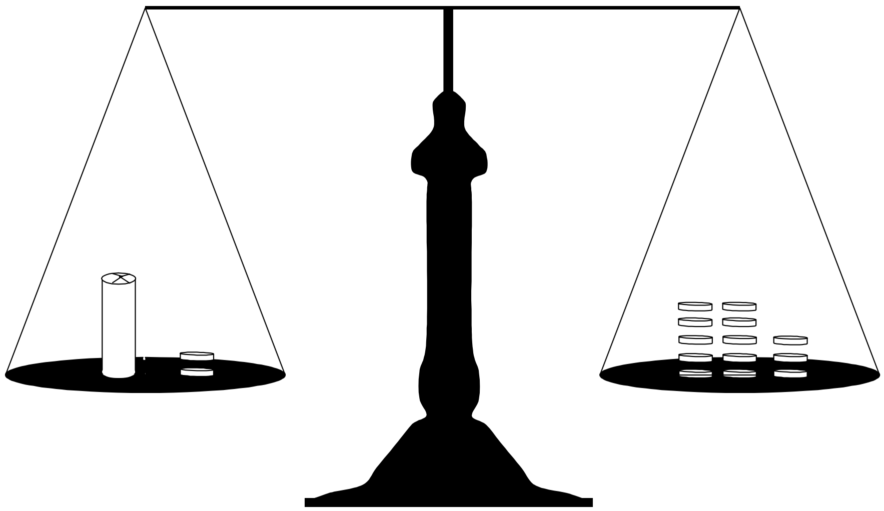
- (6 x ☐) + 4 = 16
- (8 x ☐) + 10 = 50
- (10 x ☐) + 7 = (8 x ☐) + 13
- (8 x ☐) + 115 = (5 x ☐) + 127

6. Solve the balance pictures and record steps taken with the corresponding equations.

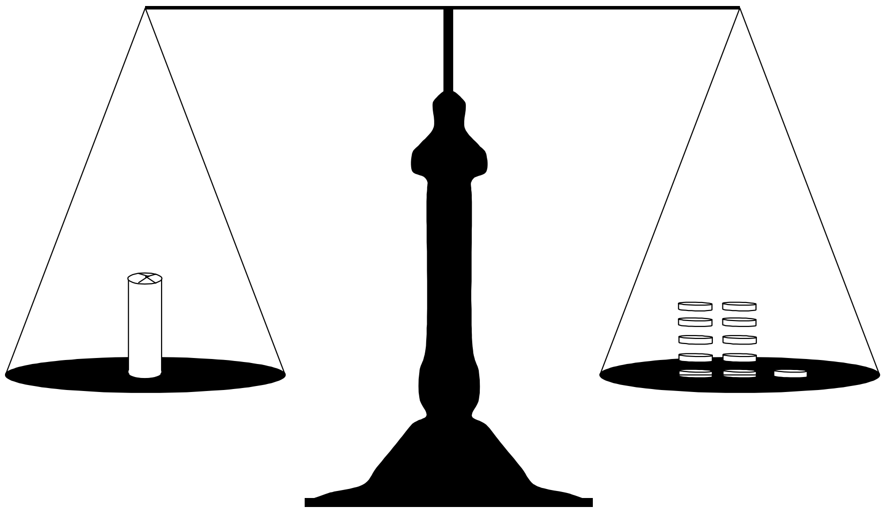
Have students use any of the balance picture/equation pairs, solve the balance picture, and record each step in a new corresponding equation. For example:

Balance Picture	Equation
	
Start with:	(4 x □) + 2 = (3 x □) + 13

☐



Remove 3 rolls from each pan: (1 x ☐) + 2 = 13



Remove 2 pennies from each pan: 1 x = 11

So, there are 11 pennies in a roll.

Check your solution in the original picture and equation.

(4 x 11) + 2 = (3 x 11) + 13

44 + 2 = 33 + 13

46 = 46

Correct!

Create more balance pictures for students to solve. For each picture, students should write an equation. Each time they remove pennies from a pan, they should rewrite the equation to match the picture. The solution should tell how many pennies are in a roll.

Extension

Have students create some pan balance problems for you to solve.

Equation Bingo

8

Overview

Mathematical Focus

- Solutions to equations

Equation Bingo is a fun way to practice solving equations.

Preparation and Materials

Before the session, gather the following materials:

- Student Page 10: Equation Pieces, with all the equation pieces cut out
- Several copies of Student Page 11: Bingo Board
- 50 small paper squares for covering numbers on the Bingo Boards

Activity

Play Equation Bingo.

Each player gets Student Page 11: Bingo Board.

Using the following numbers, players write a number in each space on their bingo boards:

–33	12	–4	10	7
–6	21	–1	0	100
–8	3	9	5	8
11	15	4	–15	–3
6	20	30	–7	50
2	–2	–9	1	–5

Place all the equation pieces in a stack, face down.

Players take turns selecting an equation piece from the top of the stack, placing the piece on the center of the table so all players can see it. All players solve the equation.

If the solution to the equation is on the Bingo Board, cover it with a small paper square.

When five spaces in a row (horizontally, vertically, or diagonally) are covered, the player shouts "BINGO!" and wins that game. The player must match his or her covered numbers with the equations that were solved.

Extension

Have students make up their own set of equations to go with the Bingo Board.

Target Numbers Cards

1

1	2	3	4	5	6	
7	8	9	10	11	12	
13	14	15	16	17	18	
19	20	21	22	23	24	25

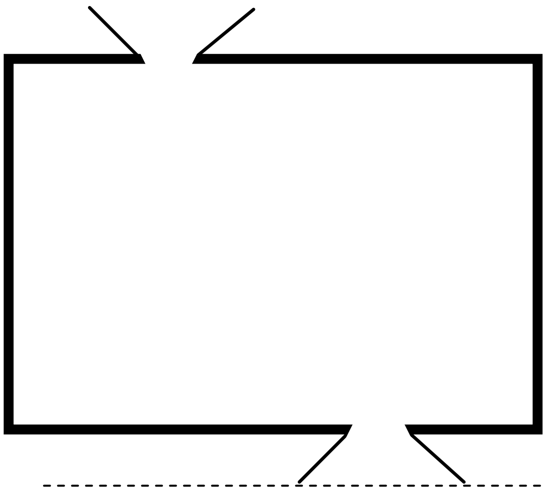
0–99 Chart

2

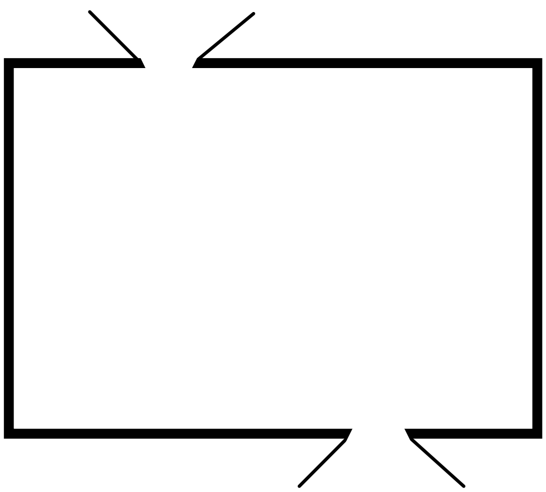
0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

3

One-Step Function Mach
Record Sheet



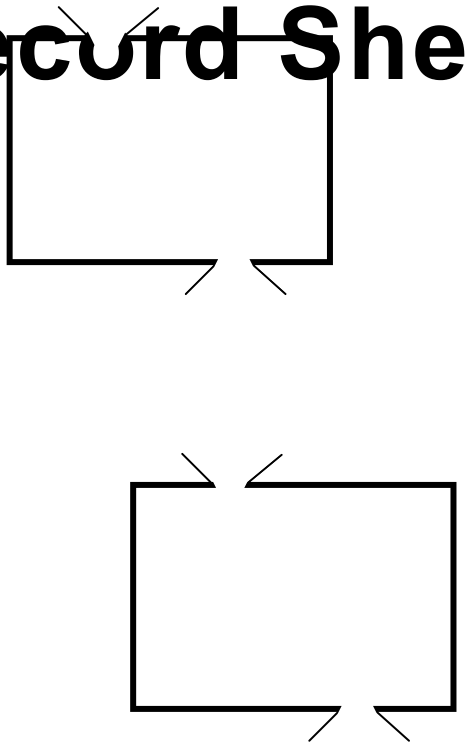
Input	Output



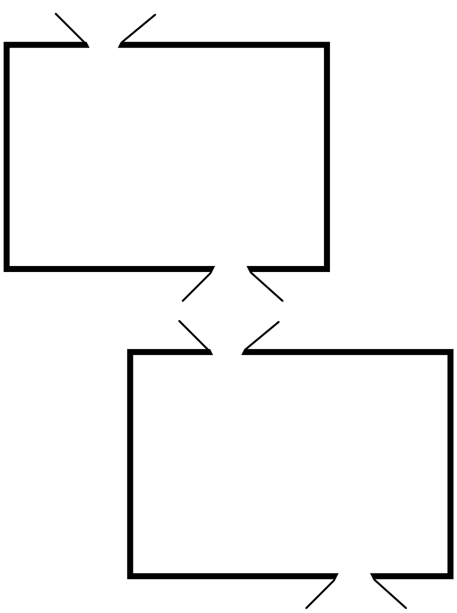
Input	Output

Two-Step Function Machine Record Sheet

4



Input	Output

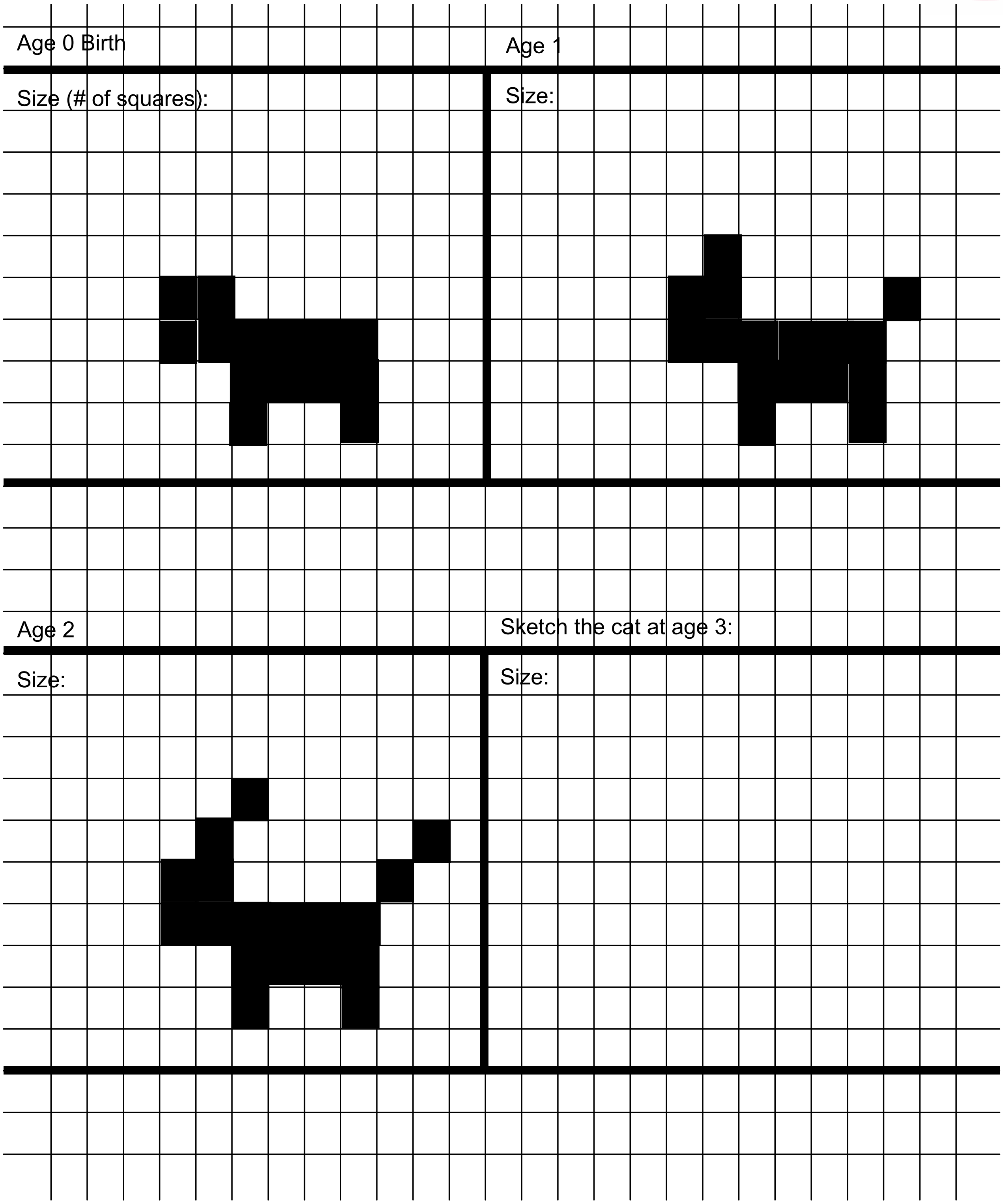


Input	Output

1-cm Grid Paper

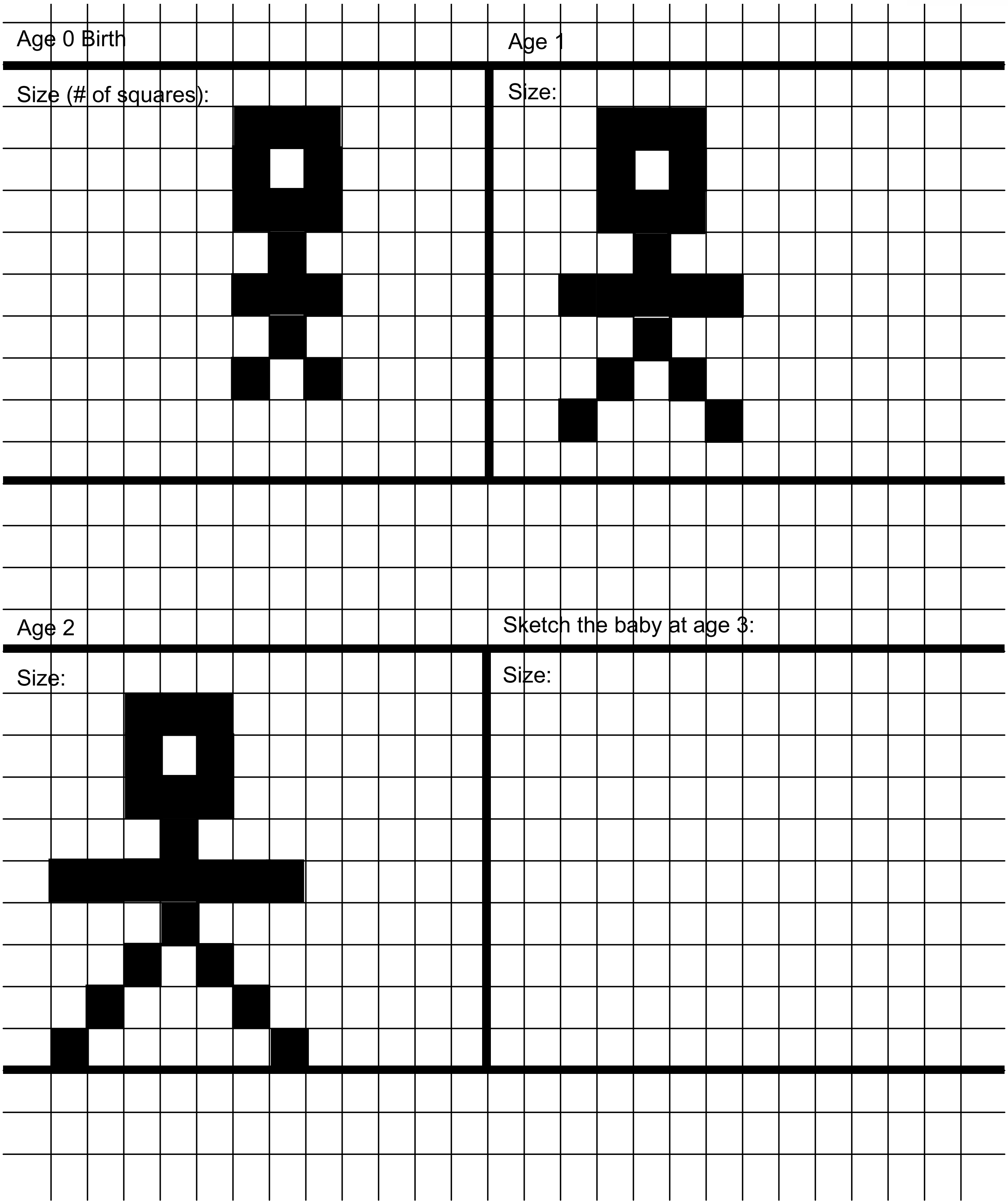
Growing Cat

6



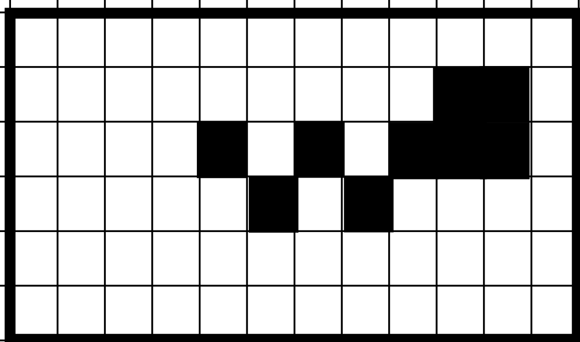
Growing Baby

7

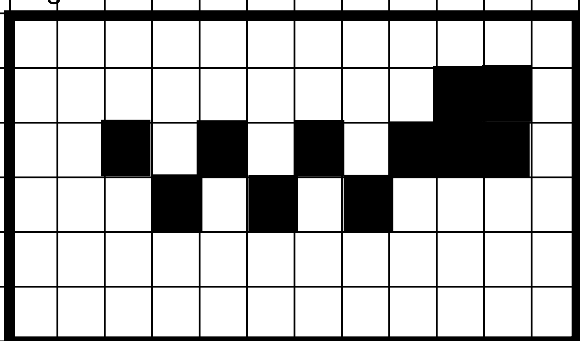


Grow-Worm

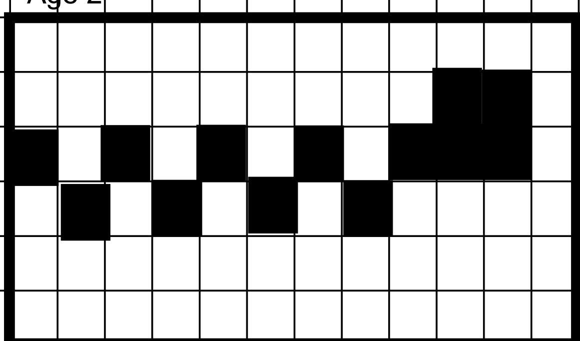
Age 0 Birth



Age 1



Age 2



Sketch the grow-worm at ages 3 and 4.

9

Input/Output Table Recording Sheet

Input	Output

Input	Output

Input	Output

Input	Output

Equation Pieces

10

$8s + 10 = 50$	$3x + 7 = x + 23$
$4p + 2 = 3p + 13$	$7n + 10 = 5n + 40$
$8y + 115 = 5y + 127$	$-6x + 10 = 4x + 40$
$-10m + 7 = 8m + 13$	$3n + 2 = 4n - 4$
$y + 9 = 3y - 1$	$-6p = 24$
$-2x + 13 = -27$	$4n = 120$
$-5 - 7c = 58$	$-8p + 3 = 59$
$4x - 100 = 100$	$5x + 15 = 15$
$20 - 5x = 10$	$17 - 4n = 24 - 6n$
$101 - 3y = 200$	$80 + 6x = 140$
$10x - 19 = 2x + 37$	$-7n + 4 = 46$
$50 - 2y = 8$	$10x + 10 = 1 + x$
$x - 66 = 34$	$5n + 22 = n - 10$
$12x + 4 = 13x + 1$	$2x = 81$

Bingo Board

11

Equation Bingo				
		FREE		