Grades 8–9

Algebra: Operations and Expressions

TABLE OF CONTENTS

Robot Game

1

Overview

Mathematical Focus

- Addition, subtraction, multiplication and division of positive and negative integers

In this activity, students explore the multiplication, division, addition and subtraction of positive and negative numbers by playing a simple game in which robots move backward and forward on a number line. They extend their understanding to division with signed numbers. Through their experience with the Robot Game, students further develop their ability to follow logical instructions.

Preparation and Materials

Before the session, gather the following materials:

- Student Page 1: Robot Game and Playing Pieces
- Student Page 2: Robot Game Spinners
- Student Page 3: Robot Game Recording Sheet, several copies
- Paper fasteners, 3
- Paper clips, 3

Cut out and assemble the Robot Game (Student Page 1) and the Robot Game Spinners (Student Page 2) ahead of time.

Notes

Encourage students to write positive and negative numbers with superscripts, $^{-}5$, i.e., rather than $^{-}5$, to distinguish between addition and subtractions signs, ($^{+}$ and $^{-}$) and positive or negative numbers.

Activity

Play the Robot Game

1. Learn about the Robot Game.

Give students the robot playing pieces and number line cut out from Student Page 1. Explain that students will play a game in which simulated robots move forward or backward some number of spaces on the game board. Use the examples below to explain how the robots work.

If the command is:	The robot moves:
FORWARD $^{+}5$	5 spaces in a *positive* direction
FORWARD $^{-}5$	5 spaces in a *negative* direction
BACKWARD $^{+}5$	5 spaces in a *negative* direction
BACKWARD $^{-}5$	5 spaces in a *positive* direction

Make sure students understand the result of each move. Ask a student to stand up and pretend that he or she is the robot. Have the student do the following:

- Walk forward $^{+}5$ steps
- Walk forward $^{-}5$ steps
- Walk backward $^{+}5$ steps
- Walk backward $^{-}5$ steps

Make sure students understand that walking forward a negative number of steps is the same as walking backward, and that walking backward a positive number of steps is the same as walking forward.

Show students the three spinners and read the rules of the game aloud:

1. Players take turns moving their robots.
2. The move depends on the three spinners. The first spinner determines whether the move is forward or backward; the second determines whether the number of steps is positive or negative; and the third determines the number of steps.
3. Record each move on Student Page 3: Robot Game Recording Sheet before moving the robot to its new position.
4. After 10 turns, the player whose robot is closest to zero wins.

2. Play the Robot Game.

Have students play the game several times. After playing a few times, encourage students to use a shorthand notation, + for forward, and – for backward.

Example:

Their initial recording will look like this:

Starting Point	Command	Movement	Finish Point
$^-7$	Forward $^-5$	5 spaces back	$^-12$
$^-12$	Backward $^-3$	3 spaces forward	$^-9$

Using the shorthand notation, their recordings will look like this:

Starting Point	Command	Movement	Finish Point
$^-7$	$+\,^-5$	$^-5$	$^-12$
$^-12$	$-\,^-3$	$^+3$	$^-9$

Suggest these variations to the students:

- Instead of starting at zero, have each player start at the same distance from zero (+10 and –10, for example). The player who stays the closest to his or her starting point (or to zero) wins.
- Play a "tournament" of 10 games. At the end of each game, count each player's score as the number of spaces between their starting and ending positions. For example, if a player started at –10 and ended at –6, the player's score is 4. After 10 games, take the average of each player's scores. The player with the lowest average score (i.e., closest to zero) wins the tournament.

3. Play advanced versions of the Robot Game that involve multiplication of signed numbers.

Have students play an advanced game by having the robot jump on the number line, which can be done by spinning the number spinner twice. Explain that the first spin is the size of each jump and the second is the number of steps for each jump. Now the students' recordings will look something like this:

Start Point	Command	Movement	Finish Point
$^-7$	Forward $2 \times {}^-5$	$^-10$	$^-17$
$^-17$	Backward $5 \times {}^-3$	$^+15$	$^-2$

When students change to the shorthand notation, their recordings will look something like this:

Starting Point	Command	Movement	Finish Point
$^-7$	$^+2 \times {}^-5$	$^-10$	$^-17$
$^-17$	$^-5 \times {}^-3$	$^+15$	$^-2$

If students are uncomfortable with the shorthand notation, remind them that $^-5 \times {}^-3$ means jump backward 5 times, 3 spaces each time. Ask them if they can explain why $^-5 \times {}^-3 = {}^+15$.

Have students try the variations from the second part of this activity, using the "advanced" commands.

4. **Solve problems involving addition, subtraction, and multiplication of signed numbers.**

Challenge students to solve problems such as the following:

$^+3 \times {}^+2 =$	$^+3 \times {}^-2=$
$^-3 \times {}^+2 =$	$^-3 \times {}^-2=$
$^-15 + {}^+3 \times {}^+7 =$	$^-15 + {}^+3 \times {}^-7 =$
$^-15 + {}^-3 \times {}^+7 =$	$^-15 + {}^-3 \times {}^-7 =$

If students are having difficulties, remind them to think about each problem as if it were a move on the game board. For example, $^-15 + {}^-3 \times {}^+7$, is the same as "Start at $^-15$ and jump backward 7 steps, 3 times." "What is $^-7 \times 3$?" [$^-21$] So, where do you end up? $^-15 + {}^-21 = {}^-36$

5. **Solve problems that include the division of signed numbers.**

Now extend the process of arithmetic with positive and negative integers so that it includes division. Ask students what they think they should do for calculations such as the following:

$^+6 \div {}^+2 =$	$^+6 \div {}^-2 =$
$^-6 \div {}^+2 =$	$^-6 \div {}^-2 =$
$^-15 + {}^+10 \div {}^+5 =$	$^-15 + {}^+10 \div {}^-5 =$
$^-15 + {}^-10 \div {}^+5 =$	$^-15 + {}^-10 \div {}^-5 =$

Teaching Tip: If the student is not sure what to do with a problem such as $^+6 \div {}^-2$ or $^-6 \div {}^-2$, remind them that any dividing by any number is the same as multiplying by its reciprocal. That is,

$6 \div 2 = 6 \times ½$, (= 3)
$6 \div {}^-2 = 6 \times {}^-1/2$ (= $^-3$)
$^-6 \div {}^-2 = {}^-6 \times {}^-1/2$ (= $^+3$).

Therefore the arithmetic rules that determine whether a product is positive or negative are the same for division as for multiplication. Just for practice, ask the student to make up and solve some division problems involving positive and negative integers.

2

Profit and Loss

Overview

Mathematical Focus

- Algorithms for the multiplication, addition, and subtraction with positive and negative numbers
- The use of a model to understand an arithmetical system

This activity extends the learning from Activity 1 by including more complex calculations. In this activity, students engage in a simple business simulation in which they are the Chief Financial Officer of Widgets, Inc. In this role, they must track the company's cash reserves and keep the rest of the company informed as to how much money they have in the bank at the end of each day. They receive and send checks and bills, and must account for the effect of each action on the company's bottom line.

Preparation and Materials

Before the session, gather these materials:

- Student Page 4: Widgets Inc.
- Student Page 5: Widgets Inc., Continued
- Student Page 6: Widgets Inc., Ledger, several copies
- Calculator

Activity

Profit and Loss at Widgets Inc.

1. Learn about the Widgets, Inc. business simulation.

In this business simulation students act as the chief financial officer of Widgets, Inc. They must keep track of the company's daily cash flow as they send and receive checks and bills. Distribute Student Page 4: Widgets Inc. and read through it with the students. Explain that for the first week they are to use the data provided on the sheet to find their profit and loss each day. For the three weeks following, you will make up the data for the company together.

2. Test your accounting skills.

Ask students to answer the questions in the Test Your Accounting Skills section on Student Page 5. (The correct answers are in italics below.)

1. Represent these transactions by arithmetic and tell how much richer or poorer you are:
   - a. Send 2 bills for $9.95 each. *[$^{-}2 \times {}^{-}9.95 = {}^{+}19.90$; $19.90 richer]*
   - b. Receive 5 bills for $20 each. *[$^{+}5 \times {}^{-}20. = {}^{-}100$; $100. poorer]*
   - c. Receive 3 checks for $17.75 each. *[$^{+}3 \times {}^{+}17.75 = {}^{+}53.25$; $53.25 richer]*
   - d. Send 4 checks for $57.50 each. *[$^{-}4 \times {}^{+}57.50 = {}^{-}230$; $230. poorer]*

2. Describe what happened in each of these transactions and tell how much richer or poorer you are:
   - a. $^{+}4 \times {}^{-}13.50$ *[Receive 4 bills for $13.50; $54. poorer]*
   - b. $^{-}2 \times {}^{+}4.75$ *[Send 2 checks for $4.75; $9.50 poorer]*
   - c. $^{+}5 \times {}^{+}15.50$ *[Receive 5 checks for $15.50; $77.50 richer]*
   - d. $^{-}2 \times {}^{-}18.50$ *[Send 2 bills for $18.50; $37.richer]*

Make sure that students understand the basic principles of these calculations before continuing, i.e.:

Receiving checks: Positive number × positive. number ⇒ Positive result

Receiving bills: Positive number × negative number ⇒ Negative result

Sending Checks: Negative number × positive number ⇒ Negative result

Sending bills: Negative number × negative number ⇒ Positive result

3. **Track cash flow for the first week.**

   Have students work through one week of the simulation, using Student Page 6: Widget, Inc. Ledger to track the cash flow for the week. Make sure students double-check each transaction and the net change for the day before recording them.

   If students are having difficulty, suggest that they make a separate table to help track each day's cash flow. They could list each type of transaction, leaving enough space to record several of each type.

   Example:

Type of Transaction	List of Transactions	Net Change
Checks Received		
Bills Received		
Checks Sent		
Bills Sent		

4. **Practice transactions some more if needed.**

   After recording one week's transactions, determine with your students whether they think they need or would like to practice some more with another week of business data. If so, you and the students should make up a list of transactions for each day; the students should then work out the profits and losses for the week.

3

The Operations Game

Overview

Mathematical Focus

- Order of operations

In this activity, students play a game with number cards in which they try to make as many different numbers as they can with the same four cards. By reorganizing the numbers and using various combinations of parentheses and arithmetic operations they can create many different numbers.

Preparation and Materials

Before the session, gather the following materials:

- Student Page 7: Order of Operations
- Index cards
- Blank piece of paper to use as a score sheet

Create 30 number cards ahead of time by cutting fifteen 3 x 5 index cards in half and writing a number from 0—9 on the blank side of each card.

Create 6 sets of operation cards ahead of time by cutting nine index cards into quarters and writing a different operation [including parantheses] on each of them, i.e., +, -, x, /, (, and).

Activity

Playing the Number and Operations Game

1. Learn about the Number and Operations Game.

Explain that the goal of the Number and Operation Game is to make as many different numbers as you can by multiplying, dividing, adding, and subtracting a set of number cards.

Distribute Student Page 7: Order of Operations. This page will help remind students of what they already know about the order of operations.

Shuffle the number cards and pick four from the top of the deck. Tell students that they must use all four number cards in combination with operation cards to make as many different numbers as they can. After forming each combination, students write down their numbers on a score sheet.

Have students start with a sample game. Ask: *If your numbers are 5, 2, 6, and 1, how many different numbers can you make?*

Examples:

$5 + 2 + 6 + 1 = 14$

$5 + 2 + 6 - 1 = 12$

$5 + 2 - 6 + 1 = 2$

$5 \times (6 + 2 + 1) = 5 \times 9 = 45$

$6 \div 5 + 2 - 1 = 6/5 + 1 =$
$1\ 1/5 + 1 = 2\ 1/5$

$6 - (5 + 2) - 1 = 6 - 8 = {}^{-}2$

$(6 - 5) \times (2 - 1) = 1 \times 1 = 1$

$(5 - 6) \times (2 - 1) = {}^{-}1 \times 1 = {}^{-}1$

2. Explore variations of the Number and Operations Game.

There are several ways to play the game. The different variations are ordered from simplest to most complex. Read through the variations and start with the one you think will be most interesting to your students. Then move to a variation your students might find more challenging. Encourage students to come up with their own variations of the game.

A. Play the game collaboratively, seeing how many different calculations you and the students can form together. Give yourselves a point for each different calculation.

B. Play the game competitively. Players write down as many calculations as they can, including the value of each one. At the end, players compare their lists. Any calculations that are on *both* lists are crossed out. Any calculations that are left after all the common ones are crossed out earn a point for that player. The player with the most points wins.

C. Play the game competitively, but only count calculations that have different numbers as their result. For example $(6 - 5) \times (2 - 1) = 1$, and (6 – 5) □ (2 – 2) = 1. These are two different calculations but they each have the same result, so in this version of the game, only one of them will count. The player with the most points wins.

D. Have each player try to find the largest number he or she can make. The player with the largest number wins.

E. Play collaboratively and try to find at least one way to calculate every number from 0 to 10, from 1 to 20, and from 1 to 100. (You could also play this variation competitively; the player who finds the most different ways wins.

F. Play collaboratively and try to find as many ways possible to make zero.

Equivalent Expressions 4

Overview

Mathematical Focus

- Equivalent forms of simple algebraic expressions

In this activity, students find and compare different algebraic rules to represent linear patterns. They begin to recognize that a visual pattern can be described by more than one algebraic rule, and that rules that describe patterns always have the same output for a given input. Students learn to show how two expressions are algebraically equivalent.
This activity has four parts.

Preparation and Materials

Before the session, gather the following materials:

- Student Pages 8, 9, and 10: Patterns and Equivalent Rules
- Toothpicks, coffee stirrers, or similar materials, 50
- Graph paper for recording patterns

Notes

This activity makes a nice follow-up to the Pattern Trains activity in the Unit on Linear Patterns. It would be appropriate to use that activity before this one, if possible.

Activity

Same Pattern, Different Rules

1 Give students a copy of Student Page 8: Patterns and Equivalent Rules. If they have already done the Pattern Trains activity in the Linear Patterns unit, explain that in this unit they will also explore algebraic rules for visual patterns, but that this time they will find and compare different rules that describe the same pattern.

Ask students to do the following:

- Describe the pattern train they see at the top of the worksheet.
- Predict what the next term of the pattern train will look like.
- Look at Billy's diagram and table and explain how Billy got his rule for the number of sticks in the nth train.
- Look at Sally's diagram and table and explain how Sally got her rule for the number of sticks in the nth train.
- Answer the questions at the bottom of the page.

(Students should be able to explain that both rules give the same result for all values of n. They may also may be able to expand the expression $4 + 3 \times (n - 1)$ to $4 + 3 \times n - 3 = 4 - 3 + 3 \times n = 1 + 3 \times n = 1 + n \times 3$.)

Note: Demonstrating that $4 + 3 \times (n - 1)$ is algebraically equivalent to $1 + n \times 3$ is more complex than it seems, as it requires knowledge of a few basic rules of arithmetic and algebra. Those who know these rules often take them for granted, but the rules may be new or unfamiliar to algebra students. If your students are not able to show the algebraic equivalence of the two expressions, you may want to review these rules with them, as they will be needed throughout this activity and the next.

- The *distributive rule for multiplication over subtraction* (or over addition) says that $a \times (b - c) = a \times b - a \times c$. Understanding of this rule is needed to say that $3 \times (n - 1) = 3 \times n - 3$.
- The *commutative rule for addition and subtraction* says that $a + b = b + a$, or $a - b = -b + a$. Understanding of this rule is needed to reverse the order of two terms so that $4 + 3 \times n - 3 = 4 - 3 + 3 \times n$.
- The *associative rule for addition and subtraction* is used to combine terms. It says that $a + (b + c) = (a + b) + c$.

Understanding of this rule is needed to say that $4 - 3 + 3 \times n = 1 + 3 \times n$.

- The *commutative rule for multiplication and division* says that $a \times b = b \times a$, and that $a \div b = 1/b \times a$. Understanding of this rule is needed to say that $3 \times n = n \times 3$.

2 Have students look at the problem on Student Page 9. This time, Billy's and Sally's rules are given, but students must determine how to draw the pattern to illustrate each rule. Give students the toothpicks, coffee stirrers, or other materials and tell them they may use them if they wish. They must also construct the tables, connecting the number of trains to the number of toothpicks (etc.) needed. Finally, students need to figure out how to convince a friend that both rules are equivalent.

3 Have students repeat the process with a slightly different pattern. This time, they must develop both rules themselves, illustrate the rules, construct the tables, and show that the two rules are equivalent. Two possible rules for this pattern are $3 + 5 \times (n - 1)$ and $5 \times n - 2$.

4 Have students construct their own patterns and work through all the steps of finding two rules and showing that they are equivalent.

Simplifying Expressions

5

Overview

Mathematical Focus

- Comparison of expressions to determine whether they are equivalent
- Use of simple algebraic operations to simplify and to complicate algebraic expressions

In this activity, students use algebraic operations to compare expressions and determine whether they are equivalent.

Preparation and Materials

Before the session, gather the following materials:

- Student Page 11: Equivalent Expressions
- Student Page 12 and 13: Simplifying and Complicating

Activity

Part 1: Which Expressions Are Equivalent?

1 Distribute Student Page 10: Equivalent Expressions. Tell students that they will start with Part A, in which they will solve a series of puzzles. Explain that for each puzzle, students may first evaluate each expression for several values of n to see if the values are the same. If two or three of the expressions have different values for a value of n, they cannot be equivalent. If students decide that two or three expressions are equivalent, they can "prove" equivalence by expanding each expression and combining terms to attempt to change one or two of the expressions look like the other. Point out that Student Page 10 also includes a list of algebraic rules that will help them simplify and complicate expressions.

Give students time to complete Part A. Encourage them to use the algebraic rules, as needed.

Solutions to Part A:

1. Answer: b and c

a: $3n + 1 = 3n + 1$;

b: $3(n + 1) = 3n + 3 \times 1 = 3n + 3$;

c: $3n + 3 = 3n + 3$

2. Answer: all three

a: $3(n - 1) = 3n - 3$;

b: $2 + 3n - 5 = 3n - 3$;

c: $3(2 + n) - 9 = 6 + 3n - 9 = 3n - 3$

3. Answer: a and b

a. $3n + 2(3 - n) - 7 = 3n + 6 - 2n - 7 = n - 1$

b. $3(n - 2) - 2(n - 1) + 3 = 3n - 6 - 2n + 2 + 3 = 3n - 2n - 6 + 5 = n - 1$

c. $3(n - 2) - 2 (n - 3) = 3n - 6 - 2n + 6 = 3n - 2n + 6 - 6 = n$

4. Answer: None

a: $3(3 - n) - 3 = 9 - 3n - 3 = 6 - 3n$;

b: $2(3 - n) - 3n = 6 - 2n - 3n = 6 - 5n$;

c. ${}^{-}n - 2(3 - 2n) = {}^{-}n - 6 + 4n = {}^{-}6 - n + 4n = {}^{-}6 + 3n$

2 Tell students that in Part B, they will make up four puzzles of their own and you will do the same. Then you and the students can swap and try to solve one another's puzzles.

Note: To help students make up puzzles, you might show them how to start with a simple expression and then "complicate" it. For example, $2n + 2$ can be complicated by adding 2 and subtracting 2: $2n + 4 - 2$. Then "factor" 2 out of the first two terms, and write $2(n + 2) - 2$. This is equivalent to $2(n + 1)$ and $2n + 2$.

To make this into a puzzle, students might want their third example to be a tricky one, one that looks like one of the others but is slightly different. For example:

a. $2(n + 1)$ b. $2(n + 2) - 2$ c. $2(n - 2) + 2$
(Only a and b are equivalent.)

OR

a. $2(n + 1)$ b. $2(n + 2) - 2$ c. $2n + 2$
(All three are equivalent.)

Have students complete Part B. Then trade puzzles with one another and try to solve them.

Part 2: Simplifying and Complicating

1 Explain to students that sometimes two expressions can be equivalent, but one will seem more complicated than the other. Write down these two expressions. Ask: *Which seems more complicated?*

$3n + 2(3 - n) - 7$ OR $n - 1$
$3(n - 2) - 2(n - 3)$ OR n

(The first expression in each is much more complicated than the second.) Explain to students that it's usually critical in algebra to make the expressions you are working with as simple as possible, as long as you can do that without changing the value of the expression, that is, by multiplying, dividing, adding, subtracting, combining terms, or looking for common factors.

Say: *Sometimes, though, it's important to be able to complicate expressions. How do you do this?* Refer students to the second example above, and explain how they could get the more complicated expression from the simpler one. Say:

- *Start with n, but write it as 3n – 2n*
- *Subtract 6 after the 3n and add 6 after the 2n to get 3n – 6 – 2n + 6.*
- *Notice that 3 is a common factor of the first two terms and –2 is a common factor of the last two terms. Use the distributive rule to rewrite the terms. Your "complicated" new expression is 3(n – 2) – 2(n – 3).*

2 Give students a copy of Student Page 9: Simplifying and Complicating. Tell students that Part 1, they will compare two columns of expressions, choose the equivalent pairs, and justify their choices. Encourage them to use the rules of algebra to decide which expression in Column A is equivalent to which expression in Column B.

If students have difficulty, suggest that they expand one of the expressions in Column A by multiplying and separating terms, then combining similar terms and simplifying the expression. They can then see which expression it matches in column B.

Solutions for Student Page 9:

A1 = B5: $3n(1 + 3/n) - 7 = 3n + 9n/n - 7 = 3n + 9 - 7 = 3n + 2$

A2 = B4: $(n + 1)(n - 3) + 2n(n - 3) = n^2 - 2n - 3 + 2n^2 - 6n =$
$n^2 + 2n^2 - 2n - 6n - 3 = 3n^2 - 8n - 3$

A3 = B1: $(n + 1)(n + 1) - 5(n + 3) = n^2 + 2n + 1 - 5n - 15 =$
$n^2 + 2n - 5n + 1 - 15 = n^2 - 3n - 14$

A4 = B6: $6(n + 3) – 3(n + 6\ 2/3) = 6n + 18 – 3n – 3 \times 20/3 =$
$6n – 3n + 18 – 20 = 3n – 2$

A5 = B2: $(n + 60)/5 – 5 = n/5 + 60/5 – 5 = n/5 + 12 – 5 = n/5 + 7$

A6 = B3: $(n + 1)(n – 1) – 2(n^2 + 1) = n^2 + n – n – 1 – 2n^2 – 2 = –n^2 – 3$

Teaching Tip

Another way to help students with these puzzles is to make up a set of cards for each one, with the steps of the solution given below. For example, problem A1 would have 4 cards, each with one of the four steps shown in the solution. The cards would all be labeled A1, but would not be numbered. Students would have to put the cards in order, and explain the algebraic operation that takes them from one card to the next.

3 If students have difficulty expanding expressions such as $(n + 1)(n – 3) = n^2 –2 n – 3$, encourage them to break the problem into three steps: $(n + 1)(n – 3) = n(n – 3) + 1(n – 3) = n^2 – 3n + n – 3 = n^2 –2n – 3$

There is a rote method for expanding products of two binomials, called the “FOIL” method: “First, Outside, Inside, and Last.” Multiply the first two terms together, then the two outside terms, then the two inside terms, and then the two last terms. Here’s how it would work for (n + 1)(n - 3):

$$(n + 1)(n – 3) = (n \times n) + (n \times – 3) + (1 \times n) + (1 \times –3)$$
$$= n^2 – 3n + n – 3 = n^2 –2n – 3$$

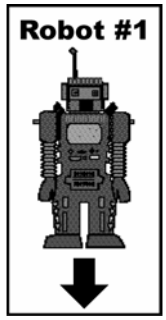
4 Tell students that in part 2 of this activity, they will make up a puzzle of their own and you will do the same. Then you and the students can swap and try to solve one another’s puzzles. Point out that two examples of puzzles are provided to help them get started.

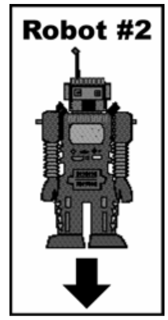
Have students complete Part 2. Then trade puzzles with one another and try to solve them.

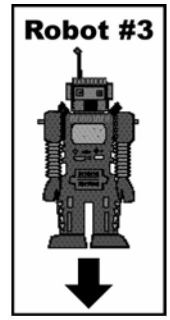
1

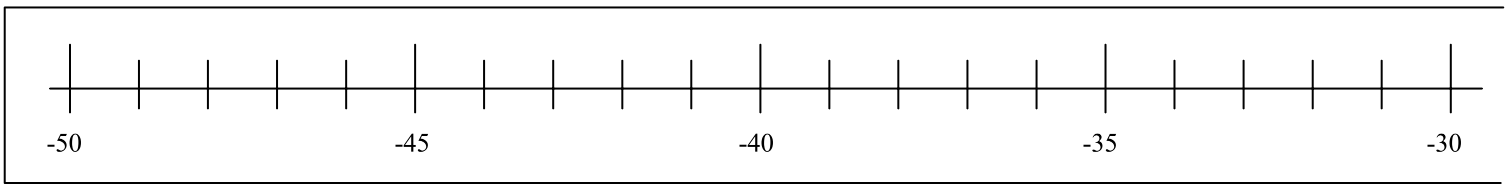
Robot Game and Playing Pieces

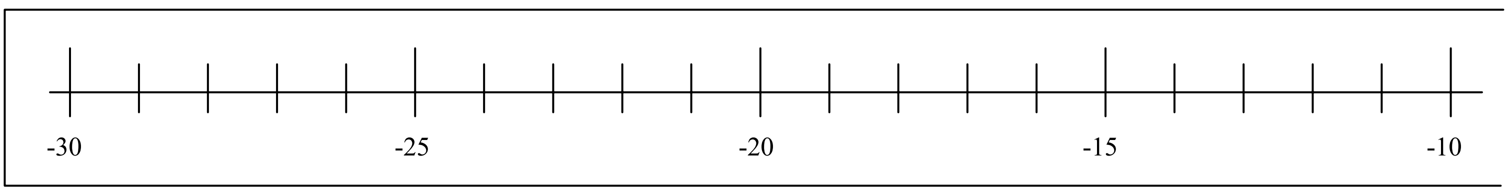
Cut out and tape these strips together, lining up the –30 from the end of the first strip on top of the –30 at the end of the second strip, and doing the same with the numbers at the ends of each strip so that you end up with a number line that goes from –50 to 50. Also, cut out the three robot playing pieces.

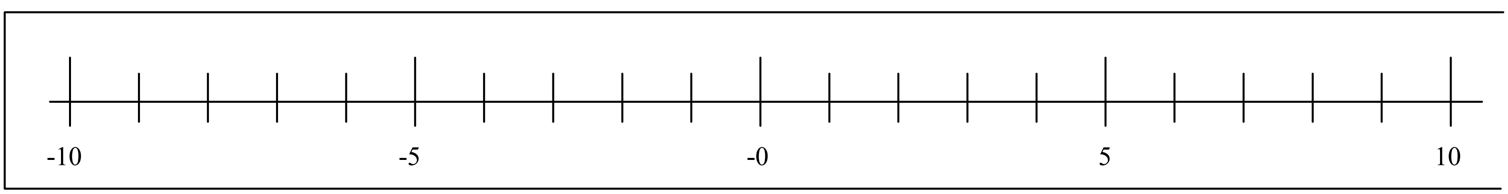


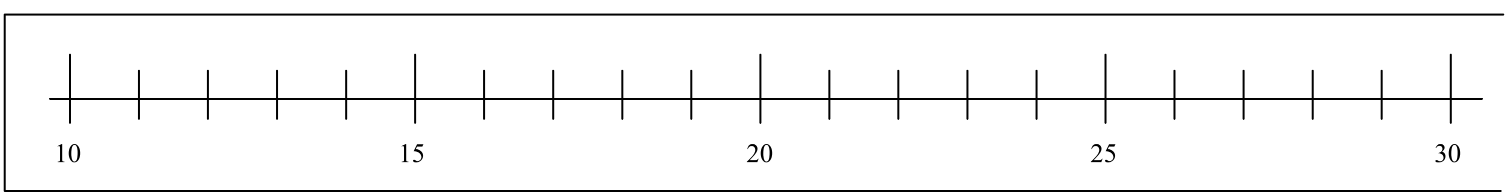


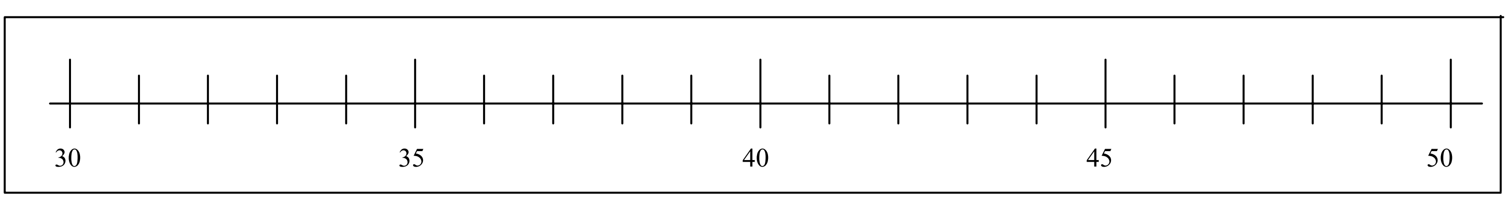










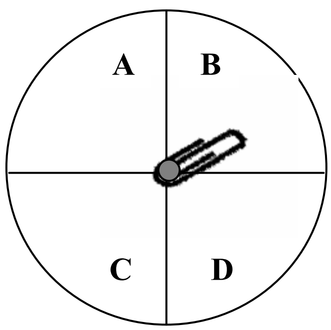


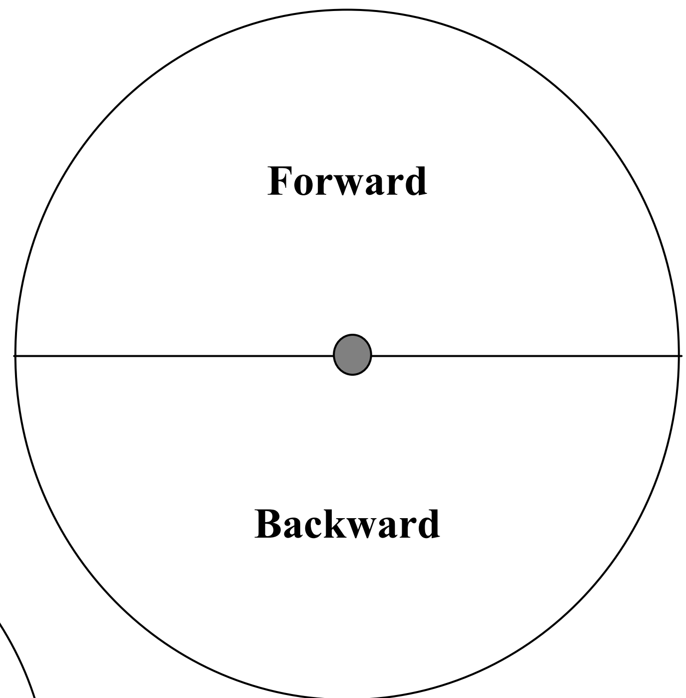
Robot Game Spinners

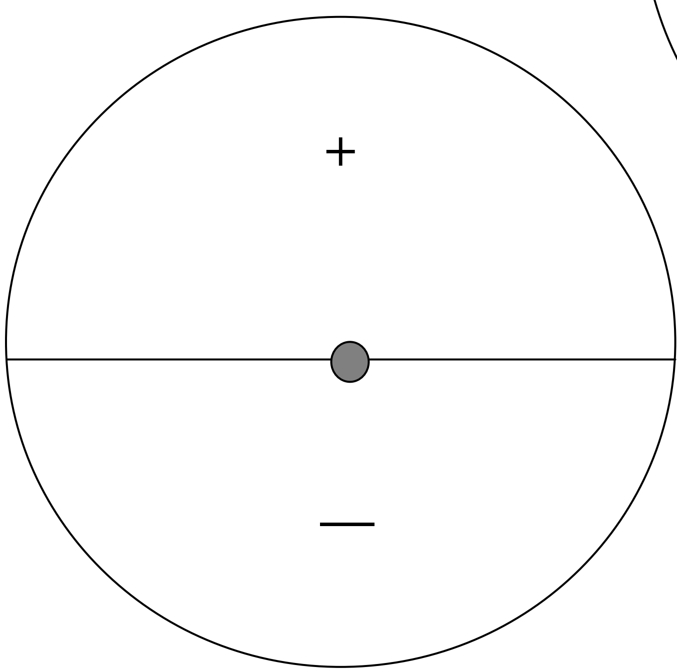
Making the Spinners:

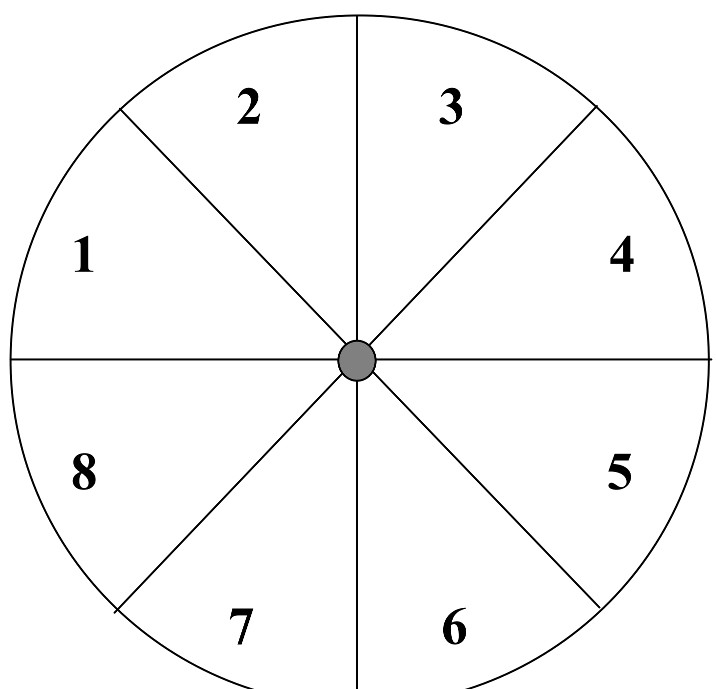
Use cardboard to make the spinners. Cut out circles. Put one end of a paperclip over the middle of each circle. Push a paper fastener through the end of the paperclip and through the center of the spinner. Close the fasteners, loosely, on the back of the spinner. (See the example, below.)

Example









Robot Game Recording Sheet

3

Move #	Starting Point	Command	Movement	Finishing Point
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Move #	Starting Point	Command	Movement	Finishing Point
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Move #	Starting Point	Command	Movement	Finishing Point
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Widgets, Inc.

4

You are the Chief Financial Officer of Widgets Inc. As such, you must keep track of the cash flow every day. The company sells widgets of different kinds and prices, and buys materials to make widgets. Your job involves sending and receiving checks and bills. All of your transactions are handled electronically which means that your account is immediately increased or decreased by the proper amount. Here are some examples of typical financial transactions:

- You receive a check when someone buys a widget.
- You receive a bill when someone sends you some materials and charges you for them electronically.
- You send a check when you order something on the Internet and pay for it ahead of time.
- You send a bill when someone orders widgets over the Internet.

Remember that all transactions are electronic and happen instantly.

Sample Transactions:

1. Suppose you receive three checks for $15.95 each. Do you have more or less money in your account? How much more or less?

[Receiving something is a *positive* transaction, represented by a + sign. A check is a positive amount of money, represented by a + sign. If you receive three checks for 15.95, you could represent the result as $^{+}3 \times {}^{+}15.95 = {}^{+}47.85$; the company has $47.85 more in the bank.]

2. Suppose you receive two bills for $35.50 each. Do you now have more or less money? How much more or less?

[A bill is a *negative* amount of money represented by a – sign. If you receive two bills for $35.50 each, you could represent the result as $^{+}2 \times -5.50 = -71$; the company has $71. less in the bank.]

3. Suppose you send five checks for $41.25 each. How much more or less do you now have in your account?

   How could you represent this transaction?

[Sending something is a *negative* transaction, represented by a – sign. If you send five checks for $41.25 each, you could represent the result as $-5 \times {}^{+}41.25 = -206.25$; the company has $206.25 less in the bank.]

4. Suppose you send two bills for $19.95 each. How much more or less do you now have in your account?

   How could you represent this transaction?

[If you send two for $19.95 each, you could represent the result as $-2 \times -19.95 = {}^{+}39.90$. The company has $39.90 more in the bank.]

Widgets, Inc. (continued)

Test your accounting skill:

1. Represent these transactions by arithmetic and tell how much richer or poorer you are:
   a. Send 2 bills for $9.95 each.
   b. Receive 5 bills for $20 each.
   c. Receive 3 checks for $17.75 each.
   d. Send 4 checks for $57.50 each.

2. Describe what happened in each of these transactions and tell how much richer or poorer you are:
   a. $^{+}4$ x $^{-}13.50$
   b. $^{-}2 \times {}^{+}4.75$
   c. $^{+}5 \times {}^{+}15.50$
   d. $^{-}2 \times {}^{-}18.50$

The first week's transactions:

Monday: Receive: 2 bills for $7.50; 3 checks for $18.95; 3 bills for $35.00; 4 checks for $9.95
Send: 3 bills for $13.50; 2 checks for $7.95; 5 checks for $15.

Tuesday: Receive: 2 checks for $18.95; 5 checks for $10.50; 4 bills for $23.50; 1 bill for $20.
Send: 4 bills for $18.95; 3 checks for $45.00; 2 bills for $13.50

Wednesday: Receive: 4 bills for $5.95; 2 bills for $54.50; 3 checks for $18.95; 3 checks for $12.50
Send: 2 checks for $36.50; 3 bills for $17.95; 2 checks for $24.

Thursday: Receive: 3 bills for $45.; 2 checks for $17.75; 5 checks for $18.95; 2 checks for $15.50
Send: 2 bills for $12.50; 4 checks for $27.75; 3 bills for $15.50

Friday: Receive: 4 checks for $7.50; 2 checks for $19.95; 3 bills for $45.
Send: 1 check for $18.; 2 bills for $25.60; 3 checks for 10.; 4 bills for $18.50

6

Widgets, Inc. Ledger

Day	Start of Day: Cash on Hand	Record of Each Transaction	Net Change	End of Day: Cash on Hand
Monday	$1,000.			
Tuesday				
Wednesda y				
Thursday				
Friday				

7

Order of Operations

This information will help you play the Number and Operations Game.

1. When you evaluate an arithmetical expression, always multiply or divide first, then add or subtract.

   Examples:

   a) $3 \times 5 – 4 = 15 – 4 = 11$
   b) $7 - 8 \div 4 + 5 = 7 – 2 + 5 = 10$

2. If an expression includes parentheses, always carry out the steps in the parentheses first, then multiply or divide, and then add or subtract.

   Examples:

   a) $3 \times (5 – 4) = 3 \times 1 = 3$
   b) $(6 \text{ x } 5) + 3 – 2 = 30 + 3 – 2 = 31$

3. Changing the position of the parentheses in an expression can change the outcome of a calculation.

   Examples:

   a) $(7 - 8) \div (4 + 5) = {}^{-}1 \div 9 = {}^{-}1/9$
   b) $(7 – 8) \div 4 + 5 = {}^{-}1 \div 4 + 5 = {}^{-}1/4 + 5 = 4\ 3/4$
   c) $7 – 8 \div (4 + 5) = 7 – 8/9 = 6\ 1/9$

4. If an expression includes parentheses inside other parentheses, carry out the steps in the inner parentheses first.

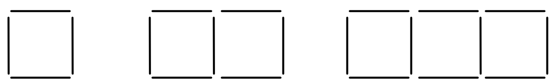
   Examples:

   a) $4 \times (3 – 2 \times [4 – 1]) = 4 \times (3 – 2 \times 3) = 4 \times (3 – 6) = 4 \times {}^{-}3 = –12$
   b) $4 \times (3 – [2 \times 4 – 1]) = 4 \times (3 – [8 – 1]) = 4 \times (3 – 7) = 4 \times {}^{-}4 = –16$

8

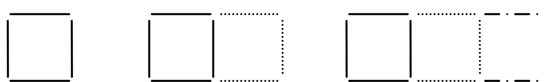
Patterns and Equivalent Rules I

Study this example and answer the questions.



Term 1 Term 2 Term 3

1. Billy said, "I started with four sticks for the first square. Then I added three more sticks each time I added a new square."



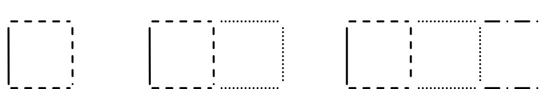
Term 1 Term 2 Term 3

Billy made this table to record his results:

Term Number	Number of Sticks
1	4
2	4 + 3 = 7
3	4 + 3 + 3 = 10
4	4 + (3 x 3) = 13
5	4 + (3 x 4) = 16
6	4 + (3 x 5) = 19
n	+ 3 x (n – 1)

2. Do you agree that Billy wrote a correct rule for this pattern?

Sally said, "I used three new sticks each time—but the first time, I added one more stick on the end."



Term 1 Term 2 Term 3

Patterns and Equivalent Rules II

9

Sally made this table to record her results:

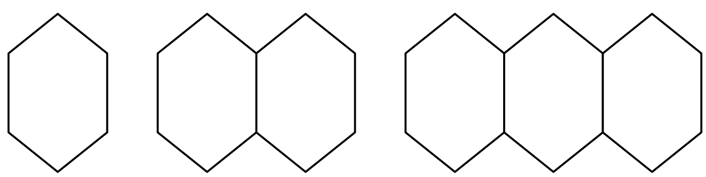
Chain Number	Number of Sticks
1	1 + 3 = 4
2	1 + 3 + 3 = 7
3	1 + (3 x 3) = 10
4	1 + (4 x 3) = 13
5	1 + (5 x 3) = 16
6	1 + (6 x 3) = 19
n	1 + (n x 3)

3. Answer these questions:

   a) Are Sally and Billy both right? Explain why or why not.

   b) Is this equation always true for every possible value of n? How can you be sure?
   $4 + 3 \times (n - 1) = 1 + 3 \times n$

4. Study this pattern chain of Hexagons:



Term 1 Term 2 Term 3

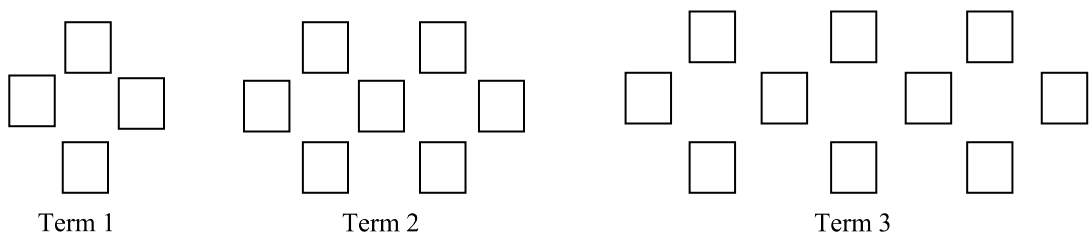
5. Build or draw the next three trains in the pattern.

Billy came up with this rule for the number of sticks in a hexagon chain that is n sticks long: $6 + 5 \times (n - 1)$. Sally came up with this rule: $5 \times n + 1$.

a. sketch the first six trains in a way that illustrates how Sally was thinking.
b. Make a table showing the number of sticks used for each of the first six trains in the pattern.
c. Which rule most accurately describes the number of sticks in a chain: Billy's, Sally's, or both of them?
d. How could you convince a friend that Billy's and Sally's rules are equivalent (i.e., give the same result for any value of n)?

Patterns and Equivalent Rules III

6. Study this tile pattern:



a. **Draw the next three parts of the tile pattern.**
b. **Make a table comparing the pattern number to the number of tiles for the first six parts of the pattern.**
c. **Find two different rules to describe this pattern.**
d. **Use drawings to show how where the rule comes from.**
e. **Show that both your rules are the same.**

7. Make up a growing pattern of your own.

a. **Draw the first six parts of the pattern.**
b. **Make a table comparing the pattern number to the number of objects used for the first six parts of the pattern.**
c. **Find two different rules to describe your pattern.**
d. **Use drawings to show where each rule comes from.**
e. **Show that both your rules are the same.**

11

Equivalent Expressions

The following algebraic rules can help you in this activity.

- The *distributive rule for multiplication over subtraction* (or over addition) says that a(b – c) = ab – ac and that a(b + c) = ab + ac. The distributive rule is also used to write the product of two binomial factors: (a + b) × (c + d) = ac + ad + bc + bd.
- The *commutative rule for addition and subtraction* says that a + b = b + a, or a – b = –b + a.
- The *associative rule for addition and subtraction* says that a + (b + c) = (a + b) + c.
- The *commutative rule for multiplication and division* says that a × b = b × a, and that a ÷ b = 1/b × a. **CAREFUL: This rule does NOT say that a ÷ b = b ÷ a!**

Part A. **For each of the following groups of three expressions, decide whether all three are equivalent, two are equivalent, or none are equivalent. Use algebra to justify your answer.**

			Answers:
1. a. 3n + 1	b. 3(n + 1)	c. 3n + 3	___________
2. a. 3(n – 1)	b. 2 + 3n – 5	c. 3(2 + n) – 9	___________
3. a. 3n + 2(3 – n) – 7	b. 3(n – 2) – 2(n –1) + 3;	c. 3(n – 2) – 2 (n – 3)	___________
4. **a. 3(3 - n) -3**	**b. 2(3 - n) - 3n**	**c. n - 2(3 - 2n)**	___________

Part B. Make up four more equivalent expression puzzles of your own.

1.

2.

3.

4.

Simplifying And Complicating

12

Part 1. Each of the expressions in Column A has a simpler equivalent expression in Column B. List the correct pairs below. Be prepared to justify your choice by using the rules of algebra to show how you can transform one expression into the other.

A	B	CORRECT PAIRS
A1. $3n(1 + 3/n) – 7$	**B1. $n^2 – 3n – 14$**	**A1.** _______
A2. $(n + 1) (n – 3) + 2n (n – 3)$	**B2. $n/5 + 7$**	**A2.** _______
A3. $(n + 1) (n + 1) – 5(n + 3)$	**B3. $–n^2 – 3$**	**A3.** _______
A4. $6(n + 3) – 3(n + 6\ 2/3)$	**B4. $3n^2 – 8n – 3$**	**A4.** _______
A5. $(n + 60)/5 – 5$	**B5. $3n + 2$**	**A5.** _______
A6. $(n + 1) (n – 1) – 2(n^2 + 1)$	**B6. $3n – 2$**	A6. _______

13

Simplifying And Complicating (continued)

Part 2. Make up a set of six equivalent expressions in both complicated and simple forms. Arrange them into a puzzle for someone else to solve. Look at the two examples below to help you get started.

Example 1: Start with a simple expression and make it more complex.

1. Start with something simple like 2n – 5
2. Add and subtract 2n: 4n – 5 – 2n
3. Add and subtract 8: 4n – 8 – 5 + 8 – 2n
4. Use the distributive rule to replace 4n – 8 with 4(n – 2): 4(n – 2) – 5 + 8 – 2n
5. Combine – 5 and + 8: 4(n – 2) + 3 – 2n
6. Check that 2n – 5 is equivalent to 4(n – 2) + 3 – 2n

Example 2: Start with a quadratic expression in factored form and simplify it.

1. Start with something like (n + 3)(n + 3) – (n + 1)(n + 1)
2. Multiply all the factors: $(n^2 + 6n + 9) – (n^2 + 2n + 1)$
2. Rearrange the expression so that you can combine similar terms: $n^2 – n^2 + 6n – 2n + 9 – 1$
4. Combine similar terms: 4n + 8
5. Use the distributive rule to replace 4n + 8 with 2(n + 4)
6. Check that (n + 3)(n + 3) – (n + 1)(n + 1) is equivalent to 2(n + 4)