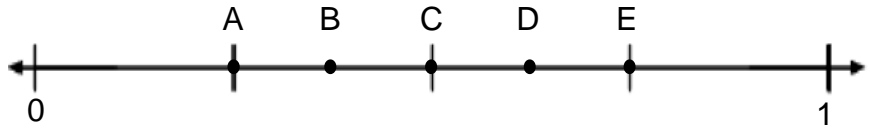


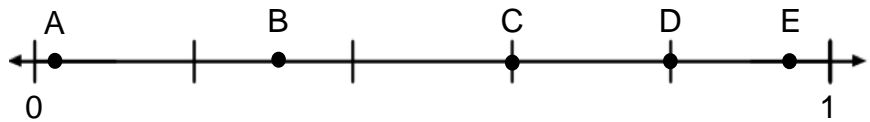
Determine which point shows the location of the fraction.

1. Choose the letter that shows the location of $\frac{3}{8}$
- A
 - B
 - C
 - D
 - E



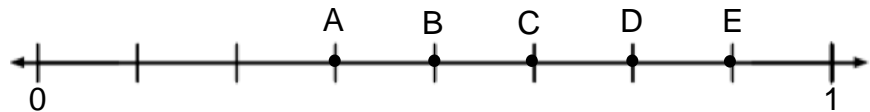
Explain why you chose this location:

2. Choose the letter that shows the location of $\frac{3}{10}$
- A
 - B
 - C
 - D
 - E



Explain why you chose this location:

3. Choose the letter that shows the location of $\frac{3}{4}$
- A
 - B
 - C
 - D
 - E



Explain why you chose this location:

Resource for Analysis of Probe #5: Locating Fractions on a Number Line

Understandings and Successful Approaches

Correct Answers:

<p>1. Choose the letter that shows the location of $\frac{3}{8}$</p>	<input type="checkbox"/> A <input checked="" type="checkbox"/> B <input type="checkbox"/> C <input type="checkbox"/> D <input type="checkbox"/> E	
<p>2. Choose the letter that shows the location of $\frac{3}{10}$</p>	<input type="checkbox"/> A <input checked="" type="checkbox"/> B <input type="checkbox"/> C <input type="checkbox"/> D <input type="checkbox"/> E	
<p>3. Choose the letter that shows the location of $\frac{3}{4}$</p>	<input type="checkbox"/> A <input type="checkbox"/> B <input type="checkbox"/> C <input checked="" type="checkbox"/> D <input type="checkbox"/> E	

Students who choose each of these correct answers and have explanations that support their choices may be applying the following:

- understanding that the points represents numbers and that each has one discrete location on a number line;
- ability to determine the whole based on the labels provided (i.e. the interval from 0 to 1 or 2 to 3 or $\frac{1}{4}$ to $1\frac{1}{4}$); and
- ability to reason about the size of the intervals of the whole, recognizing that these intervals must be of equal size.

Potential Common Misunderstandings/Misconceptions to Look For

A mixture of correct and incorrect choices may reveal a variety of misunderstandings related to locating fractions on a number line.

“Incorrect Counting” Misconception

Incorrect Counting Answer Pattern: 1)C or E, 2)B or C, 3)A

- **Counting On:** Students with this misconception often apply whole number reasoning by simply using the value of the numerator to count on from the starting point without considering which fractions the hash marks represent or the size of the intervals between them. Student may or may not count the points, instead focusing only on the hash marks.

a. Counting On: Points and Hash Marks

<p>1. Choose the letter that shows the location of $\frac{3}{8}$</p>	<input type="checkbox"/> A <input type="checkbox"/> B <input checked="" type="checkbox"/> C <input type="checkbox"/> D <input type="checkbox"/> E	
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b. Counting On: Only Hash Marks

<p>3. Choose the letter that shows the location of $\frac{3}{4}$</p>	<input checked="" type="checkbox"/> A <input type="checkbox"/> B <input type="checkbox"/> C <input type="checkbox"/> D <input type="checkbox"/> E	
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Resource for Analysis of Probe #5: Locating Fractions on a Number Line

“Incorrect Counting” Misconception, continued.

- **Other Incorrect Counting:** Student may also apply other incorrect counting strategies resulting in a different answer pattern.
 - Counting All: including the starting hash mark in their count
 - Counting Back: starting with 1 and counting backwards

“Incorrect Use of Benchmarks” Misconception

Answer Pattern: Answers will vary

- Students may apply a variety of incorrect reasoning approaches including:
 - **Incorrectly positioning the fraction in relation to a benchmark**, such as 0, $\frac{1}{2}$ or 1
For example, a student correctly locates the benchmark $\frac{1}{2}$ and writes that $\frac{3}{8}$ is close to $\frac{1}{2}$. However, he incorrectly chooses a point that is *too close* to $\frac{1}{2}$.
 - **Incorrectly locating a benchmark which leads to incorrect positioning of the fraction**
For example, a student correctly writes that $\frac{3}{8}$ is close to the benchmark $\frac{1}{2}$. However, she makes an incorrect assumption that $\frac{1}{2}$ is the middle of a 0-2 number line. This error leads to the student placing $\frac{3}{8}$ in the incorrect position.
 - **Determining the size of a fraction incorrectly based on a benchmark**
For example, a student knows that $\frac{3}{8}$ is close to the benchmark $\frac{1}{2}$ but incorrectly thinks that $\frac{3}{8}$ is larger than $\frac{1}{2}$.

For each problem, review the answers and the example approach to solving the problem. Consider how the approaches are similar and/or different from the approach you took.

<p>1 Choose the letter that shows the location of $\frac{3}{8}$.</p>	<input type="checkbox"/> A <input checked="" type="checkbox"/> B <input type="checkbox"/> C <input type="checkbox"/> D <input type="checkbox"/> E	
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Example: *Counting* Approach to Finding the Answer to Problem 1

Resource for Analysis of Probe #5: Locating Fractions on a Number Line

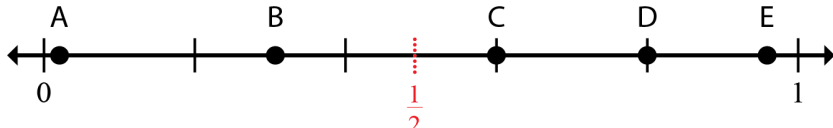
<p><i>Determine the whole and size of the intervals:</i> The original number line goes from 0-1. It is partitioned into four equal parts so each interval between hashmarks is a fourth.</p>	
<p><i>Locate the target fraction:</i> Since the target fraction ($\frac{3}{8}$) is in eighths, split each of the fourths in half to create eighths. Count and label the hashmarks.</p>	

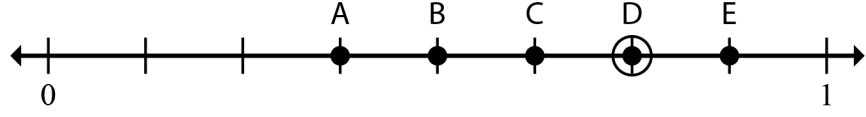
<p>2. Choose the letter that shows the location of $\frac{3}{10}$</p>	<div style="display: flex; align-items: center; gap: 20px;"> <div style="display: flex; flex-direction: column; gap: 5px;"> <input type="checkbox"/> A <input checked="" type="checkbox"/> B <input type="checkbox"/> C <input type="checkbox"/> D <input type="checkbox"/> E </div> <div style="text-align: center;"> <p>$< \frac{1}{10}$</p> <p>A</p> <p>$\frac{3}{10}$</p> <p>C</p> <p>D</p> <p>E</p> <p>$\approx \frac{9}{10}$</p> </div> </div>
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Example: Benchmarking/Reasoning Approach to Finding the Answer to Problem 2

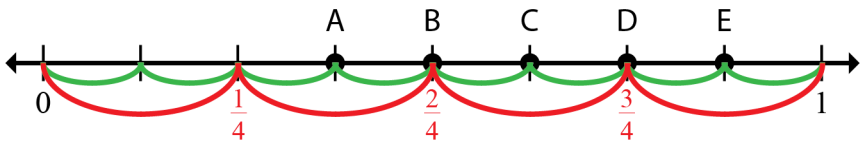
<p><i>Determine the whole and size of the intervals:</i> One whole unit on the number line is the distance from 0-1. A whole unit is partitioned into five equal parts so each interval between hashmarks is a fifth.</p>	
<p><i>Locate helpful</i></p>	

Resource for Analysis of Probe #5: Locating Fractions on a Number Line

<p><i>benchmarks:</i> Since $\frac{3}{10}$ is less than $\frac{1}{2}$, start by finding $\frac{1}{2}$ on the number line. Two points (A and B) are less than $\frac{1}{2}$. Since $\frac{3}{10}$ is closer to $\frac{1}{2}$ than to 0, B is the answer.</p>	 <p style="text-align: center;">A number line from 0 to 1. Points A, B, C, D, and E are marked. A vertical dashed red line is drawn at $\frac{1}{2}$.</p>
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<p>3. Choose the letter that shows the location of $\frac{3}{4}$</p>	<p> <input type="checkbox"/> A <input type="checkbox"/> B <input type="checkbox"/> C <input checked="" type="checkbox"/> D <input type="checkbox"/> E </p>	 <p style="text-align: center;">A number line from 0 to 1 with points A, B, C, D, and E. Point D is circled.</p>
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Example: *Counting Approach to Finding the Answer to Problem 3*

<p><i>Locate the target fraction:</i> Since the target fraction ($\frac{3}{4}$) is in fourths and the number line is partitioned into eighths, express the eighths as fourths by combining two eighths to make a fourth. Label the hashmarks and see that D is $\frac{3}{4}$.</p>	 <p style="text-align: center;">A number line from 0 to 1 with points A, B, C, D, and E. Red arcs show $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$. Green arcs show $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, and $\frac{8}{8}$.</p>
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