

Guide for Probe: Comparing Fractions

I. Understandings and Successful Approaches

Correct Answers:

1. $\frac{1}{7}$ is greater than $\frac{1}{10}$	5. $\frac{4}{5}$ is greater than $\frac{1}{5}$
2. $\frac{6}{8}$ is equivalent to $\frac{3}{4}$	6. $\frac{5}{7}$ is less than $\frac{7}{9}$
3. $\frac{4}{5}$ is less than $\frac{6}{7}$	7. $\frac{1}{3}$ is less than $\frac{7}{12}$
4. $\frac{5}{12}$ is less than $\frac{3}{4}$	8. $\frac{4}{6}$ is equivalent to $\frac{2}{3}$

Students who choose each of these correct answers and have explanations that support their choices are applying one or more successful strategies including:

- reasoning about the size of the fractions (i.e. use of benchmarks, equivalent fractions, etc.);
- drawing and/or reasoning about models/visual representations;
- reasoning about the distance or amount missing for the whole;
- use of common denominators; or
- conversions to decimals or percents

Students with a deep and flexible understanding of fractions will chose a strategy based on the specific fractions in an item rather than applying the same strategy across all items.

II. Potential Common Misunderstandings/Misconceptions to Look For

A mixture of correct and incorrect choices may reveal a variety of misunderstandings related to comparing fractions. It is important to note that each set of answers below includes one or more correct responses as sometimes the correct answer can result from incorrect reasoning. (Correct answers are underlined below.)

1. “Whole Number Thinking” Misconception

Answer Pattern: 1) less than, 2) greater than, 3) less than, 4) greater than, 5) greater than, 6) less than, 7) less than, and 8) greater than

- Students with this misconception apply whole number reasoning and make their decision based on which number in the denominator is greater.

4.	Explain your choice using words and/or pictures
<input checked="" type="checkbox"/> Greater than (>) <input type="checkbox"/> Less than (<) <input type="checkbox"/> Equivalent (=)	$\frac{5}{12}$ $\frac{3}{4}$ $\frac{5}{12}$ has a bigger denominator

2. “Smaller is always bigger” Misconception

Answer Pattern: 1) greater than, 2) less than, 3) greater than, 4) less than, 5) less than, 6) greater than, 7) greater than, and 8) less than

- These students have overgeneralized the concept that “the larger the value of the denominator the smaller the piece” to all cases without consideration of the numerator (the number of pieces).

4.	Explain your choice using words and/or pictures
<input checked="" type="checkbox"/> Greater than (>) <input type="checkbox"/> Less than (<) <input type="checkbox"/> Equivalent (=)	$\frac{4}{5}$ $\frac{6}{7}$ Really my explanation is the same for all of them small numbers are bigger in fractions.

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3. Incorrect “Gap” Reasoning Misconception

Answer Pattern: 1) less than, 2) greater than, 3) equivalent, 4) less than, 5) less than, 6) equivalent, 7) less than, and 8) greater than

- These students either compare the difference between the numerators and denominators or compare the number of pieces needed to make a whole without considering the size of the pieces.

Example a) Compares the differences between the numerators and denominators.

<p>2.</p> <div style="display: flex; align-items: center; justify-content: space-between;"> <div style="text-align: center;"> $\frac{4}{5}$ </div> <div style="text-align: center;"> <input type="radio"/> Greater than (>) <input checked="" type="radio"/> Less than (<) <input type="radio"/> Equivalent (=) </div> <div style="text-align: center;"> $\frac{6}{7}$ </div> </div>	<p>Explain your choice using words and/or pictures.</p> <div style="text-align: center;"> <p style="text-align: center;">Both are 1 part</p> </div>
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Example b) Compares the # of pieces needed to make a whole without considering the size of the pieces.

<p>6.</p> <div style="display: flex; align-items: center; justify-content: space-between;"> <div style="text-align: center;"> $\frac{7}{8}$ </div> <div style="text-align: center;"> <input type="radio"/> Greater than (>) <input checked="" type="radio"/> Less than (<) <input type="radio"/> Equivalent (=) </div> <div style="text-align: center;"> $\frac{5}{6}$ </div> </div>	<p>Explain your choice using words and/or pictures.</p> <p style="font-family: cursive;">I picked less than because $\frac{5}{6}$ is closer to 1 than $\frac{7}{8}$.</p>
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4. Other difficulties: Students may use a variety of other incorrect approaches, including drawing models or visuals incorrectly, making calculation errors, and confusing the greater than and less than signs.

Answer Pattern: Responses will vary

Example a) Draws different size wholes

<p>2.</p> <div style="display: flex; align-items: center; justify-content: space-between;"> <div style="text-align: center;"> $\frac{4}{5}$ </div> <div style="text-align: center;"> <input checked="" type="radio"/> Greater than (>) <input type="radio"/> Less than (<) <input type="radio"/> Equivalent (=) </div> <div style="text-align: center;"> $\frac{6}{7}$ </div> </div>	<p>Explain your choice using words and/or pictures.</p> <div style="text-align: center;"> </div>
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Example b) Calculation errors

<p>2.</p> <div style="display: flex; align-items: center; justify-content: space-between;"> <div style="text-align: center;"> $\frac{4}{5}$ </div> <div style="text-align: center;"> <input checked="" type="radio"/> Greater than (>) <input type="radio"/> Less than (<) <input type="radio"/> Equivalent (=) </div> <div style="text-align: center;"> $\frac{6}{7}$ </div> </div>	<p>Explain your choice using words and/or pictures.</p> <div style="text-align: center;"> <p> $5 \times 7 = 35$ $4 \times 7 = 28$ $7 \times 5 = 35$ $6 \times 4 = 24$ </p> <p> $\frac{28}{35}$ greater and $\frac{24}{35}$ lesser $(\frac{4}{5})$ </p> </div>
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