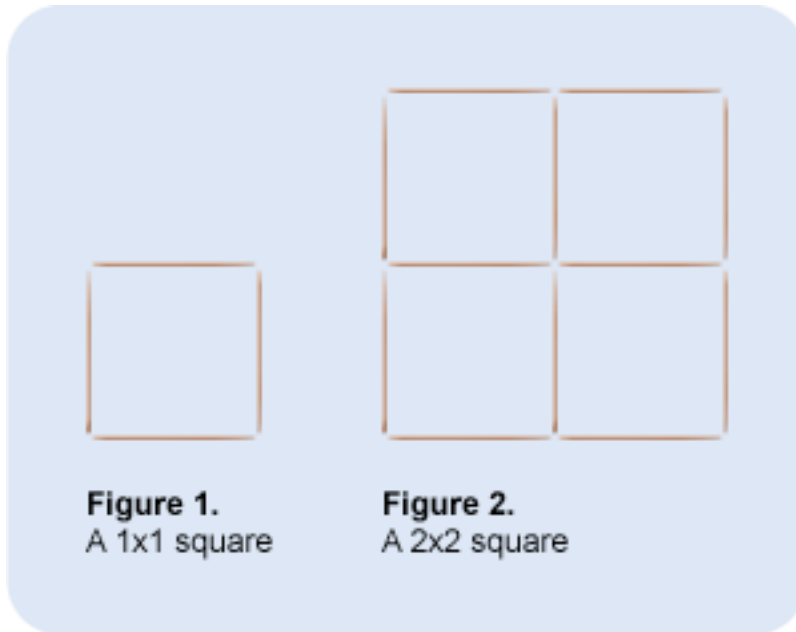


How Many Toothpicks?¹

Below are the first two figures in a sequence of squares built with toothpicks:



1. Find and record the number of toothpicks needed to build the first three figures in the sequence.
2. Continue to build or draw subsequent squares in the sequence (as many as you need) in order to find a pattern that can tell you the number of toothpicks required for each square.

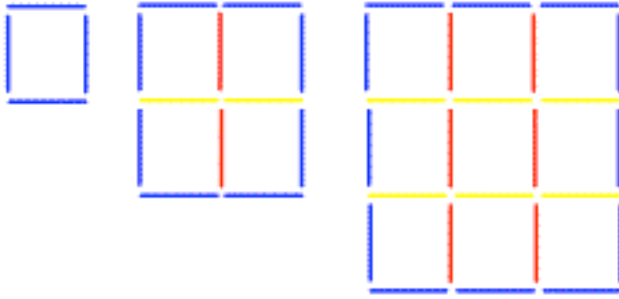
¹ Adapted from the *Seeing Math Secondary* materials, developed by the Concord Consortium and distributed by PBS TeacherLine.

Optional: Further scaffolding for entering the How Many Toothpicks? problem

If, after making a good effort to grapple with the initial framing of the problem, you and your group determine that some additional structure would be helpful, the following set of directions and questions can guide you to organize your thinking around one approach to the task (which is actually an approach taken by one group of the students we will view in the video, later in this session).

1. Use toothpicks to build the next figure in the pattern (a 3 x 3 square). Then draw Figure 3 on grid paper to keep track of your data.
2. Find and record the number of toothpicks in the perimeter of the first three figures in the pattern. How would you predict the number of toothpicks in the perimeter of an 8 x 8 square? Write an algebraic rule for determining the number of toothpicks for any $n \times n$ square.
3. Find and record the number of toothpicks in the interior of the first three figures in the pattern. How would you predict the number of toothpicks in the interior of an 8 x 8 square. Does this help you predict the number of toothpicks for any $n \times n$ square? How might you write this as a rule?
4. Predict the *total* number of toothpicks in a 4 x 4 square. A 5 x 5 square. Record your predictions in a table. Then, draw the squares on grid paper and check if your predictions are correct. Label them Figure 4 and Figure 5.
5. How would you determine the *total* number of toothpicks in an 8 x 8 square?
6. How would you determine the total number of toothpicks for any $n \times n$ square?

Student A's approach



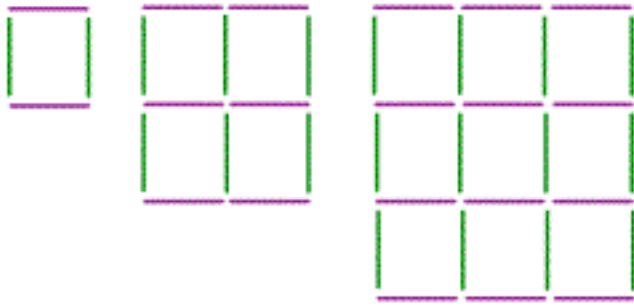
How is this student dividing up her toothpicks?

How does she use these patterns to help her find a pattern for the number of toothpicks in each of these figures?

Describe, in words, how the number of toothpicks grows.

Make use of Student A's approach to write a rule for the number of toothpicks in any figure. (It should simplify to the rule you found already, but it might not look the same initially)

Student B's approach



How is his student dividing up his toothpicks?

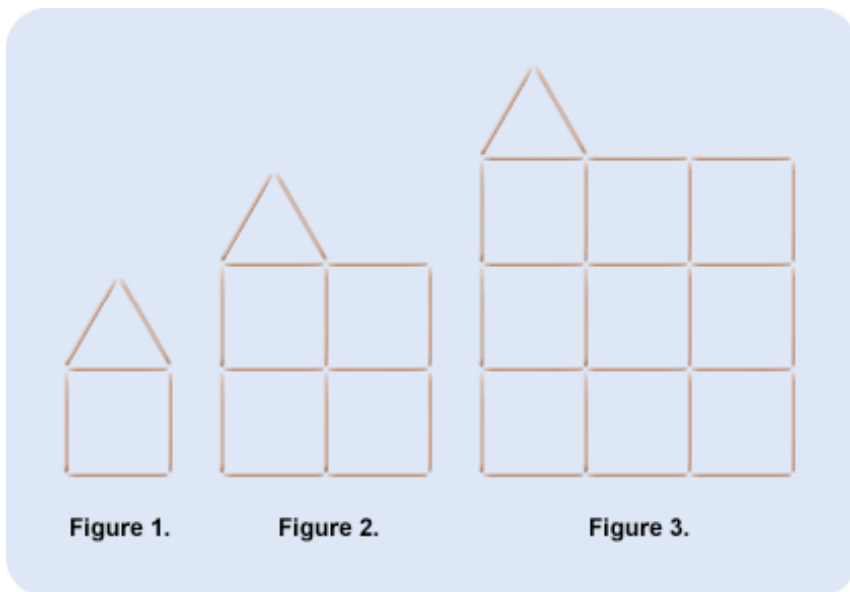
How does he use these patterns to help him find a pattern for the number of toothpicks in each of these figures?

Describe, in words, how the number of toothpicks grows.

Make use of Student B's approach to write a rule for the number of toothpicks in any figure.

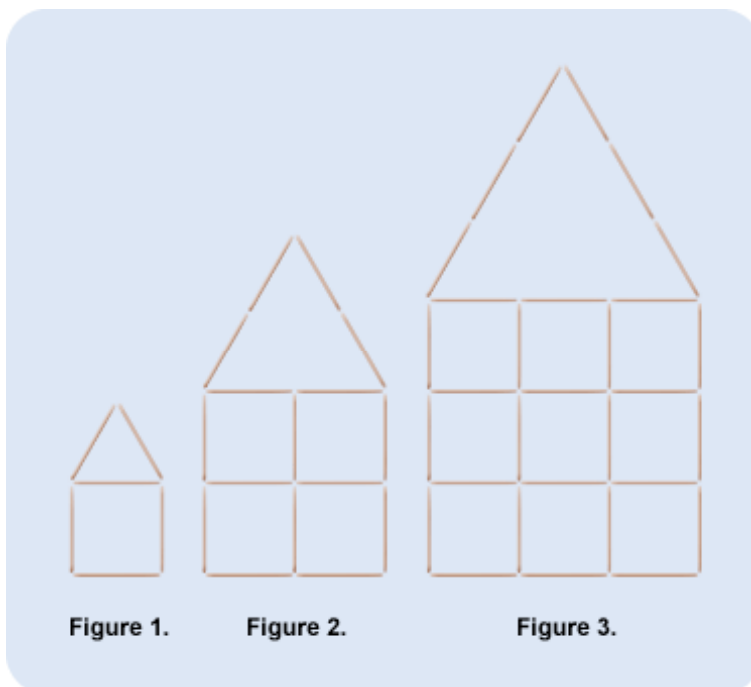
Challenge 1: A Toothpick Square with a Small Roof²

Find a rule that describes the total number of toothpicks as a function of the figure number.



Challenge 2: A Toothpick Square with a Large Roof

Find a rule that describes the total number of toothpicks as a function of the figure number.



² Adapted from the *Seeing Math Secondary* materials, developed by the Concord Consortium and distributed by PBS TeacherLine.