Lies my calculator told me:
Creating learning opportunities through calculator "mistakes"

"How could my answer be wrong? I used my calculator!" I'll bet that every teacher has heard this - or something very similar - from a student. The calculator, specifically the graphing calculator, is an incredibly important - and often misused - tool for teaching and learning mathematics, but we all bemoan the tendency of some students to believe that the calculator is some sort of magic box that knows the answer to every question. Of course, a calculator is merely a very fast - and usually fairly accurate - approximator. It doesn't know any mathematics, rather it is very good at following algorithmic commands which it has been programmed to perform. At best, a calculator only computes (or approximates) the functional operations we enter; at worst, the calculator provides misleading information.

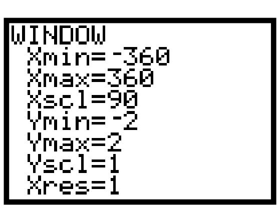
The following activity provides teachers and their students with some examples of calculator mistakes. Truly, calculator *mistake* and *lie* are misnomers, since in each example I provide, the calculator is merely doing what it was designed to do and presenting its "answer" in the form it was programmed to use. The *mistake* is in using the information blindly, without considering whether or not the answer makes sense. Many of those people who decry the use of calculators in the classroom claim that calculator use relieves the user of having to think about and do mathematics. I disagree. In fact, I claim that this activity provides evidence that, in order to use the calculator effectively, one must know *more* mathematics, not *less.* After all, without some mathematical knowledge, we would be "tricked" into believing the calculator's "lie".

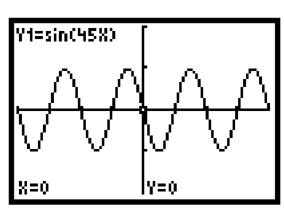
I use these examples to answer my students when they ask why we must study certain mathematical topics since "the calculator can do it for us, anyway." Of course, in order to have the calculator give us an answer, we must know enough mathematics to be able to tell the calculator what to do in the first place. But when that answer is displayed by the calculator, we must again use our mathematical understandings to interpret and make sense out of the displayed information.

The examples I provide in the accompanying activity sheet are applicable to the TI-73, 81, 82, and 83 calculators, but these "lies" are made by other brands and models, as well. In fact, the last collection of "lies" are made by ***every*** calculator, from the most basic to top of the line (with the exception of calculators that perform calculations purely symbolically). I have chosen these calculators for a variety of reasons, but chiefly, these are the calculators with which I have the most experience and, in my admittedly informal survey of colleagues at a variety of institutions, I found *Texas Instruments* graphing calculators to be the most commonly used by students and faculty.

Some graphing lies:

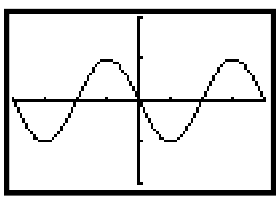
I will be measuring angles in degrees during this activity, but there are analogous "radian" examples. We'll start by graphing the function $y = \sin(45x)$ over the interval [-360,360] (while this interval is chosen somewhat arbitrarily, if it is changed dramatically, we will need to change the function to achieve the desired "lie"). Note that, at first glance, the graph is - more or less - what you'd expect: it's certainly sinusoidal, as shown in the following figures.



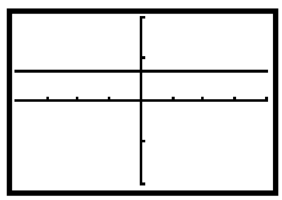


However, after further scrutiny, we realize that something is horribly amiss. First of all, we know that since sin(x) has period 360, the curve "repeats" exactly once every 360 degrees. However, sin(45x) will repeat 45 times within the interval, [0,360], since it will have period 360/45 (=8), so the period of the calculator's graph is *way off* . Also, the slope of the curve at the origin should not be negative (to see this without resorting to Calculus, evaluate sin(45x) at x = -1, 0, and 1, or consider again the relationship between the graphs of sin(45x) and sin(x)). Clearly, the calculator has made a mistake, right? When most people see this anomaly, they first check to be sure they typed the correct function into their calculator and set the MODE and WINDOW correctly (even though the MODE and WINDOW do not affect the slope of the curve) since the calculator cannot *possibly* be wrong!

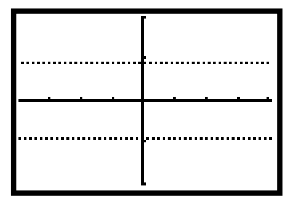
Now, if you think this is confusing. consider the second worksheet. Change the graphing window so that we're looking at the function over the interval [-350,370] and you'll get another surprise - now the curve doesn't even seem to go through the origin! It's a lot of fun to have different groups of students go up to the board to copy the calculator's graph over various intervals, like [-365,355], [-355, 365], [-345,375]. Actually, you don't have to keep the width of the interval equal to 720, but it helps to stay consistent for the sake of comparison. Nor do you have to measure you're angles in degrees, as these anomalies occur in RADIANS mode, but it is useful for the interval to contain 0 so the behavior of the function at x=0 can be examined. Some additional graphing "lies" are addressed in the activity sheet are displayed below:



sin(46x) on [-360,360]



sin(47x) on [-359,361]



sin(23.5x) on [-357,363] (dot plot)

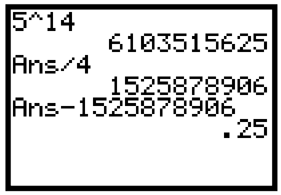
Spotting these "mistakes" requires a <u>nontrivial</u> understanding of the behavior of periodic functions (and the way a calculator plots functions). In fact, the calculator is not lying at all. If you TRACE along the curves above, the X and Y values that the calculator provides are good approximations. All the calculator does is plot points and "connect the dots"; it's just that the points it plots depend on the choice of Xmin and Xmax. In a nutshell, when graphing in CONNECTED mode, the calculator divides the window width by the number of pixels on the screen to determine which points get plotted, then connects those "dots" by filling in with a vertical string of pixels, if necessary. You may have noticed that you can always "TRACE" the x-values given by the endpoints and midpoints of the window's interval.

It is my hope that these "lies" convince at least <u>some</u> of my students to THINK before blindly trusting their calculator and that there is some value in studying the behavior of functions so as to predict the shape of their graphs.

<u>Arithmetic errors</u>:

Due to the limitations of the number of pixels on a screen, the preceding "lies" aren't that surprising, at least in retrospect. But if there's one thing a calculator can do, it's integer arithmetic, right? At least if the numbers aren't too large, that is. For example, the TI-83 can't compute the exact value of 14! (Of course, the "!" stands for factorial, not emphasis.) And we should avoid division since we might end up with rounding errors in complicated expressions. The following "lie" can be found on every calculator I've tried and was astonishing, at least initially.

Enter 5^{14} on a TI-73, 81, 82, or 83 (5^{17} on the TI-85). This particular number was chosen as the highest power of 5 which can be expressed exactly by the calculator, determined by the number of digits the calculator can display. In most scientific calculators, 5^{10} fits the bill (that is, 5^{10} is computed precisely, but 5^{11} is represented using scientific notation). When the appropriate power of 5 is divided by 4, the answer displayed by the calculator is an integer (as shown below)!



Of course, the calculator doesn't really believe that 5^{14} is even as is seen when we subtract the apparent answer from itself. I will leave it to the calculator manufacturers to explain in detail what causes this "lie". I'll just say it seems to have something to do with the fact that the actual value of $5^{14} \div 4$ has one more digit than it's able to display precisely and it won't resort to scientific notation because 5^{14} *could* be expressed precisely.

Just as this activity was being prepared for publication, a student suggested we try the MOD function of the TI-85 and 86 to determine the remainder when 5^{17} is divided by 4. To my amazement, it gave the correct answer of 1, even though when the calculator divided 5^{17} by 4, we got an integer (as in the above example). It also correctly determined that 5^{18}, 5^{19}, and 5^{20} each had remainders of 1 when divided by 4, but "claimed" that 5^{21} and 5^{22} had remainders of 2 and 0, respectively. As above, I'll leave it to the reader - or Texas Instruments programmers - to determine what's behind this "lie".

Final comments:

The reaction I get from students, after they're confronted with these examples, is interesting. Many of them become paranoid and worry that I'm going to only give them problems on which I know the calculator is going will make a mistake. I often have to reassure them that I usually won't try to trick them with a calculation or graph on which I know the calculator will "lie". Several of them do, however, "get it" - realizing that the calculator is only an approximator which must be used intelligently. When I use these examples in class, my hope is that my students will come to a greater appreciation of the calculator as a useful learning tool but I also hope that they understand that *it can't "do mathematics"*, at least by itself. And most importantly, if we're going to be able to recognize these calculator lies, we're going to have to *know some mathematics*, ourselves.

I hope these examples have been illuminating. I also want to make it clear that I'm an enthusiastic proponent of the *appropriate* use of technology in the classroom, but I think an important aspect of appropriateness is an understanding of the limitations of the technology being used. These "lies" at least get the conversation started. I've got more "lies" to share, but I'll save them for another day. Send me an e-mail message if you'd like to see my list and please send me any calculator lies you've found along the way.