Infusing Connections into Core Courses for Future Secondary Teachers

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Agenda

Today

Introduction

- Who we are; What we're doing
- Knowledge of mathematics for teaching

Engage in a sample activities as learners

- Engage in activity (don't just read it)
- Associated pedagogical activities

Discussion

- Philosophy - goals and strategies
- Designing activities

Tomorrow

Engage in another activity

Adoption and implementation issues

Final thoughts, next steps

Motivation

I'm still not sure why I had to learn about rings and fields and other such topics to be a high school math teacher.

— A veteran high school teacher

Knowledge of Mathematics for Teaching

What undergraduate mathematics is important for secondary teachers to understand?

How might an understanding of that mathematics help someone be a better teacher?

What topics in secondary mathematics provide seeds for the study of undergraduate mathematics?

How might the study of that undergraduate topic be designed?

Knowledge of Mathematics for Teaching

- Not everything a teacher needs to know ends up on the chalkboard.
- The ability "to think deeply about simple things" (A. Ross)

  What's really behind the geometry of multiplying complex numbers?
- The ability to create activities that uncover central habits of mind

  What do $5^{3/2}$ *and* $5^{\sqrt{2}}$ *mean?*
- The mining of student ideas

  When is a "partially-baked" idea worth pursuing and how might we pursue it?

Knowledge of Mathematics for Teaching (cont'd)

- The ability to see underlying connections and themes
  - Connections
    - Linear Algebra brings coherence to secondary geometry
    - Number Theory sheds light on what otherwise seem like curiosities in arithmetic
    - Abstract Algebra provides the tools needed to transition from arithmetic with integers to arithmetic in other systems.
  - Themes
    - Algebra: extension, representation, decomposition
    - Analysis: extension by continuity, completion
    - Number Theory: reduction, localization

Engaging in a sample activity

Engage in the activity as a learner.

Think about how an undergraduate might engage with the activity.

Keep track of questions and observations (share them with your working group, as well as the whole group)

Reflect on the first part of the activity

1. What might students learn by engaging in the first part of this activity?

2. What understandings of (and between) transformational geometry and linear algebra might students have *not* yet learned after engaging in the first part of the activity?

Returning to the activity

Be sure to keep track of questions and observations you have.

In particular, what student understandings would likely develop that might not develop had we stopped after the first part?

Day 2 Agenda

Continued reflection on yesterday's activity

Messages from Research and Policy Makers

Activity Analysis

Based on themes of effective professional development

Designing activities

Next Steps

Continued Reflection

1. What important mathematical ideas might be learned by engaging in this activity?
2. Why might this activity be important for ALL undergraduates?
3. Why is it important for prospective secondary teachers, in particular?
4. In what course(s) does the activity make sense?
5. Where might the activity fit into the course(s)?
6. Where might this activity lead?

Related Research

Portnoy, N., Grundmeier, T., and Graham, K (2006). "Students' understanding of mathematical objects in the context of transformational geometry: Implications for constructing and understanding proofs." *Journal of Mathematical Behavior,* 25, 196-207.

Messages from the mathematics community

Over the past 15 years, two refrains have echoed through the discourse about teachers' knowledge of mathematics:
(1) that U.S. teachers mathematical knowledge is weak
(2) that the mathematical knowledge needed for teaching is different from that needed by mathematicians.

— **Mathematical Proficiency for All Students: Toward a Strategic Research and Development Program in Mathematics Education** (RAND, 2001)

Messages from the mathematics community

The mathematical knowledge needed by teachers at all levels is substantial, yet quite different from that required by students pursuing other mathematics-related professions. . . . College courses developing this knowledge should make connections between the mathematics being studied and mathematics prospective teachers will teach.

— **The Mathematical Education of Teachers** (CBMS, 2001)

Messages from the mathematics community

Teachers need several different kinds of mathematical knowledge:

- Knowledge of the whole domain
- Deep, flexible knowledge about curriculum goals and about the important ideas that are central to their grade level
- Knowledge about the challenges students are likely to encounter in learning these ideas
- Knowledge about how students' understanding can be assessed

— **Principles and Standards for School Mathematics** (NCTM, 2000)

What does the research say about professional development that improves student achievement?

The overall message is that more research needs to be done. However, important themes are emerging,

- Engagement in mathematics needs to be central
- Important that PD content be connected to the content that teachers teach
- Content need not be *identical* to the curriculum (and should go deeper and *wider* than students would)

Resnick, L. B., (ed.), "Teaching Teachers: Professional Development To Improve Student Achievement" *AERA Research Points*, American Educational Research Association, Summer 2005

Incorporating themes of successful professional development

- What does it mean to be engaged in mathematics?
- How might we engage future (and practicing teachers) in mathematics in their core coursework?
- What is the meaning of the phrase, “PD content (should) be connected to the content that teachers teach?”
- What are the ways that we can structure core courses so that the content is connected to tasks of teaching that secondary teachers engage in?
- What do we mean when we say teachers “should go deeper and wider” than their students would?

Jig-Saw: Two activities

1. You will consider two different activities. In your groups, comment and keep notes on how your activity addresses themes of successful professional development

2. Move back to your original tables to compare observations.

Creating activities for your course

Choose a course you teach in which future teachers might be enrolled. Take a problem/topic and begin to think how you might modify it to meet the themes discussed.

Possibilities/suggestions:

A problem on group structure
A finite geometry investigation
An investigation into convergence or integrability
An investigation into factorization, divisibility
Fitting points to polynomial (or other) curves

Alternatively, think of a secondary mathematics topic and create an activity for a core course that is related.

Laws of logarithms
Asymptotes of rational functions

Where can these approaches be used?

The extremes:

- An instructor looking to insert a few problems or activities into the existing curriculum

- A department interested in revamping or creating one or more courses for prospective secondary teachers with connections built in as an integral part

Where are these approaches being used?

The "middle ground":

- Special units within the core courses of linear and abstract algebra, geometry, and number theory
- Shadow seminars attached to the core courses
- Mathematical content for secondary methods courses
- Capstone course (senior seminar)
- Freshman seminar courses for prospective teachers
- Independent study courses
- Workshops for in-service teachers (on-going study groups and 1-2 week institutes)

A few comments from participants

- It felt extremely empowering to understand and apply abstract algebra concepts that in the past I had great difficulty in comprehending.
- Liked having the math knowledge but won't use modular (arithmetic) in class. Other topics discussed were extremely helpful.
- It was fun to have to think!
- I feel that any good future teacher should take this course.
- The content was great. It was the best math ed course I have had at B.U.

A few comments from participants

- I had no physical representation of what a transformation was from [her course in] linear algebra. The first day [in this geometry class] when we saw that a reflection was just doing this [flipping over an eraser], I was pretty upset.
- I thought it [seeing connections] was very beneficial because learning it [matrix multiplication] in my linear algebra class, I didn't see any uses for it.

Our challenge (or is it an opportunity?)

- (The) material presented is interesting (i.e., to know that such topics/fields of info exist) but the information provided will have no impact on my lessons. It is at a much higher level than I will ever or have ever worked/learned at.

- I thought that these workshops would:
  1) Address specific standards from the state's list.
  2) provide me with a product that I could use in class
  3) provide me with a product I produced which I could show the state if I am audited.

Resources

Ways to Think About Mathematics: Activities and Investigations for Grade 6-12 Teachers; Benson, Addington, Arshavsky, Cuoco, Goldenberg, Karnowski; Corwin Press, 2004.
`http://www2.edc.org/wttam`

Mathematical Connections: A Companion for Teachers and Others; Cuoco, MAA, 2005

Mathematics for High School Teachers - An Advanced Perspective; Usiskin, Peressini, Marchisotto, Stanley; Prentice Hall, 2003.

Seeing the Connections: Promoting Profound Understanding of Secondary Mathematics; Benson, Cuoco, Graham, Greenes, Grundmeier, Portnoy (in preparation)

Seeing the Connections
materials are available online

http://www2.edc.org/connect/connectionslink.html
http://www2.edc.org/connect/mathconnlink.html
http://www2.edc.org/connect/gatewayslink.html

Additional materials and information:

http://www2.edc.org/wttam
http://www2.edc.org/mathproblems
http://www2.edc.org/makingmath

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