

Developing Proof Throughout High School Mathematics

Handout #1

Addition Table

| | | | | | | | | | | | | |
|----|----|----|---|----|----|----|----|----|----|----|----|----|
| 12 | 13 | 14 | | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 11 | 12 | 13 | | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | |
| 10 | 11 | 12 | | 14 | | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| 9 | 10 | 11 | | 13 | 14 | | 16 | 17 | 18 | 19 | 20 | 21 |
| 8 | 9 | 10 | | 12 | 13 | 14 | | 16 | 17 | 18 | 19 | 20 |
| 7 | 8 | 9 | | 11 | 12 | 13 | 14 | | 16 | 17 | 18 | 19 |
| 6 | 7 | 8 | | 10 | 11 | 12 | 13 | 14 | | 16 | 17 | 18 |
| 5 | 6 | 7 | | 9 | 10 | 11 | 12 | 13 | 14 | | 16 | 17 |
| 4 | 5 | 6 | | 8 | 9 | 10 | 11 | 12 | 13 | 14 | | 16 |
| 3 | 4 | 5 | | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | |
| 2 | 3 | 4 | | 6 | 7 | 8 | 9 | 10 | 11 | 12 | | 14 |
| 1 | 2 | 3 | | 5 | 6 | 7 | 8 | 9 | 10 | | 12 | 13 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

These parts of an addition table and a multiplication table may differ from tables you have seen, because they start in the lower left corner. Usually, addition and multiplication tables show smaller numbers in the upper left corner.

1. What are the missing numbers in each table?
2. Find and explain several patterns in the addition table.
3. Find and explain several patterns in the multiplication table.

Multiplication Table

| | | | | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|-----|-----|-----|-----|
| | 12 | 24 | 36 | 48 | 60 | 72 | | 96 | 108 | 120 | 132 | 144 |
| 0 | | 22 | 33 | 44 | 55 | | 77 | 88 | 99 | 110 | 121 | 132 |
| 0 | 10 | | 30 | | | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 0 | 9 | 18 | | | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 0 | 8 | 16 | | | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 0 | 7 | | 21 | 28 | | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 0 | | 12 | 18 | 24 | 30 | | 42 | 48 | 54 | 60 | 66 | 72 |
| 0 | 5 | 10 | 15 | 20 | 25 | 30 | | 40 | 45 | 50 | 55 | 60 |
| 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | | 36 | 40 | 44 | 48 |
| 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | | 30 | 33 | 36 |
| 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | | 22 | 24 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 12 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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Handout #2

1. Follow these steps.

- Choose an integer a .
- Square the integer.
- Find the prime factorization of the squared integer. How many twos are in the prime factorization?

Repeat this process with at least five integers. Use even and odd integers.

Is it possible to find an integer a , such that the prime factorization of a^2 contains an odd number of twos? If so, find the integer. If not, explain why.

2. Follow these steps.

- Choose an integer b .
- Square the integer.
- Multiply the squared integer by 2.
- Find the prime factorization of the squared integer. How many twos are in the prime factorization?

Repeat this process with at least five integers.

In general, does the prime factorization of $2b^2$ contain an even or odd number of twos, or does it depend on your choice of b ? Explain.

3. Write About It. Can a perfect square ever be twice as large as another perfect square? Use your results from Problems 2 and 3 to explain.