## BUILDING COLLABORATIVE SKILLS

Every discipline has to attend to both the content of the discipline and the broader skills that contribute to success within it. It is tempting to neglect these larger habits and skills and to hope that they are being taught elsewhere or that they materialize naturally. The investment of class time in the development of collaborative skills produces both mathematical and interpersonal growth.

Many newer curricula involve students in small-group projects. When doing so, it is important to pay attention to the dynamics that arise in such groups that can sidetrack progress. Some curricula provide a page of notes on appropriate group interaction for the teacher to distribute to the class. Such guidelines, passively received, are unlikely to influence behavior dramatically. Students pay much more attention to rules that they have had a hand in creating. For example, assessment rubrics tend to be more demanding and more closely adhered to when generated by a class. Likewise, instructions for proper conduct during group projects are usually more complete and effective when developed by the students themselves.

## UNDERSTANDING AND SUPPORTING GROUP EFFORTS

When you've done a joint paper, both co-authors do $75 \%$ of the work.

## - Mathematician Ron Graham

A class with a mathematics research strand should seek to become a community of mathematicians. Students can work on research questions together, combining their insights and supporting one another during challenging times. Students often encounter difficulties working together, but these problems can be anticipated and ameliorated through peer and selfassessments of group process and behavior.

Prior to the first small-group assignment, ask your class to spend four minutes writing responses to the question "Why do we have people work together in groups? What are the goals of cooperative efforts?" If you circulate and scan their responses as they write, you can later call on students who do not volunteer but who have good ideas. When the four minutes are up, make a list of students' ideas on the board. Feel free to add one or two reasons that you think are important at the end, if they have not been noted.

Students' answers to these questions have included the following: (1) Some tasks cannot be accomplished by an individual in a reasonable amount of time. (2) Collaboration, when effective,
is capable of producing outcomes that are more interesting, creative, and thorough (because of checking one another's assumptions and computations) than any of the group's members could produce on their own. (3) Group work requires the development of listening and communication skills that have importance outside formally structured group endeavors. (4) Group work facilitates the sharing of different ideas and perspectives. These varied approaches can make it easier to get unstuck. (5) Group work brings together the different backgrounds, and therefore different strengths, of the group members, leading to a greater appreciation for one another's talents. (6) It is more fun than working individually (and "enjoyment" is a course goal).

One optimistic individual suggested that groups were preferable "because it is better to fail with someone than alone." Another said that the purpose was to decrease the number of papers the teacher had to correct. Often, students will suggest that group work enables them to divvy up a larger task; however, although there are times when this division of labor does occur, it should be viewed with a skeptical eye. Remind students that everyone in a group is responsible for ultimately understanding each aspect of the group's project, and note that reports that result from pasting together individual efforts are rarely coherent or correct. There should be a regular meeting of the minds, and every member should be involved in editing any final product.

Repeat the above writing and discussion cycle for the second set of questions: "What issues arise when groups work on a task? What factors affect how well or poorly a group functions?" The following student concerns are typical: (1) How much time each person invests in the project-is the distribution of labor fair? (2) Is there agreement on the group's purpose? Are there common standards and an equal commitment to the task? (If not, effort may be unequal.) (3) Do the individual members have different work habits or approaches? Is each member organized and cooperative? (4) Do they have similar or different problem-solving styles? Complimentary or conflicting ideas and philosophies? (5) Is there good listening and good will, or competitiveness? Is communication honest but constructive? Is there clear expression of opinions and ideas? (6) What roles do people play in groups? What structures are possible? (7) Are the group members friends, or even friendly? (8) Is there enthusiasm for the project? Is the task understood? (9) Are individuals focused and patient?

One student observed that over-agreement can lead to less satisfactory group outcomes. Too much harmony can lead to a lack of careful checking ("Oh, their idea matches mine so it must be right!") or too narrow a range of approaches. A little ferment can be a good thing.

In one class, a student wrote that groups work well when there is a recognized leader who delegates responsibilities. In contrast, another said that groups fail when one person tries to tell everyone what to do. This disagreement highlights the need to explicitly explore students' conceptions of effective approaches to group work. Neither view is wrong, but such a combination of views left unexplored would ensure a poor experience. The class in which this difference arose brainstormed clues, signals, and feedback that would help to establish or change the relationships group members had so that a structure that served the members and the project was chosen. When students discuss troubles that can arise, they do a better job of avoiding the pitfalls.

In that spirit, have students volunteer answers to a final question: "What can you do if a group is not working well?" (Note that while the writing encourages everyone to think about the questions, it can also become too routinized. Switching to a more casual format at this point can end the discussion on a more energetic note and keep the class involved.) Past suggestions include the following: (1) Discussing the problems as a group. (2) Talking with an individual whose behavior is of concern and trying to find out why that person is behaving a certain way. (3) Taking a break. (4) Getting a moderator. (5) Encouraging one another. (6) Taking the initiative to lead and set an example rather than nagging. (7) Avoid "zapping" one another. Respond to ideas in a supportive fashion. Many kids tend to begin with criticism ("That's not right..."). Seek to first identify and highlight the value in a classmate's contributions. (8) Talking with the teacher. Encourage students to come to you as a last resort. If you notice a problem, it is reasonable to intercede with an observation. Requests to break up a group should not be honored, but new groups should be constituted for each project.

Type up and distribute the three questions and the class's responses so that they can have them in a prominent spot in their notebooks for ready consultation. If you do this activity with more than one section, a combined list will show classes what other ideas their peers came up with.

It is important for all of us to be able to provide and receive constructive feedback. Students need to feel safe in doing so and to believe that the process is geared toward their personal and academic growth. After the first extended group activity, have students provide constructive peer feedback by writing their reflections on how their group worked during the group experience, using the following questions as a guide:

- How did your group work together (an "objective" description)?
- What worked well?
- What could have worked better?
- How could it have worked better?

Promise to read the comments only to ensure that the feedback is clear and responsible, and then pass them on to the student's partner(s). Each group member should receive the same grade for a project; the feedback itself should not be graded or influence the grades of the individuals in the group. It is helpful to give feedback to the feedback-givers as well regarding the clarity and forthrightness of their responses.

Although it is not always anticipated at the beginning, students soon realize that an overriding issue affecting the success of a group project is the availability of time to work as a group. Depending on where group members live and on other life circumstances, time may not be available for meetings outside of class. Sometimes you may be able to form groups with these constraints in mind. In general, a significant amount of class time may be needed to provide work time for the groups. This time will be most productive if, at the end of each class work session, groups are required to assign and record individual homework tasks to be completed prior to their next meeting.

Group work intimately exposes students to the ideas and approaches of others. In small-group settings, students are more likely to really listen and less likely to be distracted than during whole-group discussions. Quieter students who hesitate to participate in class become more involved, and everyone is generally more willing to experiment, take risks, make mistakes, and analyze their reasoning.

There is no one best way to create groups. They can be formed based on which research question each student wants to explore. Students can have some input into the formation of groups ("Give me a list with your top ten choices"). You can group students randomly (e.g., picking them out of a hat). However, avoid attempts at tracking students into groups that are all supposedly of the same or different ability. There are too many important skills (problemposing, representation, writing, organization, proof, etc.) that contribute to research for there to be one tidy ranking within a class. Research efforts are worthwhile, in part, because they make apparent to students that mathematics is not primarily a test of computational accuracy, and that they each have a range of skills they can bring to meaningful mathematics explorations. A well-
balanced group will give each student a chance to be the expert at some task. A group that uniformly lacks a skill (e.g., using technology) will be forced to work, as a group, to gain some mastery in that area.

## DEVELOPING GOOD CLASS DISCUSSION HABITS

The above activity for developing a whole-class understanding of the reasons for, and ways to improve, small-group efforts can serve as a model for discussion of many aspects of classroom life. For example, your classes can apply the questions to issues involved in the preparation for and delivery of an oral presentation. While small groups are an efficient way to have many students talking and thinking at once, they do have the disadvantage that each group is unaware of the ideas and methods the rest of their classmates are exploring. Whole-class discussions provide a means for involving everyone with the same questions and for calling attention to important skills and concepts. These discussions can be enhanced by a preparatory exploration of discussions themselves.

The following prompts can yield a good list of guidelines for how to be involved in a discussion:

- Why do we have class discussions?
- What factors contribute to a good discussion?
- What are the students' roles during a discussion?
- What is the teacher's role?

Student responses typically include the following: (1) Discussions bring out different points of view, give you a chance to rethink your opinions, and force you to make your ideas clear enough for yourself so that you can communicate them to others. (2) Discussions succeed when there is active participation by a variety of people (and quieter kids strive to test their voice occasionally); speakers are audible; people know what they are talking about; and comments are sincere (people stick to claims that they believe). (3) When they are not speaking, students should be active listeners (make eye contact, nod, or otherwise communicate physically that they are being attentive). (4) Everyone should be courteous, respect other people's opinions, and be open to other people's ideas. (5) The teacher (or an appointed student) is a good facilitator when she calls on folks fairly, reminds people of the standards for involvement when needed, poses
thought-provoking questions, makes sure no one dominates, and clarifies comments (Kathleen Ennis, personal communication).

Note that not all of the ideas raised by students may be the best choice for a given class. For example, students often suggest hand-raising, but a more fluid discussion may emerge when people enter into discussions in a less formal, more conversational fashion. Another possibility is to encourage hand-raising, but to make it the responsibility of the last speaker to call on the next one so that everyone grows in his or her awareness of who needs to be brought into the discussion. A class is more likely to develop into a mathematics community when the teacher is neither the primary controller of discourse nor the main creator and transmitter of knowledge, but rather the facilitator for learning. During discussions, avoid as much as possible being the sole arbiter of what answers are correct and what reasoning seems valid. When students look to the teacher for the final word on a question, they are avoiding their own responsibility to struggle with the evidence for themselves.

There are several additional steps you can take to facilitate group discussions. One is to emphasize for students that they are engaged in a discussion and not a debate. Many students see discussions as a chance to show off, to impress the teacher, or to prove their superiority. Remind the class that discussion goals include learning from others and appreciating the complexity of ideas being considered. Another step is to pay attention to the physical layout of the class, which can help or hinder discussions; whenever possible, arrange the chairs (e.g., in a circle) so that students can talk to the entire class at once.

Discussion skills are an area in which students may have emotional barriers. Many teenagers develop the protective habit of starting a question with "I have a dumb question..." or "I know this is wrong, but...." It is important to privately point out this habit and encourage students to drop these comments. They need to hear that questioning is not a sign of weakness, and that good questioning is crucial to an interesting class. They also need to help their listeners take their comments seriously by not deprecating themselves at the start.

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