Mentoring is a specialized kind of teaching; mentoring via e-mail involves additional constraints. To do it successfully takes good judgment and attention, of course, but also some specific practical knowledge. To help inform you, we have compiled some information in this handbook. The broad categories are:

- Models of Mentoring
- Legal Considerations in Online Mentoring
- Issues of Teaching and Communication
- Working Around the Constraints of E-mail
- Beginning and Ending a Mentor Relationship

If issues or concerns arise that are not addressed in this Handbook, please contact the *Making Mathematics* staff.

1. Models of Mentoring

There are two ways you might mentor a Making Mathematics project.

Mentoring a teacher and a group of students. A teacher signs up with a group of students (perhaps an entire class) to work on a *Making Mathematics* project. Students' work submitted to the mentor represents the group effort. The mentor assists the teacher and group. The teacher's role might vary, from coresearcher to co-mentor. If a teacher has several groups of students working on projects, several mentors may be assigned to the same class, each working with one or two groups. The mentor may be in direct contact with the students, or the teacher may serve as conduit between the mentor and students.

Mentoring an individual student. A student fills out an application. Once accepted, this student is assigned a mentor and begins working on a project. Mentors may provide whatever structure, encouragement, advice, and mathematical knowledge seems needed and appropriate. A teacher may not be involved in the project, so mentors may find it useful to review the resources in the Support for Teachers section of the *Making Mathematics* web site.

2. Legal Consideration in Online Mentoring

Working with Minors. Both ethically and legally, working with minors over the internet involves sensitive issues. Professional conduct and good judgment cover most of what's important, but be aware also that you must not share student e-mail addresses or other personal information with anyone besides the *Making Mathematics* staff.

Keeping Records. Whether you are working alongside a teacher or directly with a student, please save all *Making Mathematics* communication with students. For your own reference, log all the correspondence to and from your students and keep

backup copies of all messages in a format (electronic or print) that you can review as needed. In addition to your own record keeping, for our research, statistical, and legal purposes, we must require that you "cc." all your e-mail messages (including the history of the student e-mail to which you are responding) to *Making Mathematics* at DMRS@edc.org. Your student is informed, upon signing up with us, that all communications are kept, and seen only by you and project staff—not shared with others.

Communication with Parents. Sometimes, parents are directly involved in the *Making Mathematics* experience from the start. They may have encouraged their child to join or, if their child is under 13, they will have given consent for their child to register for a project. Additionally, some parents may seek further involvement. Parents may want to monitor your communication with their child, remind their child to e-mail you, inform you about emotional or environmental factors, or ask their own questions about the experience or even the mathematics. Parents are a great resource for information and advice about the students you're mentoring; use them if they offer to help. But remember, if you are contacted with *questions* about the child, you must be *absolutely certain*, before you reply, that the sender is, indeed, the child's parent. Check with us if you have any questions at all. For parents who want to remain involved, quick "check-in" e-mails are a good way to keep them in the loop. For example:

Hi.

I'm Susan Johnson, Charlie's *Making Mathematics* mentor. Just wanted to send a quick note to let you know that things are going well and Charlie's making good progress on his project. You may want to read over some of our e-mails. Please let me know if you have any questions.

Sue

Ethics, Professionalism, and Unexpected Confidences. Sustained conversations, even if they start out around mathematical research, evolve into broader relationships. Students may well tell you of family trips or other interests of theirs, or ask you similar questions about you or your family. Responding appropriately is friendly and helps build a trusting relationship. It is far less likely, but still possible, that a student may offer you unexpected confidences that you find *too* personal, that make you uncomfortable, or that reveal that the student is at risk. Such confidences, like all information about your student, cannot ethically or legally be shared. Further, it may be inappropriate to give personal advice to your student in such circumstances. However, if you feel your student is at risk, or presents a risk to others, the law may actually *require* you to act. Please contact the *Making Mathematics* staff whenever the conversation veers in a direction that raises questions of safety or ethics, or even it just makes you feel personally uncomfortable.

3. Issues of Teaching and Communication

Communicating Mathematically. It is important to consider whether you are communicating at the right level of technical sophistication for your audience, whether it be an individual student, a group of students, or a teacher. A tone that

may feel precise and mercifully brief to close colleagues may feel curt and intimidating to secondary school students. While allowing yourself to present appropriate mathematical ideas and terminology as needed, you will probably find that in general, a friendly, informal style is more likely to be effective than a mathematically formal one.

Communicating mathematically is an important skill that students can improve by working on a mathematics research project. Students might have difficulty explaining their reasoning and results, especially via e-mail. Students may also be shy or embarrassed about asking questions if they don't understand, and may pretend that they understand to avoid feeling or looking stupid. You can help them by encouraging your students to explain their steps and conclusions, asking questions, and reflecting back, re-stating what they have written in your own words ("You are saying that..."). Your written clarity about your own thinking also provides an important model for your students.

Of course, make sure that your writing about mathematics is at a level that your students can understand. For example:

Matt,

Your idea to go for shortcuts and label the cards of SET sounds great! Just make sure that labels for different cards are different. Also, as we only e-mail to each other, let's make the labels as simple as possible. What about using digits or letters to indicate colors and shapes?

By the way, I wouldn't drop the study of a smaller deck quite yet: simple examples can help a lot. You wrote that "if there are only 27 cards in the deck, it seems that there can be 5 sets among 8 cards." What made you decide this? Did you find an example? Good luck, Dr. Solomon.

In a teacher-assisted mentoring partnership, teachers might help students prepare their e-mails to you. It may be helpful to compare your and the teacher's impressions about students' comprehension and ability to express themselves mathematically.

Teaching Strategies. If you are working directly with a student, you will initiate, continue, and complete the mentoring relationship.

Most students are used to mathematics problems that are solved in a matter of minutes rather than days, weeks, or months. Many do not realize that problem solving involves being stuck and getting unstuck. They may see being stuck as a hopeless situation, or a sign of failure. A mentor's job is not only to support and guide students intellectually, but also to support them through the emotional discomfort of facing a problem that is truly tough for them.

When a student gets stuck and is frustrated, a common first impulse is to provide big hints. But hints that lead directly to the answer stand in the way of learning opportunities, and may even leave a student feeling weaker, successful only *because* you supplied the help. There are many other strategies to help a student succeed without shortcutting their own triumphs through too-easy solutions.

You can help students figure out why and how they are stuck. If you can help your students clearly state what they are seeking to determine or figure out why they are stuck at that stage in the process, this alone is often enough to help them make progress. Identifying the cause of "stuckness" (e.g., "I have too many

variables," "I don't see any pattern in this sequence") is frequently all that they need in order to focus on, and resolve, the difficulty. If you help your students to ask such questions, also make explicit what you are doing and why. You may ask yourself similar questions when you are stuck: "What do I know?" "What do I need to know?" "What techniques do I have for bridging the gap?"

Students sometimes get stuck just from lack of confidence. One teacher reports:

Often, students have come for help because they were stuck with a multi-step problem. They'd show me the first step, and freeze. I'd ask them "OK, what do you do next?" and they *know*, but then after doing that step they'd stop again. I repeat my question and they do one more step, and so on until the problem is solved. They'd thank me for the help, and I'd have to say that all I did was get them to continue. I encourage them to take over that job themselves and remind them that uncertainty should not be allowed to lead to paralysis.

Process-oriented suggestions are often helpful: "You might need to organize your data in order to look for patterns." Writing about difficulties in a research log can help a student overcome them, and the importance of organizing and documenting their research is a good message to give students. Even though you and the *Making Mathematics* staff will be keeping complete records, urge your students to keep their own log of their research. For suggestions on how students can keep such a log, see the related teacher resources pages on our site.

Stuck students may ask themselves the following questions:

- Can I restate this problem in my own words and symbols?
- Can I identify the given information and conditions of the problem? What do I know about the (unstated) properties of the objects involved?
- Is any of the given information extraneous?
- Is there enough information?
- Are there parts of the problem that I do not understand?
- Can I explain what an answer to this problem would look like?
- Can I create an example that meets the conditions?
- Can I alter the conditions in some way that would get me unstuck?

There are also some "next steps":

- Try various representations of the relationships among the elements of the problem. Make the situation *visual* by drawing a diagram. (Be cautious, however: diagrams can lock students into a single way of looking at a problem draw several valid diagrams or a general one.) Make the situation *abstract* (and also more generalized) by introducing variables. Make it *concrete* by finding specific examples (numeric, algebraic, or geometric).
- Organize the information in a useful way (e.g., in ordered pairs or a table).
- Ask, "What information do I need to know in order to proceed?" or "What techniques or theorems connect what I do know with what I need to know?" Think, "If only I could assume [some fact or lemma], then I'd have a solution." This creates a new problem: how to establish that fact.
- Look for patterns, structures, and behaviors similar to ones that the student has previously studied.

Students sometimes find the open-endedness of a problem unsettling. If you think that is getting in the way, you can suggest creating a specific sub-problem by adding an additional constraint or looking at one particular case. Students may also find their assigned problem arbitrary, inelegant, or lacking in mathematical appeal. Setting the problem in historical or social context can sometimes kindle interest.

Being stuck is OK. When armed with helpful questions, it is an important stage in developing persistence and intuition. Which question is most significant? Which method should be applied? Only practice makes us better at making such decisions. Good instincts come with experience, and students need to be given the time to gain that experience. It can be valuable practice to continue working on the problem for a day or two and then take a break from it if they remain stuck. For more teaching strategies, visit the Teacher Handbook on the *Making Mathematics* website.

Students' Feelings. Students may get discouraged if they do not feel they are progressing fast enough. It is important to help them to reflect on their research experience and to realize where they are in their research (if a special case has been solved, if a conjecture has been disproved, etc.). You can let your students know that each of these steps is an important result and help them to see how these results fit into the big picture of their research project.

There are many ways to help students remain motivated and optimistic about their research. Though the actual mathematics involved in *your* research may be beyond your mentees' comprehension, they will no doubt benefit from knowing that you experience similar patterns of excitement, hard work, and confusion. You may want to tell them about your own research experiences, including some of the emotional factors of research mathematics. Sharing your own experiences sets a tone of openness and honesty and it will make it easier for students to express themselves to you.

Using Third Parties to Good Advantage. Your students' teachers or parents can provide you with additional insight. Teachers and parents communicate with students face-to-face on a daily basis. A teacher will know what students have studied previously in other mathematics classes and can observe students' approaches, level of enthusiasm or anxiety, etc. Parents clearly can provide insight about their kids. They can also support in other ways, for example by:

- helping students formulate their questions, concerns and results;
- reminding them that it's time to write to you; and
- responding to student questions arising from their work.

Teachers, and parents, by virtue of different experience and contexts, will approach the research and student differently, and multiple perspectives can enrich the research experience.

4. Working Around the Constraints of E-mail

Independent of the model of mentoring that you have chosen, your only mode of communication with students or teachers will be electronic print. Communication with secondary school students has its own special characteristics, and on-line communication can add to the challenge. **Staying in Touch and Anticipating Breaks.** The *Making Mathematics* project anticipates that mentors and students will exchange e-mails about once a week. Timing is essential: respond promptly to your students' e-mails. You do not have to wait until students respond to each of your e-mails—they might have missed your last e-mail, have misunderstood it, or be frustrated and discouraged. Your messages do not have to be always long and informative. You may keep in touch with casual "check-in" e-mails, for example:

Hi Beth,

How are you? I keep thinking about that conjecture we discussed last week. Does it work for other cases?

Regards, Jim.

or even

Hi Beth,

Haven't heard from you in a while. I hope everything's OK. Regards, J.

Regular correspondence is very important to a student. Students may check e-mail several times a day in anticipation of a new message and unexplained silence can be perceived very negatively. If there is no explanation for the delay, a young person may internalize responsibility: "Have I done something wrong?" or "He doesn't really care." If you expect a lag in correspondence, let your mentees know about it:

Matt,

I will be out of town for a meeting, so can't write until Wednesday. Take care, and keep working on that last question we discussed. I'll check back in when I return.

Dr. Solomon.

The situation also may be reversed: you may not hear from your students for a longer-than-normal amount of time. There could be many reasons: other responsibilities and activities, vacation, limited access e-mail, etc. Other reasons could be that the student is stuck and doesn't want to write with nothing new to report, or that the student has lost some interest in the problem or process. You may first try to regain contact by way of informal check-in e-mails.

Recognizing the Limits of Print. When you speak face-to-face with a person, visual clues convey tone and purpose. In print communication, there are no additional visual clues, and tone can be difficult to decipher. Sometimes, an ambiguous message from either party can lead to misunderstandings or confusion. Before sending, reread your outgoing message trying to take your student's perspective. Also, keep in mind that an e-amil's format (e.g., line breaks and spacing) are essential for readability.

E-mail-Specific Language Young people who are frequent e-mailers have developed a host of abbreviations of phrases, and symbols like :-) or :-(to convey emotions, excitement, frustration, etc. Much internet "chat room" conversation is carried out in long strings of abbreviated phrases. Many students are familiar with these sorts of symbols, and may use them in correspondence with you. For an easy reference of such e-mail-specific language, see http://www.telementor.org/hp/hp-resources/handbook.html#s7. You may need to ask your student for translations

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of *their* language, just as they sometimes (and likely for different reasons) may need translations of yours.

5. Beginning and Ending a Mentor Relationship

Beginning a Relationship with a Teacher Partner. Teachers often register for *Making Mathematics* with the purpose of including this project in an existing curriculum. Talking about that reason and discussing both of your expectations at an early stage in your communications will enhance the experience. In your first contact with a teacher partner, it is a good idea to introduce yourself and describe your background and interest in mentoring. Then, work to come to a shared understanding of the project's goals and the process you'll undertake.

The *Making Mathematics* program expects that mentor and teacher or students exchange communication at least once per week. You may need to plan for the additional time you'll need to communicate with the teacher and to reflect on the exchanges.

You and your teacher partner may want to consider:

- expectations for the students' finished product (a presentation, program, paper, or combination, etc.),
- timeline for the project, and critical dates when your time might be especially needed (or unavailable);
- project details and resources students might require,
- supplementary material (articles, web sites, etc.) that your teacher partner or students may find helpful,
- how much time the students will be able to put in, and
- whether the students have access to software (Prepare a plan B in case the students cannot access software you consider to be necessary for the project).

The nature of working with school-related constraints may require some flexibility on the part of the mentor.

Beginning a Relationship with a Student. Your first contact is very important. It sets the mood for future correspondence and lets your students know that you are a real person, not just an impersonal source of mathematics help and information. If your students are comfortable writing to you, they will be more able to share their questions, half-formed ideas, frustrations, and concerns. Your introductary letter could include a sense of your mathematical interests, why you decided to go into the field of mathematics, and some of the things you've done. Your mathematical biography should not be highly technical. Share your expectations of the project, invite students into the problem, and make it easy for them to be engaged and to respond.

If you will work directly with a student, plan on communicating with your mentees about once per week and allocate the additional time required to prepare for, reflect on, and analyze the process. You may also want to review the teacher resources included on the *Making Mathematics* web site.

Ending the Relationship. External constraints (class time, testing, vacations, etc.) often require that a project be wrapped up by a specific date. Deadlines are difficult, as the nature of mathematics research makes the timing of discoveries more or less unpredictable. There will be situations when the research has to be continued

(for example, the student is extremely interested and is making good progress), situations where intermediate results may become final results, or situations where you and your mentees develop extensions of the project. Similarly, if unavoidable reasons cause *you* to end the mentoring relationships unexpectedly early or before the work is "done," do your best to let students know as soon as you can, so that you both have time to end the relationship in a comfortable way.

When wrapping up your mentoring relationships, you'll want to let your student know about all the different possible paths the research could take. When a project comes to a close, both you and your students will reflect on what was accomplished, considering what has been achieved and what else still could be done. You can help your students to decide how the results of the research should be presented and help them with the presentation of their work (writing up their results and their process). Students may elect to submit their final project or write-up to publications such as *Mathematics Teacher* or to post them on websites such as the *Making Mathematics* website. Your students should feel proud of the work that they have done, and giving them a public outlet for presenting their work is a rewarding way to close the relationship.

Before closing your mentor relationship, provide your students with appropriate, encouraging feedback and wish them well.