A place in Babylon Problem

Have you ever wondered why there are 60 seconds in a minute and 60 minutes in an hour?

Although our number system (the *decimal* system) is based on the number 10, our time system comes from ancient Babylonian mathematics. The Babylonians used a numerical system based on the number 60.

1. Here are Babylonian numerals for 1, 4, and 9.

V = 1   (VV / VV) = 4   (VVVV / VVVVV) = 9

A *number* is a quantity; a *numeral* is how you represent the number. For example, the symbols 4, IV, and ∷ are all representations of the number we call "four." Each symbol is a different type of numeral.

(a) What does each V symbol represent?

(b) How would you write 2 using Babylonian numerals? How would you write 8?

2. Here are Babylonian numbers for 25 and 47.

(< VV / <VVV) = 25   (<< VVV / <<VVVV) = 47

(a) What does each $<$ symbol represent?

(b) How would you write 33 using Babylonian numerals? How would you write 56?

3. In the Babylonian number system, *place values* are used to represent numbers larger than 59. Before you learn the Babylonian system, think about the decimal system, which you're more familiar with.

(a) In the numeral 2,384, the 4 is in the units or ones place, and it represents four ones (4). The 8 is in the tens place, and it represents eight tens (80).

i. The 3 is in what place? What does it represent?

ii. The 2 is in what place? What does it represent?

(b) Think about the place names and how they change. On the right is the ones place. To the left of the ones place is the tens place. To the left of the tens place is . . . . What operation (such as addition, subtraction, multiplication, or division) can you use to find the value of each new place? What numbers do you use with that operation?

For example, the 1 in 3,824,138,272 is in the hundred-thousands place. It represents one 100,000. What operation can you use with 100,000 to find the value of the place the 4 is in?

4. While the decimal system is based on 10, the Babylonian system is based on 60. (It's called a *base-60* system.)

(a) How is the decimal system's base (10) shown in how you calculate the amounts of the place values?

(b) For the Babylonian system, use the same idea for place values as you did for the decimal system, except use 60 instead of 10. Find the values of the following numbers. (You might find it helpful to know that $60 \times 60 = 3{,}600$ and $60 \times 60 \times 60 = 216{,}000$.)

i. V <<<VVVVVV

ii. <<VVVVVVV <<<

iii. <<VVV VVVVVVV <<<<<

iv. <V <<<<VVVVVVVVV <<<VVVV VVVVV

5. Write the following numbers using the Babylonian number system.

(a) 92
(b) 920
(c) 3,848
(d) 299,688

6. One disadvantage of the Babylonian system is that people can sometimes misinterpret whether space between two symbols is meant to separate places or is just a little extra space within the same place. (Is V V meant to be 2 or 61?) Why doesn't the decimal system have a similar disadvantage?

7. Think about how you would write 60 in this system. The Babylonian system has no 0 or "place-holder" for place values that should be empty. Suppose someone had written the following:

My grandfather is now V, and he owns VV oxen.

(a) How old do you think the grandfather is? Explain.

(b) Could someone misread how many oxen the grandfather owns? Explain.

Hints

Hint to problems 1 and 2. The relative position of the symbols doesn't matter, just how many there are.

Hint to problem 3(b). Write the numerals for a few place value names: 1, 10, 100, and so on. Can you add, subtract, multiply, or divide each by the same number to get the next one in the sequence?

Hint to problem 4(a). Refer to your answer to problem 3(b).

Hint to problem 4(b). The first place (on the right) is the ones. The next place is the 60s, so a V in the second place from the right represents 60, and a $<$ represents 10×60. Use the operation from your answer to problem 3b, using 60 instead of 10, to find the value of the place to the left of the 60s place.

Hint to problem 5. First, find the largest place value that is *less* than the number you're trying to write.
Then write the largest numeral that you can use in that place value and still have the result be less than the number you're trying to write.
Next, find the difference between the amount represented by what you just wrote and the number you're trying to write.
Now find the numeral you need to write in the next place. . . .

For example, to write $1,590$, first recognize that it is greater than 60 but less than $3,600$, so you start with the second place (the 60s). There are twenty-six 60s in $1,590$, so the leftmost place will have the numeral for twenty-six in it. Since $26 \times 60 = 1,560$, you now need to figure out how to write the numeral for 30.

Hint to problem 6. How many symbols does each system use?

Hint to problem 7. If the decimal system had no place holder, the numbers 1, 10, and 100 would all be written the same way: 1. What age seems most reasonable for the grandfather?

Answers

The actual position of the symbols doesn't matter, just how many there are.

1. (a) 1

(b) $\vee\vee = 2 \qquad \vee\vee\vee\vee\vee\vee\vee\vee = 8$

2. (a) 10

(b) $<<<\vee\vee\vee = 33 \qquad <<<<<\vee\vee\vee\vee\vee\vee = 56$

3. (a) i. The 3 is in the hundreds place. It represents three hundreds (300).

ii. The 2 is in the thousands place. It represents two thousands (2,000).

(b) Multiply a place value by 10 to get the value of the next place to the left.

4. (a) The base (10) is multiplied by each place value to get the next one. The first place value is 1, the next is 1×10, the next is $1\times10\times10$, the next is $1\times10\times10\times10$, and so on.

(b) i. 96

ii. 1,650

iii. 83,270

iv. 2,554,445

5. (a) $\vee \quad <<<\vee\vee$

(b) $<\vee\vee\vee\vee\vee \quad <<$

(c) $\vee \quad \vee\vee\vee\vee \quad \vee\vee\vee\vee\vee\vee\vee\vee$

(d) $\vee \quad <<\vee\vee\vee \quad <\vee\vee\vee\vee \quad <<<<\vee\vee\vee\vee\vee\vee\vee\vee$

6. See solution.

7. (a) 60

(b) Yes; see solution for explanation.

Solutions

If you want to see more about Babylonian mathematics (including operations), try www.math.tamu.edu/ don.allen/history/ babylon/babylon.html.

1. (a) There is 1 V for 1, 4 Vs for 4, and 9 Vs for 9, so each V represents 1.

(b) Use two Vs for 2 and eight Vs for 8. The position doesn't matter, for example, eight in a row is the same as two rows of four.

2. (a) Without the five Vs that represent 5, there are two $<$ symbols left to represent 20. So each $<$ represents 10. The numeral for 47 confirms this.

(b) For 33, use three $<$ symbols to represent 30 and three Vs for the remaining 3. For 56, use five $<$ symbols to represent 50 and six Vs for the remaining 6.

3. (a) See answers.

(b) The values from right to left are 1, 10, 100, 1,000, and so on. Since these numbers are increasing, think about addition and multiplication first. Using addition to go from 1 to 10, you add 9; to go from 10 to 100, you add 90. Using multiplication, you multiply by 10 to go from 1 to 10, and you multiply by 10 to go from 10 to 100. This pattern holds for each place value change, so the operation is multiplication. The numbers multiplied are 10 and the value of the current place.

4. (a) See answers.

(b) The first place on the right is the ones place. The second from the right is the 60s. The third from the right has value 60×60, which is 3,600. The fourth from the right has value $60 \times 60 \times 60$, which is 216,000. Find the amount represented by the symbols in each place, and multiply that amount by the appropriate value. The number represented is the sum of those products.

i. $1 \times 60 + 36 \times 1 = 96$

ii. $27 \times 60 + 30 \times 1 = 1,650$

iii. $23 \times 60 \times 60 + 7 \times 60 + 50 \times 1 = 83,270$

iv. $11 \times 60 \times 60 \times 60 + 49 \times 60 \times 60 + 34 \times 60 + 5 \times 1 = 2,554,445$

5. (a) Since 92 is more than 60 but less than 3,600, start with the 60s place. There is only one 60 in 92, so the 60s place will have only one V in it. Take the one 60 from 92, and 32 remains. Put the 32 in the ones place (three < symbols and two Vs).

(b) Since 920 is more than 60 but less than 3,600, start with the 60s place. There are fifteen 60s in 920 (because $920 \div 60 = 15\frac{1}{3}$), so the 60s place will have one < and five Vs. Take 15×60 from 920 to get 20; that means you need two < symbols in the ones place.

(c) Since 3,848 is more than 3,600, start with the 3,600 place. There is one 3,600 in 3,848, so put one V in the 3,600 place. The remaining amount is 248, which is four 60s, plus another eight 1s. Put four Vs in the 60s place and put eight Vs in the ones place.

(d) The fourth place from the right has a value of 216,000, so start there. There is only one 216,000 in 299,688, so put one V for that place. The remaining amount is 83,688. The third place from the right is 3,600. There are twenty-three 3,600s in 83,688, so put two < symbols and three Vs in the 3,600 place. This leaves $83,688 - 23 \times 3,600$, which is 888, for the last two places. There are fourteen 60s in 888, so put one < and four Vs in the 60s place. Since $14 \times 60 = 840$, the remaining 48 go in the ones place: four < symbols and eight Vs.

6. The decimal system use is base 10 and has 10 symbols. Only one symbol may be used in any given place, so a number can't be misinterpreted.

7. (a) If the V is in the ones place, the grandfather would only be 1 year old, which is impossible. If the symbol is in the 60s place, the grandfather would be 60, which is reasonable. The next possible value for that number is 3,600, which is impossible.

Since there were no place holders, the reader of a Babylonian numeral had to determine from the context what number was meant.

(b) The number of oxen might be 2 or 120. Depending on how expensive oxen are and how many are useful to have, either of these numbers seem reasonable. (The next possible value is 7,200, which seems very unlikely.)

