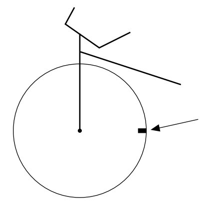
Spinning wheel 1

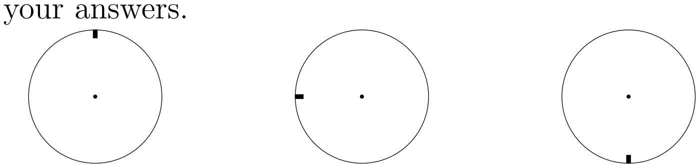


Imagine the air valve on the front tire of a bicycle is in the 3-o'clock position, as shown here.

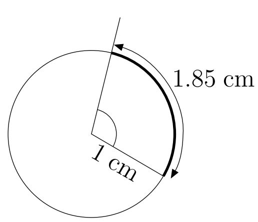
Is it possible to predict the valve's position on the wheel if you know how far the bicycle has moved (in a straight line)? Use the following problems to explore this question.

1. Bicycle wheels come in different sizes. To make calculations a little simpler at first, let's make up a unit of measure so that the bike wheel has radius 1. Call the unit a "spoke," so the wheel has radius 1 spoke.
Now suppose the bike moves one complete rotation—just enough to bring the air valve back to the 3 o'clock position. How far (in spokes) did it travel?

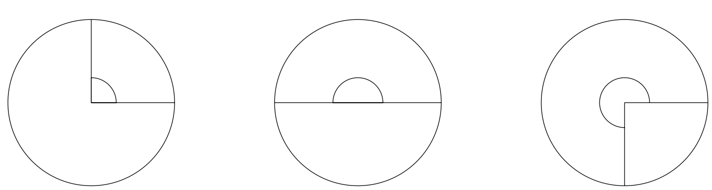
2. The first time the air valve is in each of the following positions, how far has the bike moved? Explain how you found your answers.



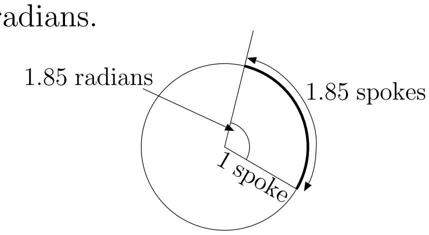
3. There is more than one way to measure an angle. You're probably used to measuring using *degrees*.
Another way uses the length of the arc on a circle of radius 1 defined by the angle, when the center of the circle and the angle's vertex are the same point:



Find the lengths (in spokes) of the arcs defined by each of the marked angles below. Assume each circle has a radius of 1 spoke.



Since you are already using "spokes" as a measure of distance, it might be a little confusing to also use it for an angle measure. Actually, mathematicians have their own name for this angle measure unit: a **radian**. If an angle cuts an arc (from a circle with radius 1 spoke) with length x spokes, that angle has a measure of x radians.

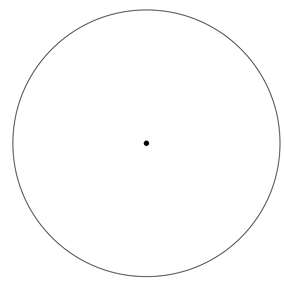


4. There are 360° in a circle. That is, imagine placing a segment with one endpoint at a circle's center. If you hold that endpoint in the same place and turn the segment until it returns to its original position, you have turned it 360°. How far has it turned in radians?

5. Suppose the air valve is at the 3 o'clock position, and then the bike moves forward $\frac{\pi}{4}$ spokes.

(a) Show about where the air valve is after the bike moves. (Assume the circle has a radius of 1 spoke.)

(b) Draw a line segment from the original position of the air valve to the center of the wheel. Then draw a segment from the position you marked to the center of the wheel. What is the measure, in radians, of the angle formed by these two segments?



6. For each of the following distances, mark the valve's approximate position after the bike has traveled the given distance. (If you put all these on the same drawing, be sure to label them.)

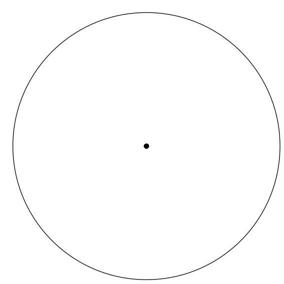
(a) $\frac{\pi}{6}$ spokes

(b) 2 spokes

(c) 4 spokes

(d) $\frac{7\pi}{4}$ spokes

(e) 3π spokes



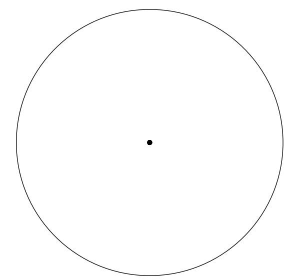
7. Imagine a line segment from the center of the wheel to the air valve. For each distance in problem 6, give the angle measure, in radians, that the line will have moved.

8. For each of the angle measures in problem 7, give the equivalent degree measure. How can you easily convert from radians to degrees? From degrees to radians?

9. You might be wondering how all of this works with real bicycle wheels. Wheels come in different sizes, but one standard size has a 26 inch *diameter.* Suppose a bike has a 26-inch wheel, and the air valve starts in the 3 o'clock position. Mark the valve's position after the bike has traveled the following distances. (Be sure to label your answers.)

(a) 7 inches
(b) 2 feet
(c) 4 feet
(d) 6 feet
(e) 10 feet

Note that 26 inches is the diameter and not the radius!



1 mile = 5280 feet

10. How many revolutions (turns of $360°$ or 2π radians) does a 26-inch wheel make when it travels 1 mile?

Hints

Hint to problem 1. Imagine laying a string in a straight line in front of the bike, with one end at the point where the bike tire touches the ground. After the bike moves forward so that the air valve is back in the 3 o'clock position, cut the string where the tire touches it. Then, imagine wrapping the string around the tire. . . .

Hint to problem 6. Try figuring out what fraction of the wheel's circumference has been traveled. For part (e), you might find it helpful to remember where the air valve should be after the bike travels 2π spokes.

Hint to problem 8. To convert from radians to degrees, you need to remember two things: there are 2π radians in a complete turn, and there are 360° in the same complete turn. If an angle is a particular fraction of a complete turn in radians, it's the same fraction of a complete turn in degrees.

Hint to problem 9. You might convert the distances to spokes (1 spoke = ? inches). Or, you might find the distance (in inches) for a complete turn.

Answers

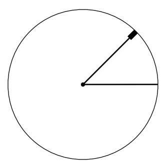
1. 2π spokes

2. In order: $\frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$ spokes. Explanations may vary. For example, the circumference of the full circle is 2π spokes, so each quarter-circle represents a distance of $\frac{2\pi}{2}$ or $\frac{\pi}{2}$ spokes. Multiply $\frac{\pi}{2}$ by the number of quarter-circles the bike has to travel to get in that position.

3. In order: $\frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$ spokes

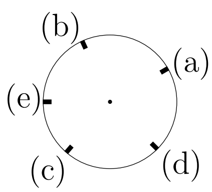
4. 2π radians

5. (a)



(b) $\frac{\pi}{4}$ radians

6.

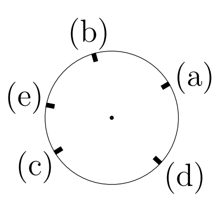


7. (a) $\frac{\pi}{6}$ radians
(b) 2 radians
(c) 4 radians
(d) $\frac{7\pi}{4}$ radians
(e) 3π radians

Teacher's Note: Students may have trouble with the concept of an angle that's greater than a complete revolution, since such an angle can't be shown with a static diagram. You might demonstrate such a turn using a pencil, holding one end fixed and rotating the pencil about that end through $1\frac{1}{2}$ revolutions.

8. (a) $30°$
(b) $\frac{360}{\pi}°$, or about $114.6°$
(c) $\frac{720}{\pi}°$, or about $229.2°$
(d) $315°$
(e) $540°$

9.



10. about 775.7 revolutions

