Checkers challenge Problem

Here are the rules for a two-player game. To play, you need a standard die (numbered 1–6) and a set of checkers.

If checkers are not available, you can use two colors of other counters (disks, blocks, or even colored paper). Use twelve of each color.

Checkers Challenge

Decide which player will use which color. Each player should roll the die. The player rolling higher goes first.

On your turn, roll the die.

- If the roll is from 1 to 4, put that many checkers into a pile between you and your opponent.
- If the roll is 5, remove one of your checkers from the pile, if possible. If that's not possible, put one of your opponent's checkers into the pile, instead.
- If the roll is 6, remove two of your checkers from the pile, if possible. If that's not possible, put two of your opponent's checkers into the pile, instead.
- Record your roll, who has more checkers in the pile, and how many more checkers that person has.

Play continues until all of one player's checkers are in the pile. The player with more checkers in the pile wins.

1. Play a game of *Checkers Challenge* with a partner.

2. How did you find who was ahead, and by how much, after each turn?

3. If you roll a 6 but can't remove two of your checkers, you must add two of your opponent's. Why do these actions have the same effect on who's ahead and by how many?

4. Craig and Juanita decided it would be easy to see who was ahead, and by how many, if they arranged their checkers in rows. At one point in their game, their pile looked like this:

Craig's red checkers 

Juanita's black checkers 

Who was ahead, and by how many?

5. Suppose the black checkers represent positive numbers and the red checkers represent negative numbers. For example, four red checkers represents -4.

(a) What positive and negative numbers are represented in Craig and Juanita's pile?
(b) Juanita put two more black checkers in the pile. How can you combine your answer to problem 4 with the number that two black checkers represents to find by how many Juanita was ahead?
(c) By how many was Juanita ahead?
(d) Craig rolled a 6 and took two of his red checkers out. What number is represented by those red checkers?
(e) Write a calculation (such as $5 + (-3)$ or $3 - 1$) that combines your answers to parts (c) and (d) to find by how many Juanita was ahead.

6. Remember that removing two of one color had the same effect as putting in two of the other color. What does that tell you about how to find $4 - (-3)$? What is $4 - (-3)$?

7. Here's a variation on *Checkers Challenge.*

Numbers Challenge
Decide which player will use positive numbers and which will use negative numbers. Each player should roll the die. The player rolling higher goes first. Start with a score of 0.

On your turn, roll the die.

- If the roll is from 1 to 4, add a positive or negative number to the score, depending on if you're using positive numbers or negative numbers. For example, if you're using negative numbers and roll a 3, add -3 to the score.
- If the roll is 5, subtract 1 or -1 from the total score. If you're using negative numbers, subtract -1.
- If the roll is 6, subtract 2 or -2 from the total score. If you're using negative numbers, subtract -2.
- Record your roll and the total score. Be sure both players agree on the score!

Play until a total of twenty rolls have been made (ten for each player). If the score is positive, the player using positive numbers wins. If the score is negative, the player using negative numbers wins. If the score is 0, the game is a tie.

(a) Play the game twice so that each player uses both positive and negative numbers.
(b) Explain how *Checkers Challenge* and *Numbers Challenge* are the same game (except for when each ends).

Scoresheet

Checkers Challenge

Example:

Roll	3	4	6	1	
Action	put in 3 red	put in 4 black	removed 2 red	put in 1 black	
Who's Ahead by How Many	Craig, by 3	Juanita, by 1	Juanita, by 3	Juanita, by 4	

First player: _____ Second player: _____

Roll								
Action								
Who's Ahead by How Many								

Roll								
Action								
Who's Ahead by How Many								

Roll								
Action								
Who's Ahead by How Many								

Roll								
Action								
Who's Ahead by How Many								

Scoresheet

Numbers Challenge

Example:

Roll Number	1	2	3	4	5
Roll	3	4	6	1	
Action	$+(-3)$	$+4$	$-(-2)$	$+1$	
Score	-3	1	3	4	

First player: _____ Second player: _____

Roll Number	1	2	3	4	5	6	7	8	9	10
Roll										
Action										
Score										

Roll Number	1	2	3	4	5	6	7	8	9	10
Roll										
Action										
Score										

First player: _____ Second player: _____

Roll Number	1	2	3	4	5	6	7	8	9	10
Roll										
Action										
Score										

Roll Number	1	2	3	4	5	6	7	8	9	10
Roll										
Action										
Score										

Hints

Hint to problem 3. Imagine you *could* remove two checkers, and then focus on how many one player is ahead by. What happens to that number if you take away two of that person's checkers? If you put in two of the other person's checkers?

Hint to problem 5. For part (b), first consider what number is represented by the black checkers. How many more black checkers are there before Juanita puts two more in? How many more are there after she puts them in? What mathematics operation (such as addition, subtraction, multiplication, or division) gives the same result as putting checkers into the pile?

Think about part (e) in the same way. What mathematics operation is represented by removing checkers? What number is represented by the checkers begin removed?

Hint to problem 6. Try representing the subtraction problem using checkers. Create a pile with which the appropriate color is ahead by 4. What does it mean (in terms of the checkers) to subtract -3?

Hint to problem 7. If you have trouble with your calculations, try representing the score with checkers. Should you put checkers in or take them out? Which color?

When you're explaining why the games are the same, focus on the four items with bullets in the rules for each.

Answers

1. Results will vary.

2. See solutions.

3. See solutions.

4. Juanita was ahead by 3.

5. (a) See solutions.
(b) Add her old lead, 3, and the new 2.
(c) She was ahead by 5.
(d) -2
(e) Her new lead was $5 - (-2)$.

6. 7; see solutions for explanation.

7. (a) Results will vary.
(b) See solutions.

Solutions

1. Results will vary.

2. One method is to count the number of each color. The player with the larger number in the pile is ahead. Subtract the smaller number from the larger number to find by how many.

Another method is to arrange the checkers in pairs, one of each color. Count any checkers left over. That's by how many that player is ahead.

Yet another method is to keep a running total. When a player puts in a number of checkers, the number that the winning player is ahead by will go up or down by that number. (Whether it's up or down will depend on which color is being put in.) If the total goes down to 0 before all the checkers for the turn have been put in, the number starts going back up again, but the other player is now ahead. Removing checkers works the same way. Removing checkers of the winning player makes the total go down.

3. If you could remove two checkers and you're ahead, it makes the number you're ahead by go down by two. If your opponent puts two in, they have more checkers so the amount your ahead goes down that way, too. If you could remove two checkers and you're behind, you get even further behind. If your opponent adds two, they have more checkers so you get further behind that way, too. In each case, the amount of change is 2, so there's always the same effect.

4. The first five of each color can be paired together. There are three unpaired black checkers, so Juanita was ahead by 3.

5. (a) There are five red checkers, which represent -5. There are eight black checkers, which represent 8.

(b) Putting two more in adds 2 to Juanita's number. She was ahead, so adding 2 increased her lead over Craig. Add her old lead, 3, and the new 2.

(c) Since $3 + 2 = 5$, Juanita was ahead by 5.

(d) Red checkers are negative, so two red checkers represent -2.

Teacher's Note: You may want to go through the rules with the class before they start playing, maybe even modeling a round or two. The scoresheets may help some students organize their work.

(e) Craig took away 2, so he subtracted -2 from the Juanita's lead. Her new lead was $5 - (-2)$.

6. Removing means you're subtracting from the total number in the pile, and putting in means you're adding. If removing one color (such as red) has the same effect as putting in the other (black), then subtracting a negative number has the same effect as adding a positive number. So $4 - (-3) = 4 + 3 = 7$.

7. (a) Results will vary.

(b) In *Checkers Challenge,* if you roll 1 to 4, you put your own checkers into the pile. That means you're adding your type of number to the score. If you roll a 5 or 6, you remove your own checkers, if you can. That means you're substracting from the score. If you can't remove your own checkers, you have to put in some of your opponent's checkers—the effect on the score is the same, and when you're subtracting a negative number, at least, it's easier to think of it as adding the corresponding positive number. Then, in both games, you have to figure out who's ahead and by how many. In *Numbers Challenge*, the score tells you by how many the winning person is ahead, if you ignore the sign. The sign tells you who is ahead. For example, if the number is negative, the person with negative numbers (red checkers) is ahead.

