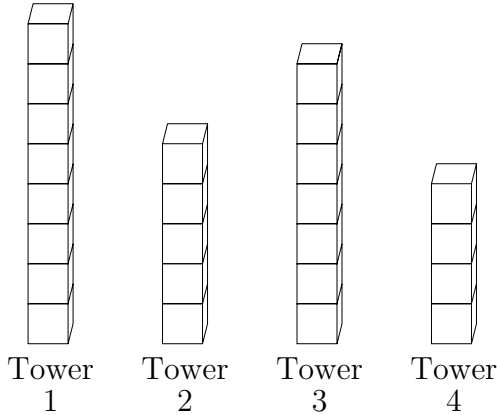


## Mean towers

Inspired by the Nuffield Mathematics Project (John Wiley and Sons)

For this problem set, you need 24 cubes.

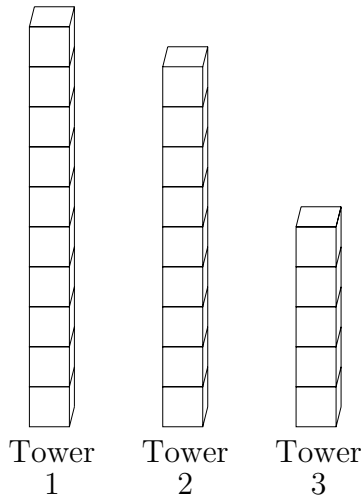
- Build the following four towers with your cubes. Think about how you might rearrange the cubes to give each tower the same height. Every time you move cubes (one or more) from one tower to another tower is called a *move*.



- What's the fewest number of moves that will give each tower the same height? Describe the moves.
- The height of the towers when they're all the same height is called the **mean height** of the towers. What is the mean height?

If you could cut the cubes, you could have mean heights that aren't whole numbers. For these towers, though, you won't need to cut cubes.

- Build these three towers.

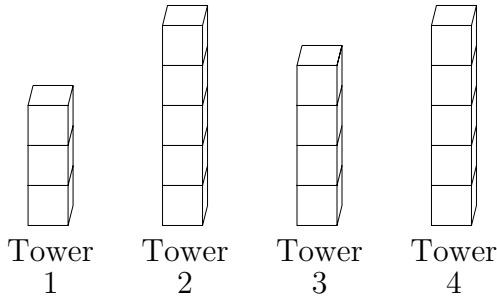


- Without moving any cubes, find the mean height of these towers. That is, if you were to move cubes so that each tower has the same height, what height would

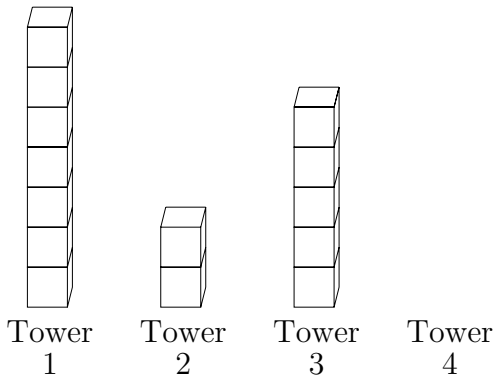
- they all have?
- (b) Describe how to give each tower the same height in the fewest moves.
  - (c) Could you have found the mean height without building the towers? What information is absolutely necessary to find the mean height?

3. Now look at the following four towers, which use only some of the cubes.

You can build them if you like, but you might not need to.



- (a) How many cubes must be added to each tower to give each a height of 6? What is the total number of cubes you must add?
  - (b) Suppose you add the total number of cubes you found in part (a), but only to the second tower. What is the mean height of the towers then?
  - (c) How many cubes (total) must be added to give a mean height of 9?
4. Here are three towers of varying heights. A fourth tower is meant stand beside them, but it hasn't been built yet. Suppose that after the fourth tower is built, you can move the cubes so that each has a height of 7. How many cubes must the fourth tower have?



The word *mean* is often used with things other than tower heights. Any group of numbers has a mean. To find the mean, you can pretend that each number gives the height of a tower with cubes that can be cut into fractions.

5. Suppose some friends each have several marbles. To play a game, they have to start with the same number of marbles, so they combine all the marbles and split them evenly.
  - (a) At first, there are four friends who have 2, 8, 6, and 12 marbles. How many marbles will each player have?
  - (b) Two more people join for the second game. One person has 8 marbles, but the other doesn't have any. How many marbles will each player have for the game?
6. Mrs. Brant gives her students five tests each grading period. To get at least a B for the grading period, each student needs a mean for the five tests of 88 or higher.
  - (a) Craig's first four test scores are 97, 88, 90, and 80. What does Craig need on his last test to get a B?
  - (b) To get an A, a student has to have a test mean of at least 94. The highest score on a test is 100. Can Craig get an A? Explain how you know.
7. Find the mean of this set of numbers:  $\{1, 4, 4, 9, 10\}$ .
8. Describe how to find the mean of a group of numbers, no matter how large the numbers are or how many there are.

Building towers to represent these scores probably isn't possible. Look back at patterns in the tower problems to figure a better way to answer these questions.

These questions are a lot like the question in problem 4.

## Answers

1. (a) Two moves is the fewest: move 2 from Tower 1 to Tower 4, and then move 2 from Tower 3 to Tower 2.  
(b) 6
2. (a) 8  
(b) Move 2 from Tower 1 to Tower 3, and then move 1 from Tower 2 to Tower 3.  
(c) Yes; you need to know the total number of cubes and the number of towers.
3. (a) You have to add 3 to Tower 1, 1 to Towers 2 and 4, and 2 to Tower 3. The total to add is 7.  
(b) 6  
(c) 19
4. 14
5. (a) 7  
(b) 6
6. (a) 85 or higher  
(b) No. To have a mean of 94, there has to be a total of 470 points. Craig has a total of 355 points so far, so he would need 115 on the last test—more than the highest possible score.
7. 5.6
8. Add the numbers together and divide by how many numbers there are.