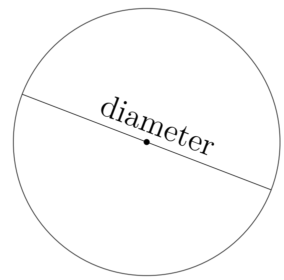
Measuring pi (π)

The sum of the side lengths of a figure like a square, a rectangle, a triangle, a pentagon (five sides), and so on, is called the figure's **perimeter.** Another way to think of the perimeter is the distance an ant would have to walk if it walked all the way around the figure.

For a circle, this distance is called the *circumference.* The distance across the circle through its center is called its *diameter.* In ancient times, thousands of years ago, mathematicians knew that the ratio

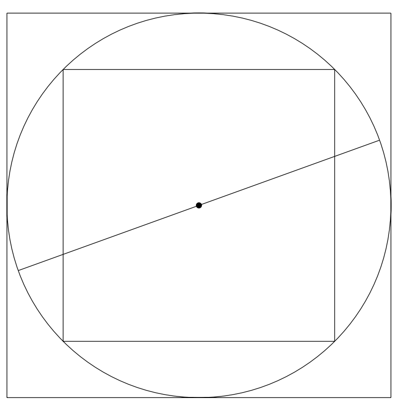


$$\frac{\text{circumference}}{\text{diameter}}$$

is always the same in every circle, no matter what its diameter is! Today, that value is known as π, the Greek letter "pi."

But just what *is* the value of π?

1. Here is a circle. A square has been drawn just outside it (circumscribed) and another has been drawn just inside it (inscribed).



(a) Measure the sides of the two squares, and the diameter of the circle.

(b) Use the measurements to find the perimeters of the two squares.

(c) The circumference of the circle is between the perimeters of the two squares; that is, it is greater than the perimeter of the small square and less than the perimeter of the large square. Why does this make sense?

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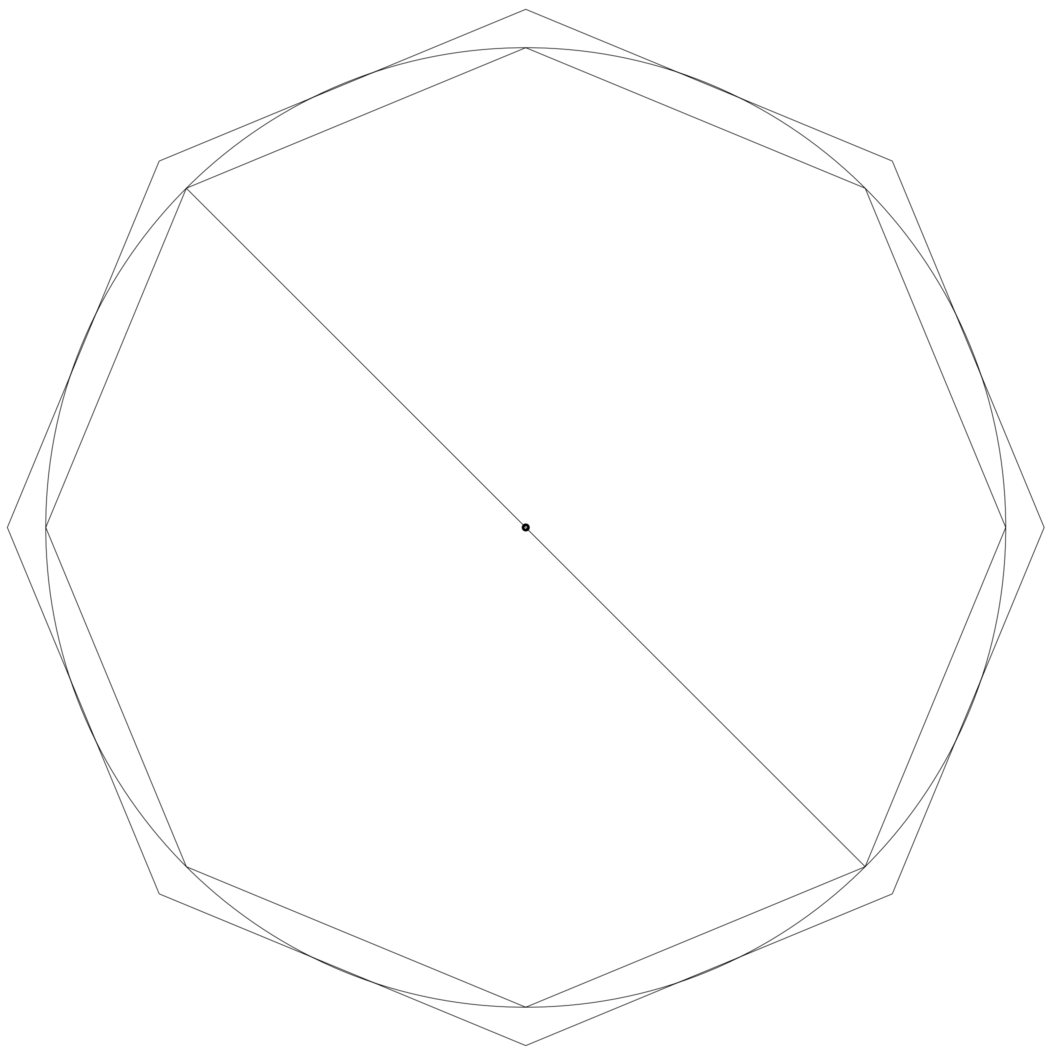
(d) Find the ratios $\frac{\text{perimeter of square}}{\text{diameter of circle}}$ for the two squares.
(e) What do you think is true about the ratio $\frac{\text{circumference}}{\text{diameter}}$? Give as good of an *estimate* of π as you can.

2. Now do the same thing with octagons instead of squares. Here, the sides of the small octagon are all the same length, and the sides of the large octagon are all the same length.



(a) Find the perimeters of the two octagons and the diameter of the circle.
(b) Find the ratios $\frac{\text{perimeter of octagon}}{\text{diameter of circle}}$ for the two octagons.
(c) Give as good of an estimate of π as you can.

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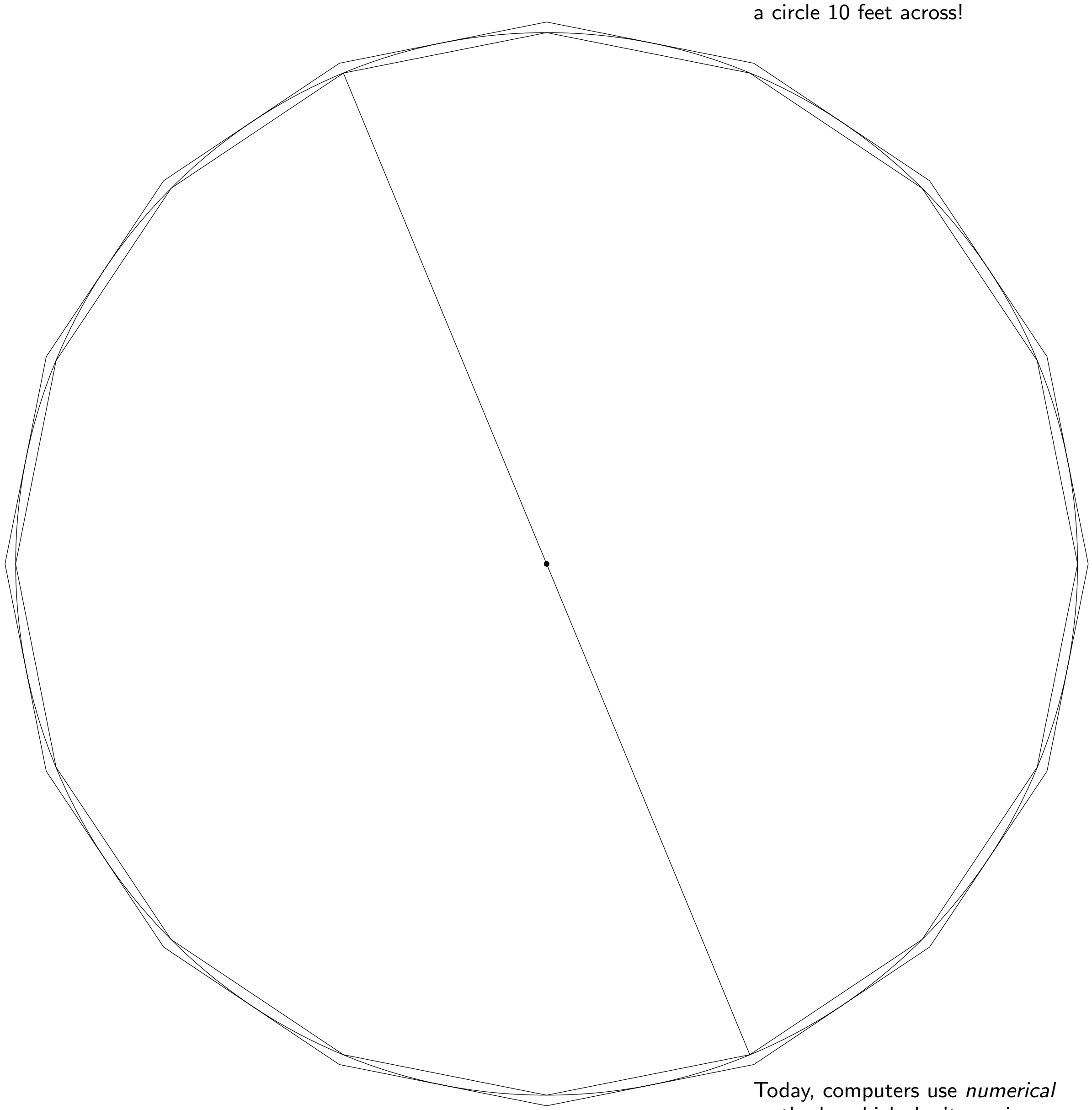
3. Next are 16-gons! These figures have 16 sides each. In each figure, the sides have the same length. Use the method of problems 1 and 2 to estimate π from these figures.

By estimating π this way, you could take a long time to find more than the first few digits accurately. An early attempt to find pi using this method required a circle 10 feet across!



4. The actual value for π isn't measurable. The number π is *irrational*, which means the decimal places go on and on without repeating or stopping. The actual value is 3.14159265.... How close did you get?

Today, computers use *numerical* methods, which don't require actually looking at figures like these, to calculate π to many digits. Calculating many digits of π is a useful way to find errors in computer hardware and software. In 1995, over 6 *billion* digits to the right of the decimal point were calculated.

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Hints

In general, make your measurements as accurately as you can. Here are some things that will help:

- Choose your units as small as possible. Compare the markings on whatever rulers you have to decide if you should measure in millimeters or inches (or any other scale you might have available). Use the side whose marks are closer together.
- Try to use sides of the figure that don't have other lines coming in. The more lines coming in to a point, the easier it is for your eye to get confused and misread the ruler. You may not always be able to choose side with no other lines, but if you can, do so.
- Don't always round to the nearest mark. If you think the end of a side sits right between two marks, or in the middle but maybe a little closer to one side than the other, use half or even quarter units. (For example, you might decide a length looks more like 20.5 mm than 20 or 21, or more like 52.25 than 52.)

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Answers

Answers are given in millimeters, although students may use other scales as well. A millimeter is smaller than $\frac{1}{16}$ inch, so unless students have finer rulers than that, they should probably use millimeters.

1. (a) The small square has sides of about 36 mm; the large square has sides of about 51 mm. The radius of the circle is also about 51 mm.

(b) The perimeter of the large square is about 204 mm; the perimeter of the small is about 144 mm.

(c) Since the circle fits between the two squares, it makes sense that its circumference is between the perimeters. Imagine that the large square were made of a loop of string, and the circle is a can. By pulling an end of the loop of string, you can make the square fit snug against the can. The extra amount of string you pulled away is the difference between the perimeter and the circumference. You can do this again with the circle as the string and a box for the square. The circle will fit around the box after you pull the string.

(d) The large square's ratio is 4; the small square's ratio is about 2.82.

(e) The ratio $\frac{\text{circumference}}{\text{diameter}}$ should be between the ratios for the two perimeters, since the circumference is between the two perimeters and they're all being divided by the same number. Estimates of π should be between 2.82 and 4. Using the average (arithmetic mean) of the two gives about 3.41.

If you look carefully at the large square and the circle, you should see that the length of a square's side has to be the same as the circle's diameter.

This reasoning works for the other polygons in this problem set, too.

Teacher's Note: Some students may want to leave their estimate as this interval. Depending on your students, you may want to ask them if, say, between 2.8 and 3.9 is likely, then would between 2.7 and 3.8 work well, too. Ask them, if you *have* to give only one value—knowing that it may not be very accurate—what would you say?

2. (a) Using 52.75 mm and 48.75 mm, the perimeters are 422 mm and 390 mm. The diameter is about 127 mm.

(b) The ratios are about 3.32 for the big octagon and about 3.07 for the small one.

(c) Estimates should be between 3.07 and 3.32. Using the average gives about 3.195.

3. Estimates for π should be between about 3.12 and about 3.19. Using the average gives about 3.16.

Teacher's Note: The side lengths are about 30.6 mm and about 30 mm—less than a millimeter of difference! These lengths are so close together, you may want to really push students not to just round to the nearest millimeter as they measure.

4. Answers will vary. The estimate of 3.16 came within 0.02 of the actual value, or within about 0.6% of the actual value. This seems like a pretty good estimate, although nowhere near 6 billion digits of accuracy!

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