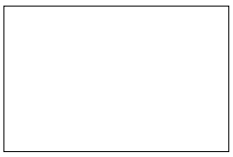
Perspective in art

In art, an important concept is that of *perspective*. Using perspective, an artist can give a two-dimensional painting or drawing a feeling of depth—as if it were three-dimensional.

1. Try this:

- Start with this rectangle. A point has been included above the rectangle. (This is called the *vanishing point*.)



- Using a straightedge and a pencil, construct line segments from the vanishing point to each vertex of the rectangle. Make these light, because you'll want to erase part of them later.
- Construct the midpoint of each of those line segments.
- Construct a rectangle, using the midpoints for vertices.
- Erase the segments from this new rectangle to the vanishing point. You can also erase the segments that define the back of the box you just created, and maybe the midpoints as well. Redraw the other lines as needed.

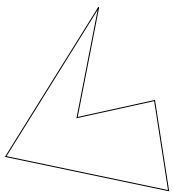
2. As you drew your box, you used similar figures. The first four steps of problem 1 is sometimes called the *projection method* of creating similar figures.

(a) What were the similar figures?

(b) What was the scale factor from the larger to the smaller?

(c) Use the method to draw a pentagon similar to this one.

The scale factor from figure A to figure B is the ratio $\frac{\text{length of side of B}}{\text{length of corresponding side of A}}$.



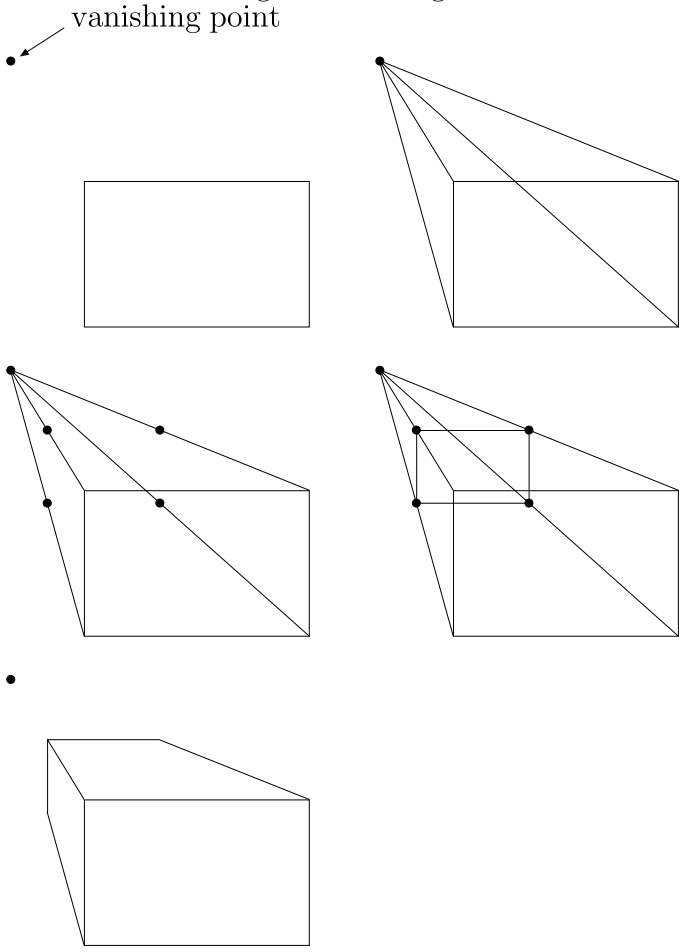
3. Below are more rectangles, but without a vanishing point. Use them (or other rectangles) to explore the following changes to the process. Then write a short report explaining the effect of the changes on how the final drawing of the box looks.

- Put the vanishing point in a different place: further away, below, to either side, or even inside the original rectangle.
- Instead of finding the midpoint of each segment, find points some fraction (not $\frac{1}{2}$) of the way from the vertex to the vanishing point. For example, find points $\frac{1}{3}$ of the way from the vanishing point to each vertex.

If you have access to dynamic geometry software, it may be more interesting to use that software and simply move the vanishing point around.

Answers

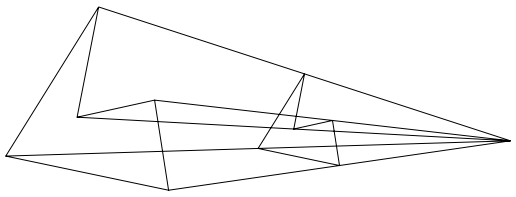
1. Here is the drawing at each stage:



2. (a) The similar figures were the original rectangle and the new rectangle (the back of the box).

(b) The scale factor from the larger to the smaller is $\frac{1}{2}$.

(c) The location of the similar pentagon may vary:

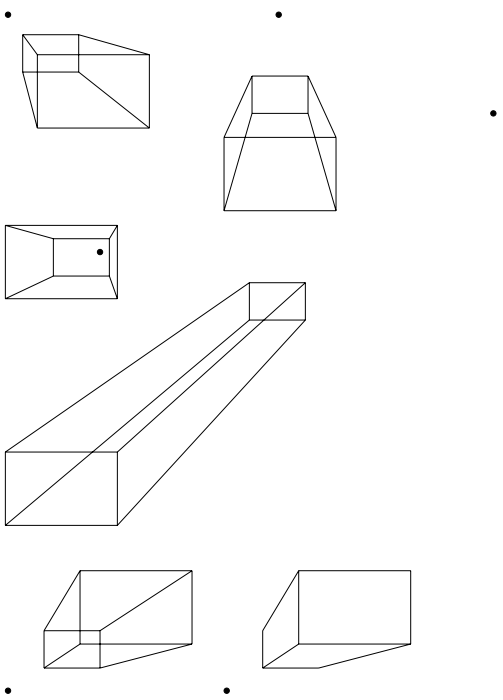


Students who have used this method before may decide to use something other than the midpoints. If so, ask them to show that their result is similar to the original pentagon.

3. There is no definitive answer to this problem; different people will see the rendering of three-dimensional object in different ways.

Here are some examples of explorations moving the vanishing point (but still using midpoints to create the similar rectangle). Most have all box edges showing. The two on the bottom show the same perspective, with and without the "hidden" edges.

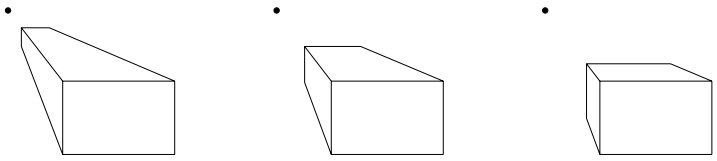
Teacher's Note: Dynamic geometry software is highly recommended for this exploration. Have students create additional segments between the two rectangles and then hide (not delete) the segments from the original rectangle to the vanishing point. Then when students move the vanishing point, the object seems even more three-dimensional, and the changing perspective becomes even more apparent.



The following explorations keep the vanishing point in the same place but use different scale factors: $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$, respectively. (The ratio

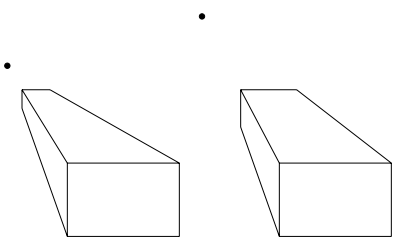
$$\frac{\text{distance from vanishing point to new rectangle}}{\text{distance from vanishing point to original rectangle}}$$

gives the scale factor.)



Interpretations of these explorations may vary. The direction of the vanishing point affects the perspective from which the observer "sees" the box: Points below and to the left of the original rectangle give a more bottom-left view, for example. If the vanishing point is inside the original rectangle, you seem to be looking inside a box. When the vanishing point is farther away, the box seems longer.

Using different scale factors is perhaps more effective to represent distance (or length), however. With smaller scale factors, the new rectangle becomes smaller and so seems farther away. Using a more distant vanishing point may instead have the effect of distorting the object—for example, the rectangular prism may seem oblique (slanted) instead of simply longer. Or, such a vanishing point may just shift the perspective to be less front-on (more above, below, or to either side). The left box below uses a scale factor of $\frac{1}{4}$, the right one uses a scale factor of $\frac{1}{2}$. In both, the new rectangles are in approximately the same place.



Teacher's Note: Allow students a lot of leeway in their reports. People do see these drawings in different ways, and describing exactly what you see can be very difficult. You may prefer to discuss this as a class rather than require written reports.

