The size of an atom
Problem

The smallest unit of an element, such as hydrogen, oxygen, or gold, is the atom. Each atom is made up of electrons, protons, and neutrons. Because these particles are so small, scientific notation is helpful when talking about chemistry.

Write all your answers for the following problems in scientific notation, unless the directions say otherwise.

1. A proton has a mass of about 1.67252×10^{-24} grams. A single electron has about 9.1091×10^{-28} grams of mass.
  (a) How many electons does it take to have the same mass as one proton?
  (b) A hydrogen atom has one proton and one electron. Find the mass of a hydrogen atom.

2. Oxygen atoms have 8 protons and 8 electrons.
  (a) Find the mass of 8 protons.
  (b) The most common type of oxygen atom has eight neutrons. A neutron has mass 1.67482×10^{-24} grams. Find the mass of 8 neutrons.
  (c) Find the total mass of this type of oxygen atom.
  (d) Oxygen is how many times more massive than hydrogen? (Answer using standard notation, not scientific notation.)

3. Water consists of small particles called **molecules.** One molecule of water contains two hydrogen atoms and one oxygen atom.
  (a) Find the mass of one molecule of water.
  (b) Water is how many times more massive than hydrogen? (Answer using standard notation, not scientific notation.)
  (c) Compare your answer to part (b) with your answer to problem 2, part (d).

4. One gram of hydrogen has about 6×10^{23} atoms. To help you understand just how much this is, consider that a small paper clip has about a gram of mass, which weighs about 2.205×10^{-3} pounds. For this problem, write your answers using both scientific notation and standard notation.

(a) Some adults weigh around 150 pounds. About how many paper clips would it take to weigh that much?

(b) A car might weigh around 3,000 pounds. How many times heavier is a car than a 150 pound person?

(c) About how many paper clips are needed to weigh as much as a 3,000 pound car?

(d) About how many 3,000 pound cars would it take to weigh as much as 6×10^{23} paper clips? (Answer in scientific notation only.)

(e) There are almost 300 million people in the United States, including children.

i. About how many cars (on average) would each person in the United States need to own for the combined weight to be as much as the weight of 6×10^{23} paper clips?

ii. Compare your answer for part i to the number of people in the United States.

(f) There are about 6 billion people in the world.

i. About how many cars (on average) would each of those people need to own for the combined weight to be as much as the weight of 6×10^{23} paper clips?

ii. Compare your answer for part i to the number of people in the United States.

Remember, 6×10^{23} hydrogen atoms have about the same mass as only one paper clip!

Hints

Hint to problem 1. To decide what operation to use, think of a similar situation with numbers that are easier to work with. For example, how many five-pound bags of sugar does it take to weigh as much as a twenty-pound box of paper?

Hint to problem 4, part (d). There are two good approaches for this:

i. First find the weight of the paper clips. Should you multiply or divide by the weight of the car?

ii. Use your answer to part (c) to find how many groups of that number of clips are in 6.23×10^{23}.

Hint to problem 4, part (e). Use your answer to part (d).

Answers

1. (a) 1826.1, 1.826×10^3
(b) 1.67343×10^{-24} grams

2. (a) 1.33802×10^{-23} grams
(b) 1.33986×10^{-23} grams
(c) 2.67861×10^{-23} grams
(d) about 16 times

3. (a) 3.01329×10^{-23} grams
(b) about 18 times
(c) See solutions.

4. (a) 6.8027×10^4 paper clips, 68,027 paper clips
(b) 20 times
(c) 1.361×10^6 paper clips, 1,361,000 paper clips
(d) 4.578×10^{17} cars
(e) i. 1.526×10^9 cars per person, 1,526,000,000 cars per person
ii. See solutions.
(f) i. 7.629×10^7 cars per person, 76,290,000 cars per person
ii. See solutions.

Solutions

1. (a) Divide the mass of a proton by the mass of an electron: $(1.67252 \times 10^{-24}) \div (9.1091 \times 10^{-28}) = 1826.1 = 1.826 \times 10^3$.

(b) The mass of an electron is 9.1091×10^{-28} grams, or $0.00091091 \times 10^{-24}$ grams. Adding this to the mass of a proton, 1.67252×10^{-24} grams, gives 1.67343×10^{-24} grams.

2. (a) Multiply $8(1.67252 \times 10^{-24}$ grams) by first multiplying 8×1.67252, which is 13.3802. To move the decimal one place to the left, you *divide* by 10, so you need to multiply the 10^{-24} by 10. This gives 1.33802×10^{-23} grams.

(b) Multiply $8(1.67482 \times 10^{-24}$ grams) in the same way to get 13.986×10^{-24} grams. This is equal to 1.33986×10^{-23} grams.

(c) To get the mass of 8 electrons, multiply $8(9.1091 \times 10^{-28}$ grams) to get 72.8728×10^{-28} grams, which is equal to $0.000728728 \times 10^{-23}$ grams. Add this to the answers to parts (a) and (b) to get 2.67861×10^{-23} grams.

(d) $(2.67861 \times 10^{-23}) \div (1.67343 \times 10^{-24}) \approx 16.007$, so oxygen is about 16 times more massive than hydrogen.

3. (a) Add the masses of two hydrogen atoms (problem 1) and one oxygen atom (problem 2):
$2(1.67343 \times 10^{-24}$ grams $+2.67861 \times 10^{-23}$ grams $= 3.01329 \times 10^{-23}$ grams.

(b) $(3.01329 \times 10^{-23}) \div (1.67343 \times 10^{-24}) \approx 18.007$, so water is about 18 times more massive than hydrogen.

(c) Water is only a little more massive than oxygen (18 times compared to 16 times). This makes sense, because water is oxygen with two hydrogen atoms, so it's only two hydrogen atoms more massive than oxygen is.

4. (a) Divide the total weight by the weight of one paper clip to find the number of paper clips: 150 lbs $\div(2.205 \times 10^{-3}$ lbs) $= 6.8027 \times 10^4$ paper clips, which is 68,027 paper clips.

(b) Divide the weight of the car by the weight of a person: $3{,}000 \div 150 = 20$.

(c) Again, divide the total weight by the weight of one paper clip to find the number of paper clips:

3,000 lbs $\div(2.205 \times 10^{-3}$ lbs$) = 1.361 \times 10^6$ paper clips, which is 1,361,000 paper clips.

(d) This can be calculated by finding the total weight of the paper clips and then dividing by 3,000 (the weight of one car); or by dividing the total number of paper clips by the number needed to weigh 3,000 pounds (part b). Using the first method as an example, the total weight is $(6.23 \times 10^{23}) \times (2.205 \times 10^{-3}$ lbs), which is 1.3737×10^{21} lbs. Dividing this by 3,000 gives about 4.578×10^{17} cars.

(e) i. You know the number of cars from part c, so now you need to distribute them among the U.S. population by dividing 4.578×10^{17} cars by 300 million people, giving 1.526×10^9 cars per person, which is 1,526,000,000 cars per person.

ii. $(1.526 \times 10^9) \div (3 \times 10^8) \approx 5.09$, so each person would own about 5 times as many cars as there are people in the United States.

(f) i. Distributing the 4.578×10^{17} cars to 6 billion people (by dividing) gives 7.629×10^7 cars per person, which is 76,290,000 cars per person.

ii. $(7.629 \times 10^7) \div (3 \times 10^8) \approx \frac{1}{4}$, so each person would own about $\frac{1}{4}$ times as many cars as there are people in the United States. In other words, every group of four people would own as many cars as there are people in the United States.

