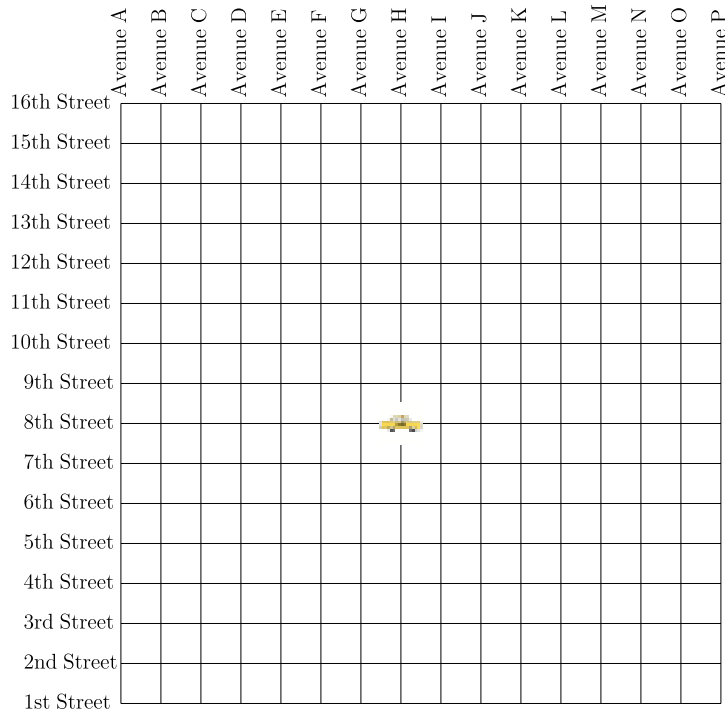


Taxicab figures

Anna drives a taxi in Grid City. All the streets are laid out in equal sized blocks, like a grid. In this picture, Anna is in her taxi at the intersection of 8th St. and Avenue H.



Anna's city gives us an interesting geometry that's a little different from the geometry you're used to.

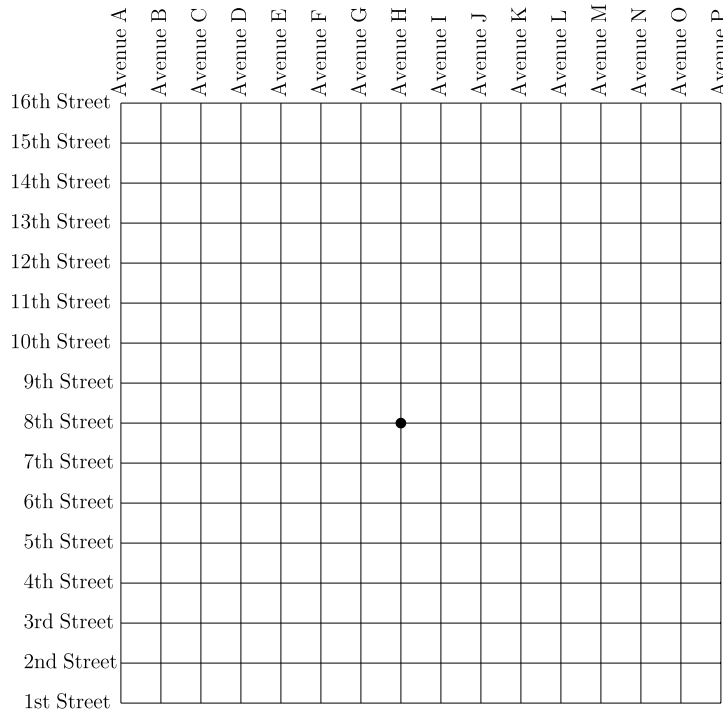
1. When a customer tells her a destination, Anna thinks about the nearest intersection to it and figures how many blocks she'll have to drive to get there. Anna thinks of each intersection as a **point**—and there are no points other than intersections.

Find the following points on the city above, and describe a route for Anna to take to each point.

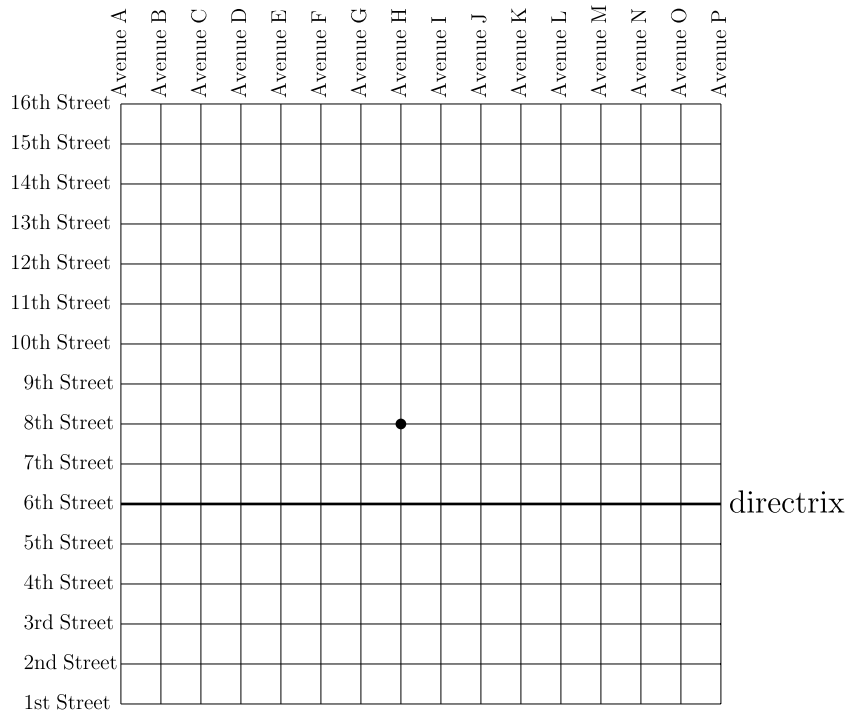
- (a) 3rd St. and Avenue L
 - (b) 10th St. and Avenue K
 - (c) 9th St. and Avenue B
2. Find three different routes for Anna to take to 4th St. and Avenue D. Make one a different length than the other two.
 3. What is the shortest distance Anna would have to travel to get to 4th and D?

4. A route that Anna can travel to a destination *without repeating a block* is a **line** between where she is and that destination point.
- Can there be more than one line between two points?
 - What kind of line should Anna use to figure the distance she has to travel to a destination?
5. A *circle* is the set of points that are a particular distance (called the radius) from another point (called the center).

Anna would want to use a shortest path between two points, so by *distance* we mean the length of a shortest path.



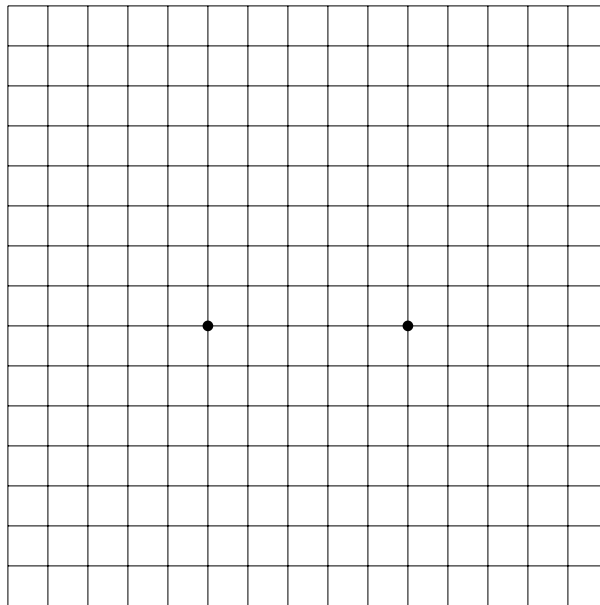
- Mark all the points that are 1 block from 8th and H.
 - Mark all the points that are 4 blocks from 8th and H.
 - What does a circle look like in “taxicab geometry”? Describe it as fully as you can.
6. A *parabola* is the set of points whose distance to a fixed line (called the directrix) is the same as the distance to a fixed point (called the focus). The distance from a point to a line is the shortest distance from the point to any point on the line. For example, the distance from 8th and H to Avenue C is 5, because 8th and C is 5 blocks away. To get to any other point on Avenue C would require traveling more than 5 blocks.
- On the following grid, mark the points that form the parabola with 6th St. as the directrix and 8th and H as the focus.



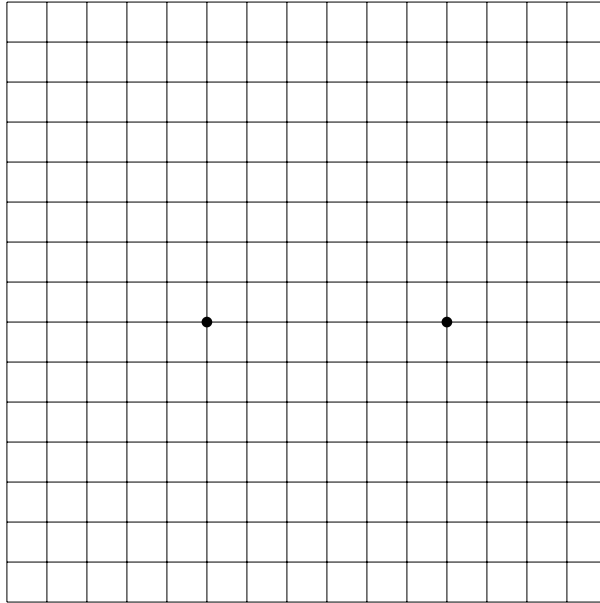
7. An *ellipse* is the set of points whose distances to two fixed points (the foci) have a constant sum. (For example, a point A is on the ellipse with foci B and C and constant d if $AB + AC = d$.)

Foci is the plural of *focus*.

Mark the points whose distances to the marked intersections add to 9.

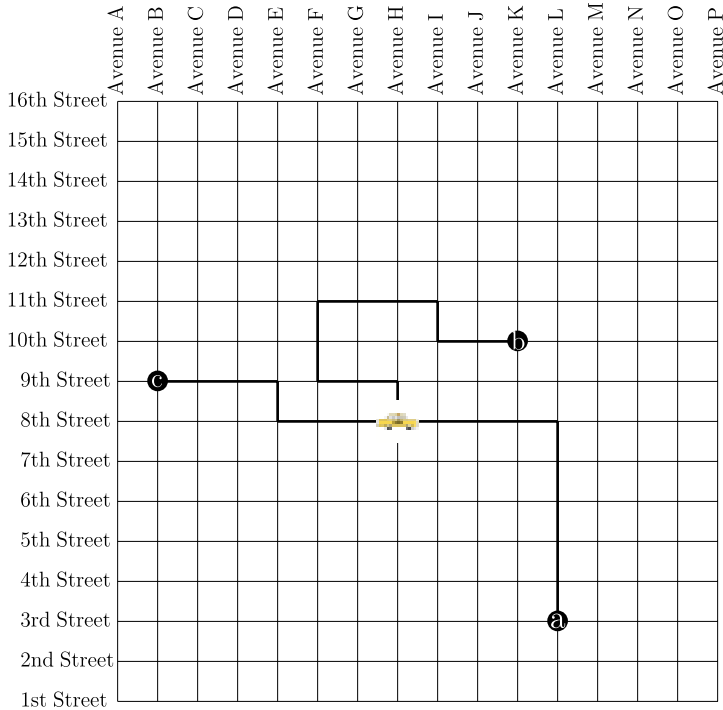


8. A *hyperbola* is the set of points whose distances from two fixed points (foci) have a constant difference.
Mark the points whose distances to the marked intersections have a difference of 4.

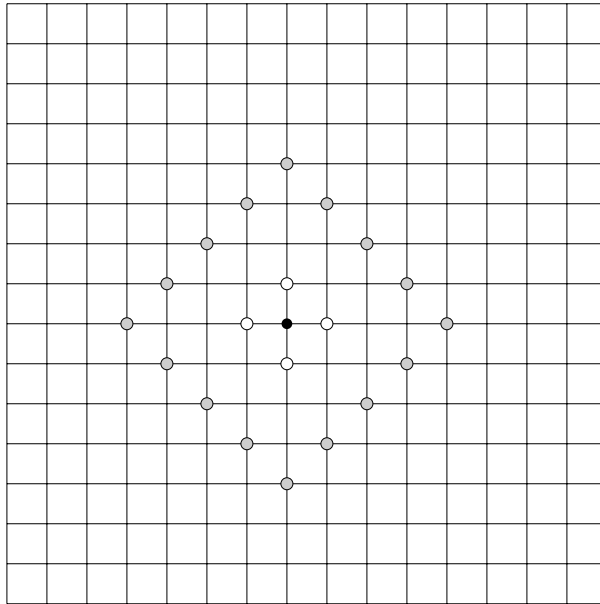


Answers

- Routes can vary greatly, as long as it begins at 8th and D and ends in the proper place. Students may draw the routes or describe them. (For example, go down 5 streets and then right 4. OR, go South 5 streets, turn *left*, and go another 4 streets.) An example is given for each.

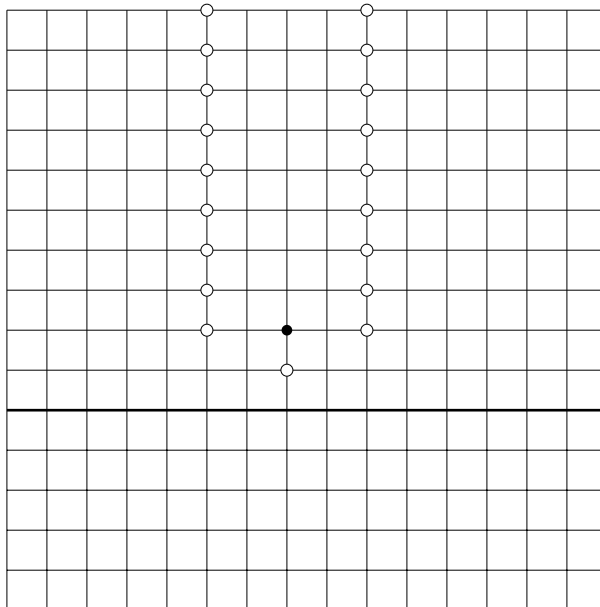


- Short routes will go down 4 streets and left 4 streets. They might do this directly (4 down and 4 left, or 4 left and 4 down) or in steps (for example, 3 down, then 1 left, then 1 down, then 3 left). Longer routes will include at least one right or down.
- 8 blocks
- (a) Yes. (The routes in problem 2 are examples.)
(b) One with at most two directions, one each of up/down and left/right.
- Parts a and b are shown in the following grid. Part a points are white, part b points are grey.

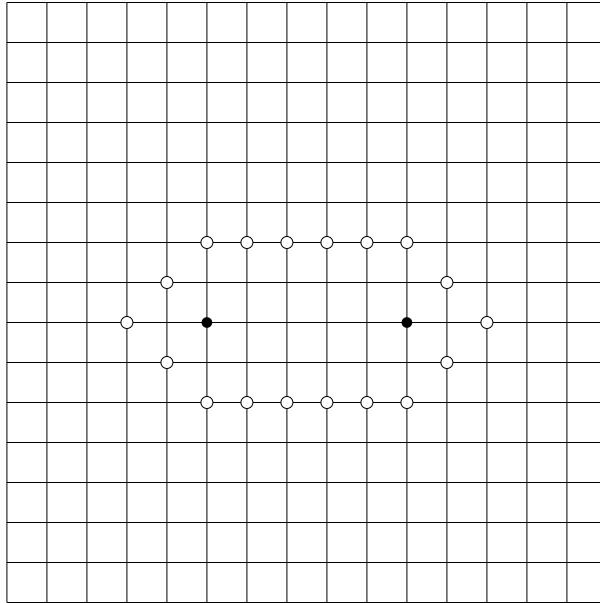


- (c) In this geometry, a circle forms a Euclidean square, with vertices on the four axis. The points making up the sides have slopes of either 1 or -1 . If the radius is r , there will be $r + 1$ points on each side, including both vertices. (Thus the circle will have $4r$ points in it.)

6. The parabola looks like this:



7. The ellipse looks like this:



8. The points in the hyperbola form two parallel Euclidean lines:

