

## What is a function? Problem

The idea for this problem sequence was suggested by Michelle Manes

In algebra we often talk about relationships between variables. We also talk about functions. Here are some questions to think about: Are relationships and functions the same things? Do relationships and functions exist only in algebra? The following problems help to answer these questions. The sidenote reminds you how to distinguish between a function and a relationship.

For each of the following examples decide whether it is a function, and explain your decision. For each example that *is* a function, describe the domain and the range.

1. Input: a number  
Output: seven more than that number
2. Input: a whole number  
Output: whether the number is even or odd
3. Input: a country  
Output: the capital of that country today
4. Input: a person  
Output: the name of that person's cousin
5. Input: the radius of a circle  
Output: the area of that circle
6. Input: the length of one side of a triangle  
Output: the perimeter of that triangle
7. Input: a person  
Output: the height of that person
8. Input: a number  
Output: always the number 186
9. Create one more example of a function and one more example of a relationship that is not a function.

Some people think of a function as a machine that takes an input and produces an output. An input can be of any sort: a number, a geometric shape, a person, a country, or anything else. But for a particular input there is always a **particular** output. For example, if the input is a number, and the output is twice a number, this is a function. Another example of a function would be: the input is a point on a coordinate plane and the output is a set of  $x$ - and  $y$ -coordinates for this point.

If there are two or more different outputs possible for one input, we are talking about a relationship, but not a function. An example of a relationship that is not a function is an input is a number and an output that is a factor of this number.

The set of allowable inputs to a function is called the **domain** of that function. The set of all outputs of the function is called the **range** of the function.

## Hints

**Hint to problem 7.** You can think of the two different situations. One is, you measure the height of many people just once. Another one is, you measure the heights of different people a number of times over a number of years.

**Hint to problem 8.** Think about each input-output pair. Is there a single output for a given input?

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## Answers

See solutions.

## Solutions

1. It is a function, because if  $x$  is a number (an input), there is only one number which can be expressed as  $x + 7$  (an output). The domain of this function is all real numbers. The range is also all real numbers.
2. It is a function, because the number is either even or odd, there is only one output for every input. The domain of this function is all whole numbers. The range is the two categories: even and odd.
3. It is a function, because for every country, there is only one capital. The domain for this function is all countries in the world. The range for this function is all capitals of the countries in the world.
4. This is not a function. A person can have more than one cousin with more than one name.
5. This is a function. A circle with a given radius has only one area. The domain for this function is any positive number. The range is also any positive number.
6. This is not a function. If only one side of the triangle is given, other sides can vary, and there could be various perimeters.
7. You can consider it a function or not a function depending on how you look at it. If you think about the height of a person throughout that person's life, then it is not a function, because for each person there are many possible heights. If you consider many different people at a single point in time, then it is a function. At a certain point in time every person has a unique height. If you consider it as a function, the domain is all people. The range is all possible heights these people have.
8. It is a function, because for any number, you have only one output, that is the number 186. The domain of this function is all real numbers. The range consists of the single number 186.
9. Answers will vary.

You can say that the height is a function of the two variables: person and time. If the person is the same, but the time varies, it is a function of time. If the time is the same, but the person varies, it is the function of a person.