

Sweet ice cream Problem

Alison created her own brand of ice cream for a project. In order to figure out the right amount of sugar for the best taste, she tried a number of different mixtures of liquid sugar with cream. To prepare liquid sugar, she dissolved 4 lb of sugar in a gallon of water. Then she mixed a number of cups of liquid sugar with a number of cups of cream, trying different combinations.

Did you ever try to dissolve 4 lb of sugar in one gallon of water? Or 1 lb of sugar in a quarter gallon?

Each of the problems below, shows two sets of sugar-cream combinations to mix. Cups of sugar are colored, and cups of cream are white. For each pair, predict which will taste sweeter. Explain your reasoning.

You can actually try to make these mixtures to check your predictions.

1.



Mixture A



Mixture B

2.



Mixture A



Mixture B

3.

Mixture A



Mixture B



4.

Mixture A



Mixture B



5.

Mixture A



Mixture B



6.

Mixture A



Mixture B



7.

Mixture A



Mixture B



Alison liked the taste of Mixture A from problem 6 the most. She decided to make ice cream for other people.

8. For her family, Alison decided to use 10 cups of her mixture. How many cups of liquid sugar and how many cups of cream should she use in order to make it to taste the same as Mixture A in problem 6?
9. For her sister Jane's birthday party, Alison decided to make 15 cups of ice cream. How should she make it to taste the same as Mixture A in problem 6?
10. For her class, Alison decided to make 18 cups of ice cream. How should she make it to taste the same as Mixture A in problem 6?

Alison's friends Leila, Sasha, and Kendall helped her. They disagreed about how to make 18 cups of ice cream for their class. They argued:

Alison: Let's just add 6.5 cups of sugar and 6.5 cups of cream to Mixture B. Increase everything by 6.5.

Sasha: Wait: 18 is 5 times 3.6. That means you should take 3.6 more sugar and 3.6 times more cream. That's 7.2 cups of sugar and 10.8 cups of cream.

Leila: Think about proportion of sugar in the mix. There are 2 cups of sugar for 5 cups of mix, that is, the proportion of sugar is $\frac{2}{5}$. Since $18 \times \frac{2}{5} = 7.2$, use 7.2 cups of sugar for 18 cups of the mix.

Kendall: Look: $3 - 2 = 1$, so you want to keep the difference between cream and sugar at 1. Since there are 18 cups total, and there is one cup more cream than sugar, we should use 8.5 cups of sugar and 9.5 cups of cream.

11. Which of the above methods will really produce ice cream that taste the same as Mixture A in problem 6? Explain.

12. Which methods are the same? Explain.

Hints

1-10. See what part of the whole mixture sugar makes.

Answers

1. Mixture A is sweeter. For explanations for all problems, see solutions.
2. Mixture A is sweeter.
3. Mixture B is sweeter.
4. Mixture A is sweeter.
5. Mixture B is sweeter.
6. Mixture B is sweeter.
7. Mixtures A and B have the same sweet taste.
8. Alison needs to use 4 cups of sugar and 6 cups of cream.
9. Alison should use 6 cups of sugar and 9 cups of cream.
10. Alison should put 7.2 cups of sugar and 10.8 cups of cream into the mix.
11. Sasha's and Leila's methods will produce 18 cups of ice cream that taste the same as Mixture A in problem 6.
12. Sasha's and Leila's methods are the same. Alison's and Kendall's methods are the same.

Solutions

You can reason about these problems in many different ways, but they should all give you the same answer for which mixture is sweeter in every problem.

1. Mixture A is sweeter, because in it sugar makes up half of the mixture, while in Mixture B sugar makes up a quarter of a mixture. Another way to reason is this: for one cup of sugar in Mixture A there is one cup of cream, while in Mixture B for one cup of sugar there are three cups of cream.
2. Mixture A is sweeter, because for one cup of cream there are two cups of sugar, while in Mixture B for one cup of cream there is only one cup of sugar. Another way to reason is this: in Mixture A, sugar makes up $\frac{2}{3}$ of the mix, while in Mixture B sugar makes up $\frac{1}{2}$ of the mix. Since $\frac{2}{3} > \frac{1}{2}$, Mixture A is sweeter.
3. Mixture B is sweeter, because for one cup of cream there are three cups of sugar, while in Mixture A for one cup of cream there are two cups of sugar. Another way to reason is this: in Mixture B, sugar makes up $\frac{3}{4}$ of the mix, while in Mixture A sugar makes up $\frac{2}{3}$ of the mix. Since $\frac{3}{4} > \frac{2}{3}$, Mixture B is sweeter.
4. Mixture A is sweeter, because for two cups of cream there are three cups of sugar, while in Mixture B for two cups of cream there are two cups of sugar. Another way to reason is this: in Mixture A, sugar makes up $\frac{3}{5}$ of the mix, while in Mixture B sugar makes up $\frac{2}{4}$ of the mix. Since $\frac{3}{5} > \frac{2}{4}$, Mixture A is sweeter.
5. Mixture B is sweeter, because in it sugar makes up $\frac{3}{5}$ of the mixture, while in Mixture A sugar makes up $\frac{4}{7}$ of a mixture. Since $\frac{3}{5} > \frac{4}{7}$, Mixture B is sweeter .
6. Mixture B is sweeter, because for two cups of sugar there are two cups of cream, while in Mixture A for two cups of sugar there are three cups of cream. Another way to reason is this: in Mixture B, sugar makes up $\frac{2}{4}$ of the mix, while in Mixture A sugar makes up $\frac{2}{5}$ of the mix. Since $\frac{2}{4} > \frac{2}{5}$, Mixture B is sweeter.
7. Mixtures A and B have the same sweet taste. There are four cups of sugar for two cups of cream in the Mixture A,

NOTE FOR TEACHERS: Each of the solutions (1-7) suggests two approaches to solving these problems. One of them is the common sense approach, which compares numbers of cups of sugar and cream in mixtures in informal way. In many problems it is the easiest and the most natural way to reason. The second approach represents the amount of sugar as a fraction of the amount of a whole mixture. Then it compares fractions in Mixture A and Mixture B. It is a more formal way, and a more universal way: using this way, students will be able to tell which mixture is sweeter in every case, even if the other way does not work. In the beginning, you may want to encourage students to use both ways (or other *correct* ways) they may come up with. It is also important that at the end students understand and are able to use the formal method of comparing fractions.

that is there are two cups of sugar for each cup of cream. It is the same in Mixture B. Another way to reason is this: in Mixture A, sugar makes up $\frac{4}{6}$ of the mix, while in Mixture B sugar makes up $\frac{2}{3}$ of the mix. Since $\frac{4}{6} = \frac{2}{3}$, mixtures A and B have the same taste .

8. Ten cups is twice 5 cups, so Alison has to increase every component two times. She needs to use 4 cups of sugar and 6 cups of cream.
9. Fifteen cups is three times 5 cups, so Alison has to increase every component three times. She needs to use 6 cups of sugar and 9 cups of cream.
10. Eighteen is 5 times 3.6. So Alison should take 3.6 more sugar and 3.6 times more cream: she has to put 7.2 cups of sugar and 10.8 cups of cream into the mix.
11. Sasha's and Leila's methods will produce 18 cups of ice cream that taste the same as Mixture A in problem 6. For the same taste they need to increase all the components proportionally.
12. Sasha's and Leila's methods are the same, they change every component proportionally. Alison's and Kendall's methods are the same, because they keep the difference between the amounts of cream and sugar the same.