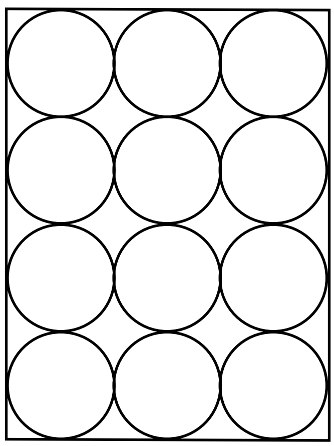
Soft Drink Package Efficiency

Adapted from *Mathematics: Modeling Our World* by COMAP.

Soft drinks are often sold in packages of twelve. The package is usually made of paperboard. This package is not 100% efficient because the cans do not fill all the space that is available in the package. (Note: A standard soft drink can has a radius of approximately 3.2 cm and a height of approximately 12 cm.)



1. What percentage of the package space is filled by the cans?

2. Does a 9-pack with a square cross-section use package space more or less efficiently? That is, does this package use a higher percentage of the available package space than the 12-pack?

3. Which package is more efficient if the criterion is package material used per can? That is, which package uses less paperboard per can?

4. Experiment with other packages that have a rectangular cross-section and in which the cans are not stacked in more than one layer. What can be concluded
- about the efficiency of package space use?
- about the efficiency of package material use?

Hints

Hint to problem 1. What is the volume of the package? What is the volume of the twelve cans?

Hint to problem 2. What is the volume of the package? What is the volume of the nine cans?

Hint to problem 3. What is the surface area of each package?

Answers

1. Approximately 78.5%.

2. The efficiency is the same.

3. The 12-pack is more efficient than the 9-pack.

4. In terms of package space used by the cans, the efficiency is always about 78.5%. In terms of package material used per can, the more cans in the package, the better the efficiency, given that cans are arranged in a package in a rectangle of a close-to-square form.

Solutions

1. The volume of the twelve cans is $12(\pi \times 3.2^2 \times 12) \approx 4632.5$ cm^3. The volume of the package is $12(6 \times 3.2)(8 \times 3.2) \approx 5898.2$cm^3. The cans use $\frac{4632.5}{5898.2} \times 100 \approx 78.5\%$ of the package space.

2. The volume of the nine cans is $9(\pi \times 3.2^2 \times 12) \approx 3474.4$ cm^3. The volume of the package is $12(6 \times 3.2)(6 \times 3.2) \approx 4423.7$ cm^3. The cans use $\frac{3474.4}{4423.7} \times 100 \approx 78.5\%$ of the package space.

3. The surface area of the 12-pack is $2 \times 12(6 \times 3.2) + 2 \times 12(8 \times 3.2) + 2(8 \times 3.2)(6 \times 3.2) \approx 2058$ cm^2. The 12-pack contains approximately 171.5 cm^2 of paperboard per can. The surface area of the 9-pack is $2 \times 12(6 \times 3.2) + 2 \times 12(6 \times 3.2) + 2(6 \times 3.2)(6 \times 3.2) \approx 1659$ cm^2. The 9-pack contains approximately 184.3 cm^2 of paperboard per can.

4. Consider a package of m by n cans, each can of a radius r and height h.

- The total volume of cans is $m \times n(\pi \times r^2 \times h)$. The volume of the package is $h(m \times 2r)(n \times 2r)$. The cans use $\frac{m \times n(\pi \times r^2 \times h)}{h(m \times 2r)(n \times 2r)} \times 100 = \frac{\pi}{4} \times 100 \approx 78.5\%$. So, regardless of the number of cans in a package as well as the size of cans, the efficiency of package space used by cans is the same.
- The surface area of an $m \times n$ package is $2(m \times 2rh + n \times 2rh + 2mr \times 2nr) = 4rh(m+n) + 8r^2mn$. There are $m \times n$ cans in a package, so a pack contains $\frac{4rh(m+n)+8r^2mn}{mn} = \frac{4rh(m+n)}{mn} + 8r^2$ paperboard per can. The second addend in this sum does not depend on the number of cans at all, but the first one does. To make the efficiency better, mn (number of cans in a package) must be big, while $m + n$ (the sum of dimensions of the package) must be as small as possible. For example, a package of 100 cans is more efficient when the cans are arranged in 10 rows of 10, not in 20 rows of 5. At the same time, it is more efficient in terms of paperboard amount to make 1 package of 100 (10×10) cans than to make 4 packages of 25 (5×5) cans.

