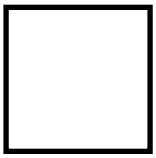
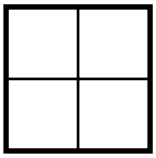
Cut a square into squares

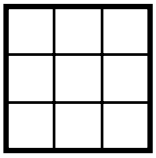
Consider one square:



It can be cut into four squares:



Or, it can also be cut into nine squares:



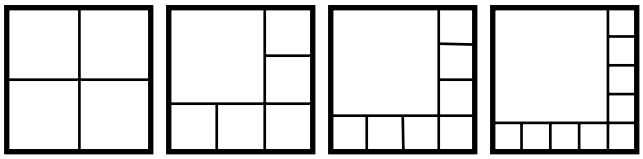
So, a square can be cut into four squares, and it can be cut into 9 squares. Can it be cut into 10 squares? Into 100 squares? Into any number of squares?

The general questions are:

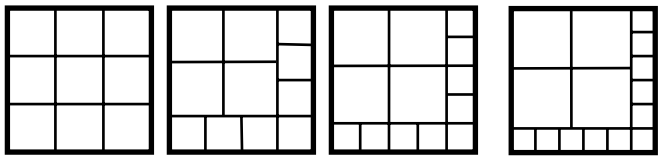
- Into how many squares can a square be cut?
- Into how many squares can a square not be cut?

Hints

Pattern 1:

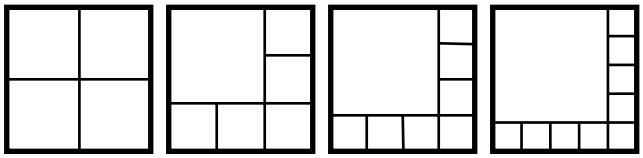


Pattern 2:

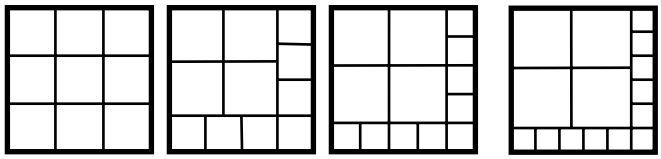


Answers

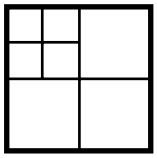
Pattern 1:



Pattern 2:



Patterns 1 and 2 show that any even number of squares starting with 4 and any odd number of squares starting with 9 are possible. One square is also no problem. So far, we've shown how to cut a square into any number of squares except 2, 3, 5, and 7 . This is how to obtain 7:



The other three numbers are impossible.

