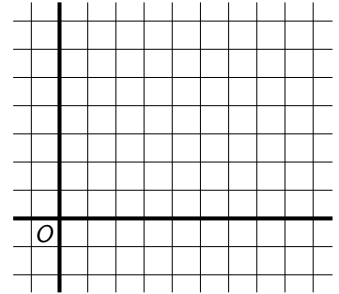


## Lattice Point Polygons Problem

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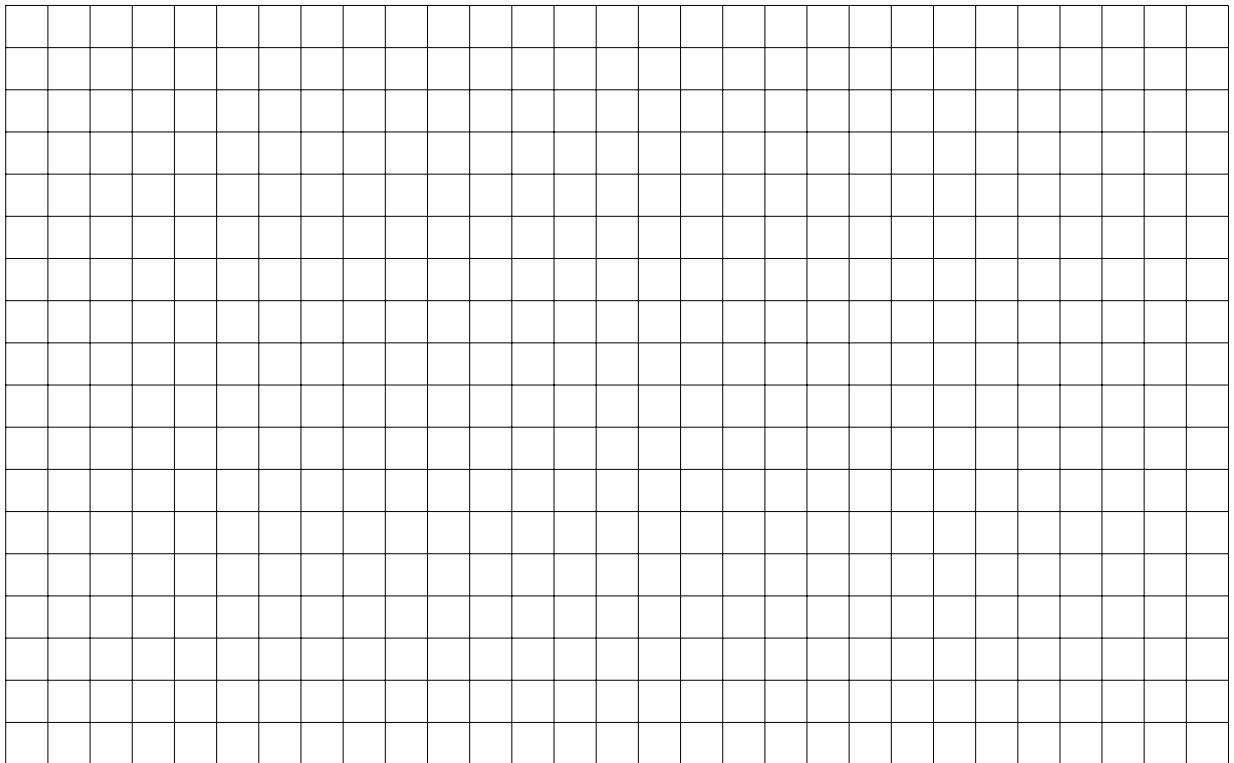
1. Plot the points  $A(1, 2)$ ,  $B(4, 6)$ ,  $C(8, 3)$ , and  $D(5, -1)$  on the grid shown here, and connect them to create quadrilateral  $ABCD$ .
2. What kind of quadrilateral is  $ABCD$ ? Prove your claim. (That is, *show* that  $ABCD$  is what you *say* it is.)



The coordinates of the vertices of  $ABCD$  were all integers, and *all of those integers were different*.

3. Using the same rules—only integer coordinates and, for any one polygon's vertices, no integer repeated—try to make other polygons, like a right trapezoid, isosceles trapezoid, scalene triangle, kite, and so on.

Your work will be judged on the quantity, quality, appearance, and complexity of your polygons, and on how clearly you show how you know what each polygon really is.



## Hints

**Hint for problem 2:** You will need to show the lengths of the sides. You will also need to show that at least one of the angles is a right angle.

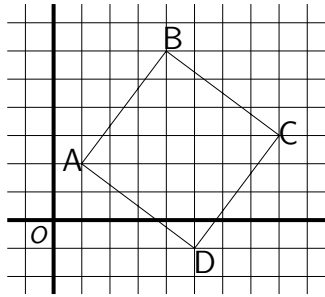
**Hint for problem 3:** You will need to show when sides are parallel (have the same slope) or perpendicular.

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# Answers

## Solutions

1.



2. Quadrilateral  $ABCD$  is a square. All four sides are diagonals of  $3 \times 4$  rectangles, so they are all 5 units long. By rotating the  $3 \times 4$  rectangles and noticing the effect on the diagonals, or by comparing the slopes ( $\frac{4}{3}$  and  $-\frac{3}{4}$ ) of the sides of  $ABCD$ , we see that the sides are perpendicular.
3. This is part of one student's solution. One of the figures can not be allowed, because it violates the rules.

