Where's the (decimal) point?—part 2

When required to compute the product $2.38 \div 0.0125$ by hand, many people move the decimals in both numbers to the right so that the divisor (the number they are dividing by) is an integer, and compute $23,800 \div 125$, instead.

When dividing one number by another, the number you are dividing into is called the *dividend*, the number you're dividing by is called the *divisor*, and the answer is called the *quotient*.

1. Carefully explain how you know that $2.38 \div 0.0125 = 23,800 \div 125$ *without* actually computing either quotient.

2. Carefully explain (and demonstrate) how to compute the following quotients by hand. Be sure to specify what quotient you are actually computing in each case. (For example, to see what $2.38 \div 0.0125$ equals, you *compute* $23,800 \div 125$.)
(a) $0.000585 \div 0.18$
(b) $0.03312 \div 2.4$

3. In problem 1, you explained why the "move the decimals in both numbers to the right the same number of places and then compute that quotient instead" method works when computing $2.38 \div 0.0125$. Now explain why this method works works in *all* cases.

One way to start your explanation is to say what is going on when you "move" the decimal to the right a certain number of places.

4. Describe a method that allows the user to compute the quotient of two numbers expressed in scientific notation. Demonstrate your method by computing the following quotients by hand.
(a) $(5.85 \times 10^{11}) \div (1.8 \times 10^{7})$
(b) $(1.098 \times 10^{-4}) \div (3.05 \times 10^{-7})$
(c) $(3.312 \times 10^{-2}) \div (2.4 \times 10^{7})$

Hints

Hint to problem 1. What *mathematical* operation can you use to move the decimals in 2.38 and 0.0125 four places to the right? What effect does that have on the quotient of the new numbers? Can you express $23800 \div 125$ in terms of 2.38 and 0.0125?

Hint to problem 3. What mathematical operation moves the decimal a given number of places to the right? If you perform this operation on both the dividend and divisor (the number you're dividing *into* and the number you're dividing *by*), what is the effect on the quotient?

Hint to problem 4. One way to do this is to rewrite the quotient without scientific notation, but that's not always easy to do. Is there a way to compute the quotient of the decimal numbers first, then deal with the powers of 10? In part (a), what power of 10 completes the equation below?

$$(5.85 \times 10^{11}) \div (1.8 \times 10^{7}) = (5.85 \div 1.8) \times 10^{?}$$

Another way to do this is to rewrite the quotient as

$$(585 \div 18) \times 10^{power}?$$

Solutions

1. Since $2.38 \times 10,000 = 23,800$ and $0.0125 \times 10,000 = 125$, the operation that corresponds to moving the decimal 4 places to the right in 2.38 and 0.0125 is multiplication by 10,000. This means that

$$\begin{aligned}
23,800 \div 125 &= (2.38 \times 10,000) \div (0.0125 \times 10,000) \\
&= \frac{2.38 \times 10,000}{0.0125 \times 10,000} \\
&= \frac{2.38}{0.0125} \cdot \frac{10,000}{10,000} \\
&= \frac{2.38}{0.0125} \\
&= 2.38 \div 0.0125,
\end{aligned}$$

so the two quotients are equal.

2. (a) Instead of computing $0.000585 \div 0.18$, move the decimal in each number two places to the right and compute $0.0585 \div 18$, instead:

$$\begin{array}{r|l}
 & .00325 \\
\hline
18 & 0.05850 \\
 & \underline{54} \\
 & 45 \\
 & \underline{36} \\
 & 90 \\
 & \underline{90} \\
 & 0
\end{array}$$

Since $0.0585 = 0.00585 \times 100$ and $18 = 0.18 \times 100$, you can use the strategy from problem 1 to see that

$$\begin{aligned}
0.0585 \div 18 &= \frac{0.000585 \times 100}{0.18 \times 100} \\
&= \frac{0.000585}{0.18} \\
&= 0.000585 \div 0.18,
\end{aligned}$$

so $0.00585 \div 0.18 = 0.00325$, too.

(b) Instead of $0.03312 \div 2.4$, compute $0.3312 \div 24$:

$$\begin{array}{r|l} & .0138 \\ \hline 24 & 0.3312 \\ & \underline{24} \\ & 91 \\ & \underline{72} \\ & 192 \\ & \underline{192} \\ & 0 \end{array}$$

Therefore, $0.033312 \div 2.4 = 0.0138$, too.

3. The solution to this problem is a generalization of the one given for problem 1. Shifting the decimal point in a number to the right a given number of places is mathematically the same as multiplying the given number by $10^{\#\text{ of places}}$ (10 raised to the power equal to the number of places to be moved). Then $(\textit{dividend} \times 10^{\#\text{ of places}}) \div (\textit{divisor} \times 10^{\# \textit{ of places}})$ can be rewritten in fraction notation as $\frac{\textit{dividend}\times 10^{\#\text{ of places}}}{\textit{divisor}\times 10^{\#\text{ of places}}}$, which simplifies as follows:

$$\begin{aligned} \frac{\textit{dividend} \times 10^{\#\text{ of places}}}{\textit{divisor} \times 10^{\#\text{ of places}}} &= \frac{\textit{dividend}}{\textit{divisor}} \times \frac{10^{\#\text{ of places}}}{10^{\#\text{ of places}}} \\ &= \frac{\textit{dividend}}{\textit{divisor}} \times 1 \\ &= \textit{dividend} \div \textit{divisor}, \end{aligned}$$

so the original quotient, $\textit{dividend} \div \textit{divisor}$, is equal to the quotient of the numbers you get by shifting their decimals a given number of places to the right, as long as the shift is the same in both numbers.

Teacher's note: The explanation given purposely avoids algebraic symbolism, but such symbolism is convenient when explaining why $a \div b$ equals $(a \times 10^n) \div (b \times 10^n)$:

$$\begin{aligned} \frac{a \times 10^n}{b \times 10^n} &= \frac{a}{b} \times \frac{10^n}{10^n} \\ &= \frac{a}{b} \times 1 \\ &= a \div b, \end{aligned}$$

4. There are many methods for computing the quotient of two numbers expressed using scientific notation. One is demonstrated and explained with the example

$$(1.0293 \times 10^{57}) \div (2.35 \times 10^{18}).$$

- Rewrite the quotient as a fraction:

$$\frac{1.0293 \times 10^{57}}{2.35 \times 10^{18}}.$$

Teacher's note: If your students have taken algebra or are comfortable with algebraic symbolism, you can also explain the method (and the reason it works) as below. $(a \times 10^n) \div (b \times 10^m) = \frac{a\times 10^n}{b\times 10^m}$, which simplifies to $\frac{a}{b} \times \frac{10^n}{10^m}$. By the law of exponents, this equals $(a \div b) \times 10^{n-m}$, so you can compute the quotient by dividing the decimal parts and dividing the powers of 10.

- Rewrite the fraction as a fraction of the decimal parts times a fraction of powers of 10:

$$\frac{1.0293}{2.35} \times \frac{10^{57}}{10^{18}}$$

- Compute the quotient of the decimal parts using the method you've already used for dividing decimal numbers, then simplify the fraction of powers of 10 by using the law of exponents:

$$0.438 \times 10^{57-18} = 0.438 \times 10^{39}$$

- Move the decimal and adjust the power of 10 as needed to get the form you prefer for your answer (scientific notation or otherwise):

$$4.38 \times 10^{38}$$

  Since the decimal part of the answer was *multiplied* by 10 (to get from 0.438 to 4.38), the power of 10 was decreased by 1 (that is, 10^{39} was *divided* by 10).

There are other ways to solve these type of problems, as demonstrated in the solutions to parts (a)–(c).

(a) First, use the fact that

$$(5.85 \times 10^{11}) \div (1.8 \times 10^{7}) = (5.85 \div 1.8) \times 10^{4},$$

which, using the "move the decimals" method, is equal to $(58.5 \div 18) \times 10^4$. Since $58.5 \div 18 = 3.25$, the final answer is 3.25×10^4, or 32500.

Here's another method, starting with $(58.5 \div 18) \times 10^4$:

$$(58.5 \div 18) \times 10^4 = (5.85 \times 10^4) \div 18 = 585000 \div 18,$$

which also equals 32500, of course.

One final method:

$$\begin{aligned}
(5.85 \times 10^{11}) \div (1.8 \times 10^{7}) &= (585 \times 10^{-2} \times 10^{11}) \div (18 \times 10^{-1} \times 10^{7}) \\
&= \frac{585 \times 10^9}{18 \times 10^6} \\
&= \frac{585}{18} \times \frac{10^9}{10^6} \\
&= 32.5 \times 10^3 \\
&= 3.25 \times 10 \times 10^3 = 3.25 \times 10^4.
\end{aligned}$$

(b) As in the last example,

$$\begin{aligned}(1.098 \times 10^{-4}) \div (3.05 \times 10^{-7}) &= (1.098 \div 3.05) \times 10^{-4-(-7)} \\ &= (1.098 \div 3.05) \times 10^{3} \\ &= (109.8 \div 305) \times 10^{3} \\ &= 0.36 \times 10^{3} \\ &= 3.6 \times 10^{2}, \text{ or } 360.\end{aligned}$$

Alternatively,

$$\begin{aligned}(1.098 \times 10^{-4}) \div (3.05 \times 10^{-7}) &= (1.098 \div 3.05) \times 10^{3} \\ &= (1.098 \times 10^{3}) \div 3.05 \\ &= 1098 \div 3.05 \\ &= 109800 \div 305 \\ &= 360.\end{aligned}$$

And the last method:

$$\begin{aligned}(1.098 \times 10^{-4}) \div (3.05 \times 10^{-7}) &= (1098 \times 10^{-7}) \div (305 \times 10^{-9}) \\ &= \frac{1098}{305} \times \frac{10^{-7}}{10^{-9}} \\ &= 3.6 \times 10^{2}\end{aligned}$$

(c) Now, for the last quotient,

$$\begin{aligned}(3.312 \times 10^{-2}) \div (2.4 \times 10^{7}) &= (3.312 \div 2.4) \times 10^{-2-7} \\ &= (3.312 \div 2.4) \times 10^{-9} \\ &= (33.12 \div 24) \times 10^{-9} \\ &= 1.38 \times 10^{-9}\end{aligned}$$

Alternatively,

$$\begin{aligned}(3.312 \times 10^{-2}) \div (2.4 \times 10^{7}) &= (3.312 \div 2.4) \times 10^{-9} \\ &= (3.312 \times 10^{-9}) \div 2.4 \\ &= 0.000000003312 \div 2.4 \\ &= 0.00000003312 \div 24 \\ &= 0.00000000138.\end{aligned}$$

And the last method:

$$\begin{aligned}(3.312 \times 10^{-2}) \div (2.4 \times 10^{7}) &= \frac{3312 \times 10^{-3} \times 10^{-2}}{24 \times 10^{-1} \times 10^{7}} \\ &= \frac{3312}{24} \times 10^{-5-6} \\ &= 138 \times 10^{-11} \\ &= 1.38 \times 10^{-9}.\end{aligned}$$

