Topic Study: Functions & Graphs

Background

This Topic Study focuses on graphing in the coordinate plane and includes sections dealing with mathematical context, pedagogy, essential questions to elicit student thinking and prompt mathematical understanding, potential errors and misconceptions associated with the topic, technology integration considerations, and possible topics for collegial dialogue. The tools that are the primary focus during this study include the ***General*** and ***Simple Coordinates Game,*** the ***Maze Game,*** and ***Graphit*** (Shodor Education Foundation, 1997-2005), ***Points in a Coordinate Planes (Activity A, B, & C)*** (Explorelearning, 2005), and the ***Coordinate Geoboard*** (National Library of Virtual Manipulatives, 1999).

Context

Understanding graphs and tables as representations

Students' familiarity with reading bar graphs accustoms them to reading coordinate graphs primarily – and sometimes solely – vertically, on the y-axis. The idea of reading the x-axis horizontally as well as reading y-axis vertically is quite new to many students. To then understand that a point on a graph represents <u>two</u> pieces of information simultaneously – one from x-axis and one from the y-axis – can often take awhile for students to grasp.

Therefore, prior to working with tables and graphs, it is helpful if students have had some experience with situations that generate pairs of values. A good set of introductory explorations can be found in visual patterns problems, such as "Spotting Numbers" and related tasks, in which you can pair the number of the figure with some characteristic of the pattern, such as number of total tiles in each iteration of the pattern or the perimeter of each iteration of a pattern of stacked squares.

Making inferences from different representations

Before students can work across different representations, such as tables and graphs, they must first have a solid understanding of what each representation is showing. It is important to give students experiences with questions such as:

- When you look at a table of values that represent a situation, what can you figure out about how the situation is behaving?
- When you look at a graph that represents a situation, what can you figure out about how the situation is behaving?

Relating the different representations

Once students understand what a given representation shows, it is important to help students understand the relationships between the representations. You may want to pose questions to students such as:

- How can you predict what a graph will do based on the corresponding numbers in a table?
- How can you look at the graph and know something about the corresponding numbers that will be generated for the situation?

- What can you learn from one representation that you can't learn as easily from the other?
- Why do these representations work well together?

Pedagogy

Begin by posing a central question: (Example: "How can you predict what the graph will do based on the numbers in the table?") This is not necessarily a question that students will try to answer until the end of your work in this Topic after a week or two. However, it will serve as a central focus for your lessons within the topic study.

Gather some initial ideas from students, but use this as an opportunity to gather and record students' ideas rather than instruct. You can revisit students' ideas throughout the lessons and help students reflect on the ideas and make judgments about their validity. It is important to help the students do the reflecting and making the judgments themselves as the learning that results will be deeper. You can do this with the use of good questions (see "Essential Questions" for an initial list).

Help students structure a set of lesser questions that they can explore that will help answer the larger central question. For example, you might ask students to first focus on the question "When you look at a graph that represents a situation, what can you figure out about how the situation is behaving?" This lesser question can be the basis of a lesson with one of the online tools.

Give students time to explore: As they do, walk around and respond to individual questions. Instruction may be necessary if students are stuck and do not understand some basic piece of what they are looking at, or are using incorrect ideas or making faulty assumptions about what they're doing.

Gather the class back together to share their observations and conjectures: As you call on students to share what they think they've figured out, record their observations, conjectures and questions. It will be your job at the end of this discussion to help students make sense of what they've done, make connections, draw conclusions and apply formal mathematical language and notation to their ideas.

Pull It All Together: This is where some fruitful instruction can happen! Take some time to summarize what students have put forward. Make note of related ideas and talk with students about how the ideas are related. Introduce mathematical language or symbols that are relevant to the ideas. Help the class come up with words to state what they have learned – make it be a joint effort! You may want to start with their language and suggest places to use mathematical language or rephrase things to be clearer or unambiguous.

Pedagogical Considerations

- Discussion questions that help students see the relationship between different representations are vital to use in conjunction with this set of tools. The tools themselves provide great opportunities for students to create interesting graphs from corresponding tables and equations; however, the deeper mathematics of understanding the relationships between the representations will emerge only with the use of focused and thoughtful questions. See the list of Essential Questions provided below.
- Look for opportunities to *encourage students to tell you* what is happening within tables and on graphs. Consider posing questions that begin with "How does…," "Why…" and "How would you explain…." to encourage student responses that require some elaboration or explanation. Be

wary of an overdependence on questions that can be answered with a "yes" or "no," including questions that begin with "Is…," "Can…," or "Did/Does…?"

Essential Questions

To probe for understanding:

- Why are there **two** numbers for every point on the graph?
- How do those two numbers for a point appear in the table? (or conversely: How do the two corresponding numbers in a table appear on a graph?)
- If you know the coordinates of a point on a graph, how can you predict what quadrant of the graph the point will be in?

To prompt mathematical reasoning:

- What patterns do you notice in the table of values?
- How does a pattern you noticed in the table appear on the graph?
- When the graph moves (up to the right/down to the right), how do the values in the table change?
- When there is a value that does not change in a situation (such as a flat fee for something), how does that value appear on the graph? in the table? in the equation?
- When there is a value that changes by increasing or decreasing by a constant amount (such as an hourly rate for something), how does that value appear on the graph? in the table? in the equation?

Common Misconceptions

Error: A student is counting the spaces on the graph rather than counting the lines.

Related misconception: This error shows a misconception related to the representation of graphs and perhaps also of number lines. If students never understood a number line model, and that they have "counted" a number when they reach the line labeled with that number, then they may have similar misunderstandings when they consider graphs.

Error: (particularly related to distance/time graphs) A student interprets a graph of a line with negative slope to represent the motion of something going downhill.

Related misconception: It is common for students to interpret graphs of motion as a literal picture of the motion itself, rather than an algebraic representation of the motion. For instance, the graph of the motion of a stopped car over ten minutes can appear as a horizontal line segment on a distance/time graph; students may easily confuse this with motion of the car traveling in a straight line.

Technology Integration

✔ This collection of tools covers a host of "basics" about graphing: plotting points, filling in a table from a description of a situation, finding an equation for the values in the table, graphing the equation, an introduction to slope, reflections of figures on a coordinate graph, and the relationship between tables, graphs, words and equations.

- ✔ The ***General Coordinates Game***, and accompanying ***Maze Game*** and ***Simple Maze Game***, focus on practicing the skill of plotting points on a Cartesian coordinate system. Note that in the ***Maze Game*** and ***Simple Maze Game,*** students also develop spatial visualization skills as they imagine and plot out a path from a robot's starting point on the grid to the end point. You can increase the mathematical complexity of the task for some students by gradually increasing the number of mines to avoid in the ***Maze Game*** and ***Simple Maze Game.*** As students do so, they visualize increasingly complex paths through the mines and can also be encouraged to try to determine the minimum number of moves in which to traverse the path.
- ✔ ***Coordinate Grapher*** is an online geoboard that shows the coordinates of the endpoints of the segments you select. There are several activities provided in the tool that focus students on the concept of slope. In particular, there are two activities that involve creating the reflection of a polygon over the line y=x. While mathematically interesting and challenging, the necessary follow-up questions regarding the relationship of the slopes of the reflected lines are not included; you will need to provide them yourself.
- ✔ ***Points in the Coordinate Plane (A, B and C)*** provide an interactive way to understand both the mechanics of plotting points (in A) but also the relationship of points to each other (in B) as well as (in C) examining patterns that lead to an understanding of slope. Note in particular the "Exploration Guide" that accompanies each tool. These guides have well-constructed questions that encourage students to form and test their own conjectures about what is happening on the graphs.
- ✔ ***Linear Functions – Real-Life Data: 1, 2 & 3*** (PBS Interactive) are designed to work as a trio of tools. #1 shows the relationship between a graph and its corresponding table of values. Note especially the 2 questions listed at the right that focus on how the table and graph change as numbers in the situation change. #2 describes situations with missing values, and prompts you to select values that match the ones shown in blue on the graph and in the table. The focus here is on understanding how a fixed value and a changing value in the situation both are represented on the graph and in the table. In #3, you are given a situation with missing values, as well as a graph and a table, and are prompted to create the linear equation that matches the data.

Professional Community

Collegial Dialogue

Ask colleagues:

- What kind of things are your students learning more easily with the online tools than they learned from the textbook, and why do you think this is the case?
- What questions are you finding most effective with which tool?

Reflection questions:

- The online tools can provide powerful learning experiences for students. What additional offline activities or experiences do students need to support the work they do with the online tools?
- What kinds of questions can students explore themselves using the online tools?

Related Readings:

Lines By the Book

Bob Mann

MTMS, August 2004, Volume 10, Issue 1, Page 8

Coordinate Plane Set Detective

Darin Beigie

MTMS, January 2004, Volume 9, Issue 5, Page 251

Using Student Contributions and Multiple Representations to Develop Mathematical Language

Beth A. Herbal-Eisenmann

MTMS, October 2002, Volume 8, Issue 2, Page 100