Maine's Impact Study of Technology in Mathematics


# Using MISTM Assessment Items for Teacher Professional Development

A Workshop & Resource Packet for Curriculum Specialists, Teacher Leaders and Other Providers of Professional Development

Produced by:

EDC

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# WELCOME

We are pleased to offer you this workshop, ***Using MISTM Assessment Items for Teacher Professional Development:*** *A Workshop for Teacher Leaders, Curriculum Specialists and Other Providers of Professional Development*. The workshop is designed to help educational leaders foster growth in the mathematics content and pedagogical knowledge of the teachers they serve.

## Background on MISTM Project and Tools

MISTM was an experimental research study conducted between October 2003 and December 2006. It was designed to examine the impact of a technology-infused professional development program for 7th and 8th grade mathematics teachers on the mathematics achievement of rural middle school students in technology-rich (laptops) classrooms. Using an experimental design with randomized assignment of schools to treatment and control conditions, the study's major outcome measure was student performance in mathematics on the eighth grade Maine Education Assessments (MEA). Additional measures included pre- and post-tests in mathematics for students; assessments for teachers aimed at learning about content knowledge and classroom practices; teacher and student use of technology tools; and practices of school principals in supporting changes in mathematics instruction.

## Further Applications

As researchers analyzed results from the MISTM student assessments, they began to see potential for a broader application of these items. Researchers realized that the assessment items, along with related documents, processes and technology tools, could provide a rich professional development experience for mathematics teachers. These realizations led to the creation of this workshop.

We hope that participants will leave this workshop with new ideas, enthusiasm, and a practical framework for strengthening the design and delivery of professional development.

## Workshop Author and Funding

This workshop was developed at Education Development Center, Inc. For information about Education Development Center visit: http://edc.org.

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# INTRODUCTION AND OVERVIEW

This guide is designed to support the facilitator of the workshop *Using MISTM Assessment Items for Teacher Professional Development*. While this guide focuses more heavily on the content of the workshop than on facilitation skills, facilitators with a range of experience can use these materials to prepare and conduct the workshop.

## Who Should Participate in this Workshop?

This workshop is intended for curriculum specialists, department chairs, teacher leaders, and others who provide professional development to middle grades mathematics teachers. It focuses on how middle school math teachers *analyze student work*, *diagnose student understanding and difficulties*, *formulate questions,* and *identify appropriate instructional strategies*. Workshop participants, by examining teachers' work, explore a framework for designing and delivering professional development.

## Workshop Design

*Using MISTM Assessment Items for Teacher Professional Development* is designed to help providers of professional development to promote growth in teachers' knowledge of mathematics content and pedagogy. To achieve this purpose, participants:

- Use the MISTM Teacher Assessment items to explore pedagogical issues such as questioning techniques and using rubrics for formative and summative assessment.
- Learn how examining student responses to math tasks can provide opportunities to assess and build *teacher* content knowledge of mathematics.
- Use MISTM Teacher Assessment items as a framework for designing professional development.

## Workshop Goals

Upon completion of the workshop, participants will be able to:

- Evaluate teacher responses to assessment items in order to determine teachers' conceptual and pedagogical understanding and difficulties.
- Develop and apply a rubric to assess teacher work and identify possible next steps for professional growth.
- Recognize the role of questioning techniques in probing for student understanding, and create professional development that fosters skill in questioning techniques among teachers.
- Apply the design format of the workshop to professional development needs or goals relevant to their own settings.

# How to Prepare for the Workshop

This guide provides information and materials to assist the user in planning and facilitating this workshop. The facilitator will need to address a variety of issues, including the issues listed below.

## Logistics

**4 Weeks or longer in advance**, the facilitator should:

- Establish a date and a 4-hour block of time for the workshop.
- Recruit participants. A minimum of 8 and a maximum of 20 is suggested. Participants should have responsibility and experience in providing professional development to middle grades math teachers.
- Secure a location for the workshop. A computer, projector, and screen are required. In addition, a white board (or an easel and chart paper) is needed and you may want to use an overhead projector.
- Review this facilitation guide and the PowerPoint presentation (PPT file); do workshop activities; select warm-up and introduction activities; personalize presentation slides as needed.

**1-2 Weeks in advance**, the facilitator should:

- Send reminders to participants
- Confirm meeting space and time
- Photocopy workshop handouts
- Practice the presentation; do all workshop activities

**1-3 Days in advance**, the facilitator should:

- Check on how to use computer projector
- Set up tables or desks to provide for partner work
- Make partner and/or grouping assignments, if necessary
- Ensure that workshop handouts are ready
- Do a final "practice run," using all of the workshop materials

## Workshop Materials

This Facilitation Guide includes most materials required for the workshop, including background information, facilitation tips, the workshop agenda, and handouts.

In addition to this document, you will need the presentation for the workshop. The presentation is provided on the CD or Web site where you found this guide.

## FACILITATION TIPS

The following is a list of suggestions to keep in mind when you are facilitating the workshop. As a facilitator, consider how to stimulate discussion among participants and get them actively involved. Since you are guiding their work, you need not take on the role of expert.

- Learn about the group you are facilitating. What are their areas of expertise? What professional development experiences have they had?
- Be flexible. Plan in advance, but be ready to change or adapt to meet the needs of the group.
- Watch your pace! Pacing suggestions are provided as guidelines, but try to gauge the needs of the group.
- Know yourself! Do you tend to move and talk quickly? Slowly? Want the details or just the big picture? As you prepare, mark reminders to yourself such as "keep this brief" or "slow down."
- Avoid simply reading through the presentation slides as you present. Practice the presentation so that you are able to use the information on the slide to remind you of the key ideas.
- Be attentive to how much interaction and discussion participants are having, and make adjustments to prompt and promote discourse.
- Be clear about the purpose and intended results of the session and keep these goals central as you present.
- If you are unfamiliar with the participants, or they with each other, consider opening the session with an icebreaker activity.
- Use the group as a resource and invite participants to share their ideas and experiences.
- Listen and respond in a manner that will move the discussion forward. Posing questions is an effective way to do this.
- Anticipate the issues or problems that could arise in the workshop and plan how to respond to them.
- Consider inviting feedback at the close of the session to help you improve your facilitation or modify the workshop. An optional feedback form is included as Handout 14.

## WORKSHOP AGENDA — SECTION 1

Introduction, Project Background, and "Candy Dots" Assessment  Total time estimate: 55 min.

| **Topic**<br>**Purpose**<br>**Duration** | **Description of Workshop Segment**<br>*What Facilitator and Participants Do* | **Materials**<br>*Slides & Handouts* |
|---|---|---|
| **Workshop Introduction**<br>Introduce this workshop in the context of the MISTM project and outline the goals and framework of the workshop, as well as the schedule for completion.<br>**Time Estimate**: 15 min. | **Introduction and Background Information**<br>• Introductions<br>• MISTM project background<br>• Workshop goal and activities<br>• Workshop topics<br>• Schedule | **Slides 1 – 10**<br>**Handout 1** *(Warm-up)*<br>Optional: distribute presentation slides |
| **"Candy Dots" Assessment Item**<br>Familiarize participants with the MISTM Teacher Assessment item *Candy Dots*, the learning goals it addresses, and how it was used with teachers.<br>**Time estimate**: 40 min. | **Getting to Know the MISTM Teacher Assessment**<br>• Goal<br>• Content focus<br>• State and national standards addressed<br>• What teachers had to do<br>**Explore an Assessment Item**<br>1. Participants do Candy Dots task that students had to do.<br>2. Participants identify and discuss learning goals, demands, and potential barriers of the task<br>3. Participants discuss how this task matches the content that the teachers they work with will be teaching. Participants discuss how teachers handle instruction of this content.<br>4. Participants discuss content and pedagogy goals and demands for the task.<br>5. Participants develop a list—what would these items tell you about what teachers know and can do? What do you expect that you would learn? (Think/Pair/Share)<br>6. Facilitator shares Rubric from the project. Participants choose either #1 or #2 of the rubric to examine and discuss with a partner. | **Slides 11 – 17**<br>**Slides 18 – 23**<br>**Handout 2** (*Patterns, Relations, and Functions Problem*)<br>**Handout 3** (*Teacher Assessment #2: Patterns, Relations, & Functions*)<br>**Handout 4** (*Examination of MISTM Teacher Assessment Item)*<br>**Handout 5** (*MISTM Rubric*) |

## WORKSHOP AGENDA — SECTION 2

Questioning, Diagnosing Difficulties, and Identifying Strategies — Total time estimate: 60 min.

| Topic<br>Purpose<br>Duration | Description of Workshop Segment<br>*What Facilitator and Participants Do* | Materials<br>*Slides & Handouts* |
|---|---|---|
| **Developing Teacher Questioning**<br>Examine the role of teacher questioning in the math classroom and to explore how questioning techniques might be developed in teachers.<br>**Time estimate**: 30 min. | **Using Assessment Items to Explore Questioning Techniques**<br>• Review the purposes of questioning in the math classroom.<br>• Discuss techniques for posing questions to students.<br>• Examine some sample teacher responses.<br>• Refine or reframe these responses to elicit more useful information from students.<br>• Review pitfalls in teacher questioning. | **Slides 24 – 34**<br>**Handout 6** *(Categories of Teacher Questions)*<br>**Handout 7** (*Evaluating Teacher Questions*)<br>**Handout 8** *(Teacher Responses to Candy Dots #3)*<br>**Handout 9** (*Refining Questions for a Specific Purpose*) |
| **Diagnosis and Strategies**<br>To outline a process for diagnosing student difficulties and identifying strategies to use in response.<br>**Time Estimate**: 30 min. | **Using Assessment Items to Practice Conducting Formative Assessment**<br>**Process:**<br>• What might be going on with this student?<br>• What other factors might explain the student's performance?<br>• What strategy will I use to address the difficulty I believe the student is having?<br>• How do I expect it will help?<br>• How will I assess how the strategy worked? | **Slides 35 – 36**<br>**Handout 10** *(Diagnosis and Next Steps Based on Evidence and Reflection on Student Work)* |

## WORKSHOP AGENDA SECTION 3

Fostering Content Knowledge and Applying Workshop Ideas Total time estimate: 60 – 80 min.

| Topic<br>Purpose<br>Duration | Description of Workshop Segment<br>*What Facilitator and Participants Do* | Materials<br>*Slides & Handouts* |
|---|---|---|
| **Fostering Content Knowledge**<br>To explore how examination of student work can provide information about what teachers know and understand about the math content.<br>To explore and share ideas about how to use this information about teachers to foster growth in teachers' content knowledge.<br>**Time Estimate**: 30 – 40 min. | **Using MISTM Assessment Item to Build Teacher Content Knowledge**<br>1. Participants examine an item designed to provide information about teacher content knowledge.<br>2. Participants identify what content knowledge is being addressed and brainstorm possible responses.<br>3. Participants review and discuss actual teacher responses to the item (card activity).<br>4. Participants discuss the potential for using student work and questions to teach math content to teachers. Can you come up with another example? | **Slides 37 – 39**<br>**Handout 11** (*Candy Dots #5*)<br>**Cards** (*made from Handout 12: Card Activity Master*) |
| **Application to PD Goals**<br>To apply workshop framework and experience to one's own professional development needs and program.<br>**Time Estimate**: 30 – 40 min. | **Applying the MISTM Framework in other Math Strands and to other Professional Development Goals or Curriculum Materials**<br>• Participants plan a professional development experience. | **Slides 40 – 41**<br>**Handout 13** (*Planning Professional Development*) |
| **Workshop Close**<br>Final remarks; credits; thank participants.<br>**Time estimate**: 5 min. | **Feedback Form**<br>• Optional Activity – *participants complete Handout 14 and return it to facilitator*<br>**Acknowledgements** | **Slides 42 – 43**<br>**Handout 14** *(Feedback Form)* (optional) |

## ADDITIONAL RESOURCES

Additional assessment items and supporting materials are provided for further exploration in a companion document entitled "Optional Additional Resources." These items allow you to extend the professional development experience into other math content areas and/or provide further practice with the content examples provided in the workshop.

# Handout 1 – Warm-up

*Write your response to one or more of the following:*

1. Describe one strategy/resource you have successfully employed recently in professional development.

2. Identify one of the biggest challenges you face in helping to promote teacher growth in math content knowledge.

3. Identify one or more challenges that you face in helping to promote teacher growth in instructional strategies.

4. What is one important topic you would like to focus on in professional development in mathematics in your work setting?

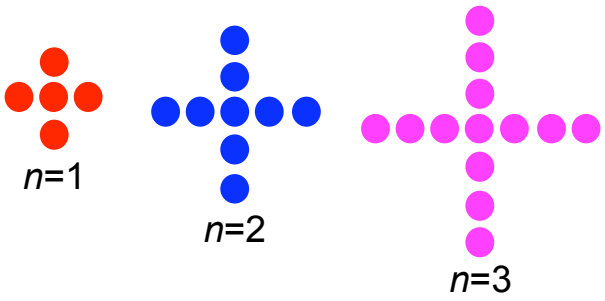
# HANDOUT 2 – PATTERNS, RELATIONS, AND FUNCTIONS PROBLEM

Sam and Maria each were given candy dots arranged in patterns as shown below. Sam and Maria decided to figure out how many dots they would have if the patterns continued.

## PART A

1. Sam filled in his table for the first 3 figures. Complete the table for the 4th and 5th figures.

| Figure (*n*) | Total Dots |
|---|---|
| 1 | 5 |
| 2 | 9 |
| 3 | 13 |
| 4 | |
| 5 | |

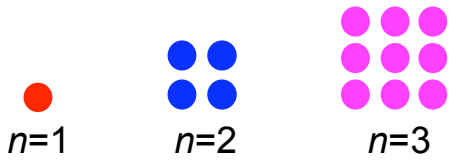



2. Write a rule with numbers and symbols to describe the number of dots for any number figure.

## PART B

1. Maria filled in her table for the first 3 figures. Complete the table for the 4th and 5th figures.

| Figure (*n*) | Total Dots |
|---|---|
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | |
| 5 | |




2. Write a rule with numbers and symbols to describe the number of dots for any number figure.

## PART C

Find how many dots are in the 10th figure for each pattern:

a. Number of dots in the 10th figure in Sam's pattern _____

b. Number of dots in the 10th figure in Maria's pattern _____

## EXAMINE

1. What math goals does this task address?

   **Part A (Sam):**

   **Part B (Maria):**

   **Part C (10th Figure):**

2. What challenges do you think this task might present for students?

   **Part A (Sam):**

   **Part B (Maria):**

   **Part C (10$^{th}$ Figure):**

## DISCUSS WITH PARTNER

1. Share your responses above.

2. Discuss how this problem compares to the kinds of problems your teachers use to teach this math content.

3. What strategies and approaches have your seen teachers use to teach this content effectively?

4. Identify one idea from your conversation to share with the whole group.

# HANDOUT 3 – TEACHER ASSESSMENT #2: PATTERNS, RELATIONS, & FUNCTIONS

## I. EXAMINE THE FOLLOWING TASK AND ANSWER THE ASSOCIATED QUESTION.

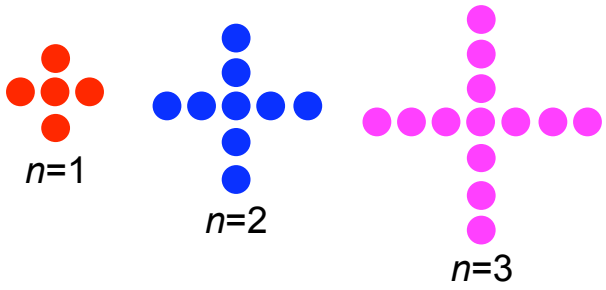
For each problem below, students were asked to read the statements, complete the table, and describe the pattern with a rule. As you answer the question that follows the student problems, consider all aspects of Part A and Part B.

Sam and Maria each were given candy dots arranged in patterns as shown below. Sam and Maria decided to figure out how many dots they would have if the same patterns continued.

**Part A.**

1.) Sam filled in his table for the first 3 figures. Complete the table for the 4th and 5th figures.

| Figure (*n*) | Total Dots |
|---|---|
| 1 | 5 |
| 2 | 9 |
| 3 | 13 |
| 4 | |
| 5 | |

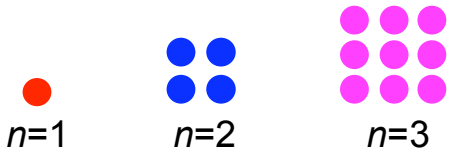



2.) Write a rule with numbers and symbols to describe the number of dots for any number figure.

**Part B.**

1.) Maria filled in her table for the first 3 figures. Complete the table for the 4th and 5th figures.

| Figure (*n*) | Total Dots |
|---|---|
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | |
| 5 | |




2.) Write a rule with numbers and symbols to describe the number of dots for any number figure.

1. What do students need to know and be able to do to answer all aspects of Part A and Part B of the task presented above?

## II. EXAMINE THE FOLLOWING TASK AND ANSWER THE ASSOCIATED QUESTION.

Part C, below, is an extension of the tasks done in Parts A and B. Students are asked to find the number of dots in the 10th figures of Sam's and Maria's patterns. Consider all aspects of Part C as you answer the associated question below the student task.

**Part C.**

1.) Find how many dots in the 10th figure for each pattern.

**a. Number of dots in the 10th figure in Sam's pattern** _____

**b. Number of dots in the 10th figure in Maria's pattern** _____

2. What do students need to know and be able to do to answer all aspects of Part C above?

## III. EXAMINE THE FOLLOWING SAMPLE OF STUDENT WORK AND ANSWER THE ASSOCIATED QUESTIONS.

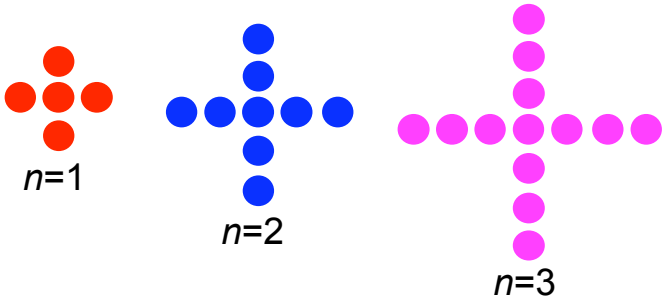
For each problem below, students were asked to read the statements, complete the table, and describe the pattern with a rule. As you answer the questions that follow the student problems, consider all aspects of Part A and Part B.

Sam and Maria each were given candy dots arranged in patterns as show below. Sam and Maria decided to figure out how many dots they would have if the same patterns continued.

**Part A.**

1.) Sam filled in his table for the first 3 figures. Complete the table for the 4th and 5th figures.

| Figure (*n*) | Total Dots |
|---|---|
| 1 | 5 |
| 2 | 9 |
| 3 | 13 |
| 4 | ***17*** |
| 5 | ***21*** |



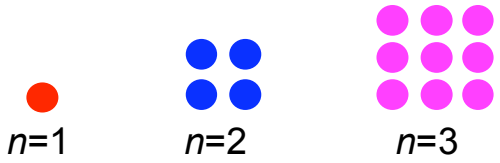

2.) Write a rule with numbers and symbols to describe the number of dots for any number figure.

***The plus sign adds 4 dots to its area every time n increases. You multiply everything by 4+1.***

**Part B.**

1.) Maria filled in her table for the first 3 figures. Complete the table for the 4th and 5th figures.

| Figure (*n*) | Total Dots |
|---|---|
| 1 | 5 |
| 2 | 9 |
| 2 | 13 |
| 4 | ***16*** |
| 5 | ***25*** |




2.) Write a rule with numbers and symbols to describe the number of dots for any number figure.

***every pattern doubles and then you add one. Each number has an equal amount of dots on each side then you add one for the bottom dot.***

3. Given the student work above, what questions could you ask the student to get a better understanding of what the student is thinking related to the task? Explain why you chose these questions.

4. Give any strategies you would use with this student given the student work shown above. Explain fully why you chose these strategies using evidence from the student work.

# Handout 4 – Examination of MISTM Teacher Assessment Item

## Candy Dots

### Think

Review items 1–4 on Handout 3 that teachers needed to complete as part of the assessment.

Think about what knowledge and skills teachers need in order to complete these items successfully, and jot down some of your thoughts:

- Math Content:

- Instructional/Pedagogical Skills and Knowledge:

### Pair

Discuss what success would look like within the areas of knowledge that you and your partner identified. Generate a list of criteria that could be used on a rubric to describe strong responses to these items.

### Share with Whole Group

One element that you would want to see on a rubric used to evaluate a teacher's response.

# HANDOUT 5 – MISTM RUBRIC

## CANDY DOTS – (1): IDENTIFYING LEARNING GOALS

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| *Learner provides minimal to no identification of learning goals.*<br>_____<br><br>**EITHER:**<br><br>3 parts but contains error(s)<br><br>**OR**<br><br>2 or fewer parts, and may contain errors | *Learner explains goals, but flaws and omissions limit scorer's understanding.*<br>_____<br><br>**EITHER:**<br><br>At least 2 parts (see ***Meets***→) with no errors or flaws<br><br>**OR**<br><br>All required for a 3 (***Meets***) but contains error(s) or flaws in learner thinking | *Learner provides good explanation of learning goals and provides an example or reference to the problem to enhance communication.*<br>_____<br><br>**BOTH:**<br><br>1. At least 4 of:<br><br>a. Identify pattern(s)<br>b. Extend a pattern (complete a table)<br>c. Generalize a pattern to a rule<br>d. Use algebraic symbols<br>e. Recognize that Parts A and B involve different types of patterns *(linear vs. non-linear, arithmetic sequence vs. geometric sequence, etc.)*<br><br>**AND**<br><br>2. Contains no mathematical errors or flaws in learner thinking | *Learner provides clear, complete explanation of learning goals and provides multiple examples and/or references to readings to enhance the communication.*<br>_____<br><br>**BOTH:**<br><br>1. ***Meets*** criteria for a 3<br><br>**AND**<br><br>2. Either or both:<br><br>i. Describes a strategy for generalizing<br><br>ii. Uses math language accurately, consistently, and clearly *(this could include: arithmetic sequence/progression and geometric sequence/progression, linear vs. non-linear situation, constant rate of change vs. non-constant/variable rate of change [this last could also be: constant ratio of change or constant rate of change in the rate of change….etc.]* |

## CANDY DOTS – (2): IDENTIFYING LEARNING GOALS

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| *Learner provides minimal to no identification of learning goals.*<br>_____<br><br>May contain 1 part listed in ***Meets*** but may contain mathematical errors, or explanation is unclear | *Learner explains learning goals, but flaws and omissions limit scorer understanding.*<br>_____<br><br>**EITHER:**<br><br>At least 1 part listed in ***Meets***<br><br>**OR**<br><br>***Meets*** criteria for a 3, but with mathematical **errors/flaws** in thinking | *Learner provides good explanation of learning goals and provides an example or reference to the problem to enhance the communication.*<br>_____<br><br>**Mentions BOTH that student:**<br><br>1. Must apply a strategy **for the given number of days** (10) *(must note that student needs to use the given number, using clear phrasing such as "the number 10," "the number of days given;" "the given number,"— or even "any given," "a given")*<br><br>**AND**<br><br>2. Could use **more than 1** strategy *(mentions this, or lists at least 2 strategies, possibly including: substitute in to a rule, extend a table, draw pictures and count, etc.)* | *Learner provides clear, complete explanation of learning goals and provides multiple examples and/or references to readings to enhance the communication.*<br>_____<br><br>**BOTH:**<br><br>1. ***Meets*** criteria for a 3<br><br>**AND**<br><br>2. Either or both:<br><br>i. Refers to the task and models<br><br>ii. Explains **2 or more** strategies thoroughly |

## CANDY DOTS – (3): QUESTIONING/PROBING FOR UNDERSTANDING

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| *Learner provides minimal to no insights or questions to probe for understanding.*<br>_____<br><br>Does not meet the criteria for a 2.<br><br>**MAY INCLUDE:**<br><br>1 or no questions<br>**and**<br>no rationale given<br><br>**-and/or-**<br><br>Significant mathematical errors or flaws in thinking<br><br>**-and/or-**<br><br>Learner response not related to task | *Learner provides little insight or weak questions to probe for understanding.*<br>_____<br>__<br><br>**EITHER:**<br><br>**1 question** to clarify student understanding with a rationale given; question is related to the student work<br><br>**OR**<br><br>**2 or more questions** with **no** rationale given; **may** contain flaws or errors | *Learner provides good insight and provides questions that probe and prompt students to clarify and prompt reflection.*<br>_____<br><br>**ALL THE FOLLOWING:**<br><br>1. **At least 2** relevant questions<br><br>**AND**<br><br>2. **At least 1** question to clarify student understanding or thinking *(probes for more information about what the student knows or for the student's thought process)*<br><br>**AND**<br><br>3. **At least 1** rationale (why) is given which is mathematically accurate/sound<br><br>**AND**<br><br>4. Contains **no** mathematical errors **or** flaws in learner thinking | *Learner provides good insight and provides multiple questions that probe and prompt students to clarify misconceptions. The questions also are structured to elicit algebraic thinking.*<br>_____<br><br>**ALL THE FOLLOWING:**<br><br>1. ***Meets*** criteria for a 3<br><br>**AND**<br><br>2. **3 or more** questions, at least 3 in question form *(written as a question with a "?")*<br><br>**AND**<br><br>3. **At least 1** question must refer to the student work shown *(directly or indirectly)*<br><br>**AND**<br><br>4. Rationale given relates specifically to **at least 1** question posed *(explain why the teacher would ask this particular question, rather than merely providing a general rationale)* |

## CANDY DOTS – (4): CONTENT/CONCEPTUAL UNDERSTANDING

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| *Learner provides minimal to no solution or the solution is not related to the task. The solution addresses none of the mathematical concepts presented in the task.*<br>_____<br><br>Does not meet the criteria for a 2.<br><br>**No** relevant strategy, **or** strategy is not related to this task<br>*(examples of strategies that are **not** relevant include: trial and error, extend the table further, try harder, look again at the table values; write the rule correctly, etc.)* | *Learner provides an incomplete solution. The solution addresses some, but not all, the mathematical concepts. Some parts may be correct but a completely correct answer is not present.*<br>_____<br><br>Does not meet the criteria for a 3.<br><br>**POSSIBLY:**<br><br>1. **One or more** strategies given that meets either i. or ii. (in ***Meets***→) but may **either:**<br><br>i. contain flaws or omissions in learner thinking<br>**-or-**<br><br>ii. not build on what the student knows or shows<br><br>**OR**<br><br>2. Strategy is related to the task but is **not** complete | *Learner provides a solution that shows understanding of the problem and major concepts needed for the solution. The solution addresses all the key components of the problem.*<br>_____<br><br>**ALL THE FOLLOWING:**<br><br>**1. At least 1** clear and complete description of a strategy that **both**:<br><br>i. moves student closer to developing **rules** in **either** Part A or Part B or both *(more than identifying a student flaw—e.g. showing that the student equations are incorrect is not sufficient)*<br>**-and-**<br><br>ii. **bridges** from the table, pattern, or diagram ***of this particular task*** to the rule<br><br>**AND**<br><br>2. Builds on what the student knows or shows in the work<br><br>**AND**<br><br>3. Contains no mathematical errors or flaws in learner thinking | *Learner provides evidence of deep understanding of the problem including all the appropriate mathematical concepts required by the task. The solution addresses all the key components of the problem. The solution uses evidence and/or examples to add detail to relevant mathematical concepts.*<br>_____<br><br>**ALL THE FOLLOWING:**<br><br>1. ***Meets*** criteria for a 3<br><br>**AND**<br><br>2. Provides clear and complete explanation of strategies for **both** Part A and Part B<br><br>**AND**<br><br>3. Describes **at least 2** different strategies |

# HANDOUT 6 – CATEGORIES OF TEACHER QUESTIONS

## BLOOM'S TAXONOMY

**Knowledge:** requires student to recognize or recall information

**Comprehension:** requires low level thinking to reproduce or communicate information but not verbatim

**Application:** requires student to solve a problem by applying what he has learned from other situations or problems

**Analysis:** requires student to solve a problem through systematic examination of facts and information

**Synthesis:** requires student to solve a problem using original creative thinking and by connecting ideas

**Evaluation:** requires student to make an assessment against established standards

## DRISCOLL'S FIVE CATEGORIES OF TEACHER QUESTIONS

**Managing:** Questions designed to help the student start or stay on task. Do you need any materials? Where are you going to show your work?

**Clarifying:** Questions designed to shed light on what the student means or to help the student understand the problem. Do you know what a rule is?

**Orienting:** Questions designed to help the student get started. What's the problem asking you to do/find? Could you draw the next figure in the pattern?

**Prompting Mathematical Reflection:** Questions designed to have a student reflect on, explain or extend his or her thinking. Can you explain how you got the values in the table?

**Eliciting Algebraic Thinking:** Questions that ask students to do and undo, make generalizations, build rules for functional relationships, look for what changes, etc. What would the 15th figure look like? How many dots would it have? Can you explain how you can use the equation to solve for the *n*th figure? How are the values in the equation related to the dot patterns?

Source: Driscoll, M. (1999). *Fostering Algebraic Thinking.* Portsmouth, NH: Heinemann, 6.

# Handout 7 – Evaluating Teacher Questions

## Candy Dots Task, item 3 *(on page 4 of Handout 3)*

"Given the student work, what questions could you ask the student to get a better understanding of what the student is thinking related to the task? Explain why you chose these questions."

1. What would you look for in a good response to this assessment item? List rubric criteria for a level 3 (proficient or meets standards) response.

2. Examine actual teacher responses (Handout 8). What ideas emerge for you as you think about how individual teachers responded, and how the group of teachers, as a whole, responded to this item?

3. Discuss: How do the teachers you work with use questioning as part of their classroom practice? How might an activity—such as the one provided in the assessment—be useful in your work with teachers?

# HANDOUT 8 – TEACHER RESPONSES TO CANDY DOTS #3

## QUESTIONS FOR SAM (PART A):

1. Can you better explain 4 + 1? What does that mean? 4 what and 1 what?

2. How do you know that #5 would be 21? Are you sure? Can you explain your process?

3. Do you see from the chart that the outcome to input ratio is 4 to 1 and this is why we use $4n$.

4. How did you figure 4 times the number plus 1. How did you check your rule? Were you able to draw figures out passed 3 to check?

5. Why do you add one if it is increasing by 4?

6. There are several questions that I would ask Sam, starting with, What does he mean by adding dots to the "area"? Is there a better way to express that instead of using the word "area"? Might that be confusing to some people and why? What do you mean when you say multiply "everything"? Is there a better way to word that that would be clearer to others? What do you mean when you say multiply by 4+1? Isn't that the same as multiplying by 5? Do you mean multiply be 4 and then add one to that product? Is your answer in the form that was requested, using numbers and symbols?

7. Please demonstrate how your rule works for 4 and 5. Will your rule work when $n$ equals 0? While listening to student answer for 4 I'd be listening for $4n+1$ idea which is not clearly expressed in student's answer. There is no $n$. I'd ask about 0 to get them thinking about the pattern before what is listed in the table.

## QUESTIONS FOR MARIA (PART B)

1. What do you mean by "double"? Usually that means to multiply by 2. Is that what you mean to say? What is the "bottom dot" ?

2. How does your answer give total dots?

3. Do you see 2 patterns here: that we are squaring a number (thus, the area of a square), and we are adding the next odd number each time?

4. Show me what you did to get the numbers in the tables. Explain to me what your rule means. How does it work?

5. How did you find the area? By asking this question I can see if they understand how to get any number of steps not just the ones in the chart. What about the pattern "doubles"? Do you know what a "square" number is? By asking these I can find out if they understand squaring numbers.

6. Thinking about *n* in each figure, is the total number of dots a multiple or a factor of *n*?

7. The first question that I would ask Maria is if she checked her rule to be sure that it worked for the three figures that are given? Have you given a rule using numbers and symbols? Would you show me a couple of examples of how your rule works?

8. I'd ask the student to demonstrate how their rule works in Maria's table. (it works from 4 to 9 only). No doubt their rule makes perfect sense to them, so hopefully I'll see what they are thinking since I don't understand how the student got 16 and 25 (or 4 for that matter).

9. Do you see 2 patterns here: that we are squaring a number (thus, the area of a square), and we are adding the next odd number each time.

# HANDOUT 9 – REFINING QUESTIONS FOR A SPECIFIC PURPOSE

Select three teacher responses from Handout 8 that interest you.
Fill in the chart for each example.

| **Part** (A or B) and **Response #** | **Categorize the question(s) in the response.** Use language from Driscoll and/or Bloom's Taxonomy (see Handout 6). | **If possible, describe a situation in which this question, as is, would be useful for a teacher to ask.** | **a) Re-word or refine the question(s) to better achieve a purpose.** **b) State the purpose.** **c) Categorize the new question(s).** |
|---|---|---|---|
| | | | |
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| | | | |

# HANDOUT 10 – DIAGNOSIS AND NEXT STEPS BASED ON EVIDENCE AND REFLECTION ON STUDENT WORK

Read and reflect upon each of the examples of written work from teachers. Answer the questions that follow.

## 1. TEACHER DIAGNOSIS

"Exactly what do you multiply by '4+1'? Could you draw out the dot pattern for $n$=4"? I would ask this to see what the student meant by "multiply everything by 4+1". I would want him to clarify, it looks like he understands HOW to complete the pattern, but not how to EXPLAIN the pattern.

- What else could be going on?

- What strategy would you consider using next with this student? Why?

## 2. TEACHER DIAGNOSIS

"The student seems to have the equation, but has perhaps left out the $x$-value and means $4x+1$."

- What else could be going on?

- What strategy would you consider using next with this student? Why?

# HANDOUT 11 – CANDY DOTS #5

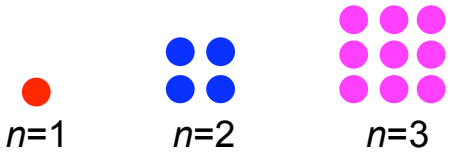
Examine the following sample of student work and answer the associated question about the scenario presented.

You are given the following response to Part B of the previous task.

**Part B.**

1. Maria filled in her table for the first 3 figures. Complete the table for the 4th and 5th figures.

| Figure (*n*) | Total Dots |
|---|---|
| 1 | 1 + 3 |
| 2 | 4 + 5 |
| 3 | 9 + 7 |
| 4 | 16 + 9 |
| 5 | 25 |




2. Write a rule with numbers and symbols to describe the number of dots for any number figure.

*Each time you multiply n by itself, or square it. That's why the pictures are all squares*

*You add the next odd number 1 + 3=4+5=9.*

5. When you are discussing the results of the task with the student, she states that she noticed that the number of dots increased by 3, then 5, then 7. She also noticed that it is the square of the figure number. She then asks why these two patterns exist with the same numbers. What might be something you could show and explain to the student to help her discover the connection?

## DISCUSS AT YOUR TABLE

What content knowledge do you think this question could assess?

What would you want to see in a teacher's response? Why?

## CANDY DOTS– (5.): CONTENT/CONCEPTUAL UNDERSTANDING

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| *Learner provides minimal to no solution or the solution does not relate to the task. The solution addresses none of the mathematical concepts presented in the task.*<br>_____<br><br>Does not meet the criteria for a 2.<br><br>Response may not be relevant to the task | *Learner provides an incomplete solution. The solution addresses some, but not all, the mathematical concepts. Some parts may be correct but a completely correct answer is not present.*<br>_____<br><br>**POSSIBLY:**<br><br>**Either**:<br>i. Explain process but does not relate the square numbers to the rate of change<br>**-or-**<br>ii. Relate the square numbers to the rate of change but does not have a process to show or describe this to the student<br><br>**AND/OR**<br><br>May contain flaws in mathematical reasoning or mathematical errors | *Learner provides a solution that shows understanding of the problem and major concepts needed for the solution. The solution addresses all the key components of the problem.*<br>_____<br><br>**ALL:**<br><br>1. Explains a process *(how to use a diagram, graph, expanded form, etc.)*<br><br>**AND**<br><br>2. Must relate the square numbers (1,4,9,16…) to the rate of change in candy dots (3,5,7…)<br><br>**AND**<br><br>3. Contains no conceptual flaws ***in learner thinking about the relationship between the square number and the rate of change*** | *Learner provides evidence of deep understanding of the problem including all the appropriate mathematical concepts required by the task. The solution addresses all the key components of the problem. The solution uses evidence and/or examples to add detail to relevant mathematical concepts.*<br>_____<br><br>**BOTH:**<br><br>1. ***Meets*** criteria for a 3<br><br>**AND**<br><br>2. **Either:**<br>i. Explanation is sophisticated *(such as relating the rate of change in candy dots to the figure number)*<br>**-or-**<br>ii. Uses 2 or more different approaches to explain the relationship *(graph and picture, picture and expanded form, graph and expanded form, etc.)* |

# HANDOUT 12 – CARD ACTIVITY MASTER

*Facilitator, photocopy onto cardstock and cut out cards to create a deck of cards. Also photocopy game boards (page 37). Give each group of 3–4 players one deck of cards and one game board.*

| | |
|---|---|
| Since there are four sides to the square and the 4th side will be complete without the last dot made on the last side to complete the new square you put around the outside, the number added will be odd(4 sides minus 1 dot will be odd). | Perhaps using dots of a different color to represent the "new" dots that are added in successive figures would help explain the pattern of adding the next odd number. The square shape would still be preserved. |
| I'd try drawing the first 5 or so figures in the pattern, each time using a different color to show the dots that were added to the previous figure. I'd then use those differently colored dots to show the student that each time the dots form an upside down "L". The number of dots in each arm of the "L" describe the dimensions of the square figure. Thus the pattern of squaring the figure number to get the number of dots. I'd also point out that their is one dot in each "L" that is part of both the length and the width. That is, the dot where the "L" bends. Since it can't get counted twice, the number of dots is always an even number minus one. This leads to the pattern: adding consecutive odd integers to the previous figure to get the number of dots in the next figure. | Let's look at the figures to see if we can see a connection between 3,5,7,9.... First you have 1 then you add 3 now you get figure 2. Then in figure 2 you have 4. In figure 3 you have 4 with 5added on, total 9. Figure 4 would have 9 then you add 7. |

| | |
|---|---|
| draw tessellations | I would help the student look at the chart and fix the chart first and then discuss the connections from figure 1 to 2 to 3, etc. When the student noticed adding the next odd number yields the next square, I would point out that s/he was finding the difference in numbers between figure 1, figure 2, figure 3,etc. You could add a third column to show this difference. |
| I would show her how the dots in the squares add up, when you look at the sides and corners, to be the total. The figure numbers are squared because the figure is a square and they're odd because there is always a corner added on plus the two sides and summing the two sides makes an even result, and the corner added on makes the total odd. | I might use the relationship of area and show how a dot can be the length of a side of a square. |

| | |
|---|---|
| It would depend their level. If it was one of my algebra kids that knows how to use foil, I would show them that it was also (*x*-1) ^2 + 2(*x*-1) + 1 and that could simplify to *x*^2. If it was one of my regular 8th graders I'd show them graphically that the new n has the old n plus two n amounts—one vertically on the side added and one horizontally on the top added and then on in the corner giving them 2n+1 being added to the old shape which are the odd numbers listed. | I can think of a couple of ways to show the relationship depending on the sophistication of the student. For a veeery good student I might try to show the relationship (*N*-1) squared, the number of squares in the previous pattern, then add (2N-1), which always gives the next odd number. (N-1) squared added to (2N-1) is equivalent to N squared. I might also use a more hands-on approach and show how the previous square could be isolated in the upper right corner of the next figure with an "L" shape of dots in the lower left corner. This "L" shape of dots is an odd number that increases by 2 every time the pattern goes to the next value of "N", hence increasing by 3,5,7,9 etc. |
| I would show the dots "appearing" in the figure to form a new larger square. Each time you need the next larger odd number | I would think that this student who sees the function in terms of n is ready to look at *n*+1 and even (n+1)^2. Depending on the student's level, I'd start by asking: if n=1, what is the next number in terms of n? Hopefully, the student would say n+1, in which case, I'd help the student see that (n+1)(n+1) is equal to n^2+2n+1. If the student is not ready for this I might start with if n=3 then you add a row and column of 3's +1 to make the next figure or in other words, 2(3)+1 =7 This is the 2n+1 or the odd numbers that the student is adding:If n=1 then 2n+1 =3 or n=2 then 2n+1 = 5 and so on.I'd also look at the equality the student wrote, which is not correct. 1+3 does not equal 4+5! |

| | |
|---|---|
| If a student is a visual learner, it's an easy connection. Start with one square (say, 2x2) then you can draw the next "layer" of dots to the right and top. Then two more dots directly to the side, two above, one diagonally up. You'll add 2n+1 dots each time. (Looking at the values of 2n+1 for n=all whole numbers that creates all odd positive integers.) Algebraically you can show that n^2+2n+1 (see previous explanation for 2n+1) is equal to (n+1)^2. I'd only use this with a student who is very advanced and picks up new things quickly, OR a student who has dealt with factoring polynomials or at least multiplying binomials together. Most 7th graders I see would understand the first explanation. My 8th grade algebra class is finally to where I'd expect the second explanation to make sense. | Because that is the rule for the sum of odd whole numbers, when you start with one. 1+3=4+5=9 |
| I would explain the mathematical relationships of fibonacci numbers. Each number is the sum of the previous two numbers. Many other relationships exist, like the square of any two alternating Fibonacci numbers is a Fibonacci number. | use manipulatives to really look at the pattern so that maybe she will start to see it |

| | |
|---|---|
| I would work with a manipulative or graph paper to draw out the first several perfect squares and show how by adding to each side of a square would double the number of dots(making an even number) but leave open the one corner dot and an even number plus 1=odd number. | Often in algebra we get the students to look at the differences between patterns. This is how they begin to recognize the pattern. Oh the difference is 1, 3, 5, 7, so the next one must be plus 9. This is sometimes an easier pattern to recognize than the fact that the numbers are squared. |
| You could ask the child what is the formula for area of a rectangle. Why is a square a rectangle? A square is a special rectangle because all sides are _____. Area of a rectangle is length times width so area of a square can be length times width or side to the second power. | I'd have her just sketch one top and one side of Figure 3 ( she'd end up with an "Ell that was 3 dots across and 3 dots down or a total of 5 dots) I'd ask her to draw the next Ell for Figure 4, where she'd have 4 dots across and 4 dots down, or a total of 7 new dots. Then I'd ask her to visually slide the second ell over the 1st and see if she noticed what she created.I also might have her use the formulas n^2 and (n+1)^2 and show the work algebraically. I might use blocks of different colors to show that to go from n=2 to n=3, I add 2 blocks horizontally and 2 vertically (these 4 blocks would be the same color) and 1 block in the corner (different color), or 2+2+1, or 2n+1. Do the same going from n=3 to n=4, you add 3+3+1, or 2*n*+1 blocks. |

*Place cards in the appropriate space below. Leave cards off the game board if they don't belong.*

This response suggests that the teacher may not understand the question or the relationship the student sees.

This response suggests that the teacher understands the relationship, but I have questions/concerns about the strategy they've chosen to use to explain it.

This response reflects an understanding of the relationship paired with a reasonable strategy.

# HANDOUT 13 – PLANNING PROFESSIONAL DEVELOPMENT

**Identify the Main Messages:** *What ideas do you want to convey in this workshop/meeting?*

**Articulate the Learning Goals:** *What would you like the participants to walk away knowing or having experienced? Identify two or three goals for the workshop/meeting.*

**Build a workshop/meeting around the activities:**

*What other information do you want to share? How will you do this?*

*What discussions would you like to have? Compose some possible discussion questions.*

**Brainstorm Activities:** *Identify two activities to support the learning goals. These could be taken directly from or adapted from today's session.*

**Assessment/Feedback:**

*How will you know if the professional development achieved its goals? What will you do to follow up and sustain the learning?*

# HANDOUT 14 – FEEDBACK FORM

1. Please rate the following:

| | Not Useful | | | | Very Useful | Give Reasons for Your Rating |
|---|---|---|---|---|---|---|
| **Overall Session** | **1** | **2** | **3** | **4** | **5** | |
| **Group Activities/ Discussion** | **1** | **2** | **3** | **4** | **5** | |

2. What was particularly useful or interesting?

3. What is one idea that you plan to apply in your school or district?

4. What suggestions do you have for improving the session?

5. What would you like us to keep in mind for future sessions of this workshop? Please let us know if you have specific questions that you would like us to address.

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