# Topic Study: Graphs – Distance/Time

## Background:

This Topic Study includes sections dealing with mathematical context, pedagogy, essential questions to elicit student thinking and prompt mathematical understanding, potential errors and misconceptions associated with the topic, technology integration considerations, and possible topics for collegial dialogue.

There are a number of graphing tools that can be used to analyze distance time graphs. A few of the tools that are aligned with this study include ***Graph It*** (Shodor Education Foundation, 1997-2005), ***Ready to Teach Qualitative Grapher*** (The Concord Consortium, 2005), and ***Distance-Time Graphs*** and ***Modeling Linear Systems (Explorelearning, 2005).*** This collection of tools can be used to model change over time in general ways then can also be used to model distance-time graphs using a number of real life examples.

## Context

*What's the important math?*

a) Interpreting graphs by relating some motion to its corresponding graph

   Important mathematical goals here include:
   - understanding how different motions appear on a graph:
   - understanding how the slope or slopes of a graph relate to the motion it represents;
   - learning to focus on the overall picture of the graph and not just on the details of the numbers and individual points.

b) Learning to plot information that involves two variables simultaneously

*Students' Difficulty with Interpreting Graphs*

While students may be able to create a graph given a table of data, they often have difficulty making sense of what the graph is showing, once it is made. Current research on students' creation and interpretation of representations of data indicates that students tend to first focus on individual data points on a graph rather than the overall shape of the graph (Konold & Higgins, 2002). This means that, given a graph, students may be able to identify significant individual points on the graph, such as the time when a car reached it's destination or when a thrown ball was at its highest point. However, they are often at a loss to relate the graph to the situation it represents, such as describing when the car sped up, slowed down or stopped for a while, or whether the ball ascended and descended at the same rate.

Therefore, it is important to give students experience with interpreting graphs that helps them focus on the overall shape of the graph and not exclusively on individual data points. One way to do this is to provide problems for students that require them to analyze graphs that have few or no numbers on the axes.

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## Pedagogy

### Pedagogical Considerations

- It can be difficult for students to think about graphical representations that use more than one variable. Students need lots of experience looking at graphs that display more than one variable to become accustomed to the way that graphs show two pieces of information at once in a single point.
- This is a terrific topic with which to use technology. Since the technology lends itself well to showing dynamic (moving) situations, students can easily look at motions and their corresponding graphs using some of the technology tools suggested here.
- You may want to begin with an initial focus on a particular motion, such as students' walking rates, and the graph of the motion over time. Have students try some simple motions, graph them and see the results, then discuss. As you discuss and analyze, then look at specific points and values on the graph.
- As you introduce various graphs for students to consider, help students focus on looking at the relationships between the information on the two axes. (Some possible questions are suggested below.) Try not to focus first on the numbers, but instead try focusing on significant places where the graph changes state, such as where the graph flattens out, starts to rise more steeply, or where it ends. The numbers can then be used to verify interpretations that students have made.
- Students benefit from seeing a wide variety of examples of distance/time graphs. Think about presenting students with examples of graphs of:
  a) distance from starting point over time
  b) total distance traveled over time
  c) distance from ending point over time
  Note that graphs of total distance traveled and distance from ending point will never have negative slopes anywhere on the graph! Be sure you can answer for yourself why this is true.
- In addition, research has shown that students benefit from seeing a variety of types of distance/time graphs for the same motion. Consider giving students situations in which they simultaneously see all three types of graphs listed above – for the same motion – and have to try to distinguish which type of graph is which for the given motion. Be sure to ask students to explain their reasoning for their choices.

### Possible Teaching Strategies

- *Establish an understanding of constant rate*
  If students have had little experience with rates, do some simple exercises in class to help them understand the idea of a constant rate. For instance, ask students to try such exercises as turning the pages of a book or tapping a pencil at a steady rate that they can recreate over several trials. This can work well in partners, with one student doing the motion and the other student timing and recording the motion.
- *Begin with experiments with motion*
  Give students time to try some different simple motions they can do in the classroom, such as walking across the room, sitting at their seats and bending back and forth at the waist (they can plot the distance of the top of their head from the floor), or
- *Have students pose some conjectures about motions*
  Ask students to predict the shapes of graphs of certain motions that they can do. Then perform

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the motions to test the conjectures. If you have a motion detector available it can be used in conjunction with graphing software to have students actually explore their own motion and simultaneously see the graphs of their motion.

Discuss whether the graph came out as expected and why or why not.

- *Hone your predictions and share*
  Spend some time as a class discussing how to make your predictions more accurate. You may want to discuss why the graph behaves in certain ways for particular motions. Summarize for the class what they are figuring out.
- *Pose some specific motion and graphing tasks to test your conclusions*
  See if your conclusions help you more accurately predict what the graph will do for a particular motion.
- *Discuss and summarize*
  Have a final discussion to pull together the final list of conclusions – what have you figured out as a class? Why does the graph behave in certain ways for certain motions?

## Essential Questions

**To probe for understanding:**

- What is happening with the motion when the graph goes up steeply here?/goes down steeply here?/rises gradually here?/falls gradually here?/stays horizontal?
- Where is there an interval of time in which the runner/vehicle is traveling the fastest/slowest? How do you know?
- (When comparing two motions on the same graph): What is happening with the motion of these two runners/vehicles when the graphs cross each other at this point? Are the runners/vehicles colliding?

**To prompt mathematical reasoning:**

- What would happen to the graph if the runner/vehicle did the same motion but did it faster/slower?
- Why is there never a part of the graph with a negative slope when measuring total distance traveled? when measuring distance from the ending point?
- Why is there never a vertical line on the graph?

## *Common Misconceptions*

***Error***: When looking at a graph of distance over time, a student interprets a horizontal line as the object moving in a straight line.

***Related misconception***: A very common misconception among middle grades students is to interpret graphs of motion as a literal picture of the motion. One good example of this is when students are asked to look at the graph of the motion of an object or person that is stopped. When they see a horizontal line, it represents stopped motion but *not stopped time*! The motion of an object that remains stationary over time is represented as a horizontal line (assuming time is graphed on the x-axis and distance on the y-axis.) This can be easily illustrated with the ***Ready to Teach Qualitative Grapher*** (The Concord Consortium, 2005).

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***Error:*** When asked a question such as “During what time interval did the car travel more slowly?,” a student will answer with a single point in time, such as “At 5:45.”

***Related misconception:*** This is not really so much an error as an indication of what level the student’s thinking is at. It is common for students to ignore intervals of time on the graph, and instead focus on a single point in time. Students who are beginning to work with data tend to focus on individual data points and not the group of data as a whole.

As you see students doing this, you can encourage them to see the data as a collection by having them list a number of individual points for which the data fits. Then model for them how to describe the group of data points as an interval of time. As students grow more accustomed to looking at intervals on a graph, you may want to discuss with them why it does not make sense to answer the question “During what time did the car travel more slowly?” with a single time. (Possible response: because it does not make sense to identify a single moment in time to describe the car’s motion. For a single point in time, you can only take a “snapshot” of where the car is within it’s larger motion.)

## Professional Community: Collegial Dialogue

Ask colleagues:

- What things have you tried in the classroom to teach this topic?
- What kinds of motion did you have your students perform experiments with? Which of those worked well and which did not?
- Which technology tools did you try and what would you recommend?

Reflection questions:

- What do your students understand about the idea of a constant rate? What questions could you pose to them to find out what they already know (or think they know)?
- What evidence did you see of your own students interpreting graphs as pictures of motion (or not?)

## Related Readings:

*A Summary of the Research on Student Graphing Misconceptions and Their Roots*
Emily R. Fagan

*Living Graphs,* David Gay and William Yslas Vélez
MTMS, November 2001, Volume 7, Issue 3, Page 172

## References and Tools

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