

From Buttons to Algebra:

Learning the ideas and language of algebra, K-12

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<http://thinkmath.edc.org>

Some ideas from the newest NSF program, *Think Math!*

from  and Harcourt School Publishers

Rice University, Houston, Sept 2007

Before you scramble to take notes

<http://thinkmath.edc.org>

With downloadable PowerPoint
at <http://www.edc.org/thinkmath/>

What could mathematics be like?

It could be spark curiosity!

Is there anything interesting about
addition and subtraction sentences?

Write two number sentences...

$$\begin{array}{r} 4 + 2 = 6 \\ 3 + 1 = 4 \\ \hline 7 + 3 = 10 \end{array}$$

To 2nd graders: see if you can find some that don't work!

What could mathematics be like?

It could be fascinating!

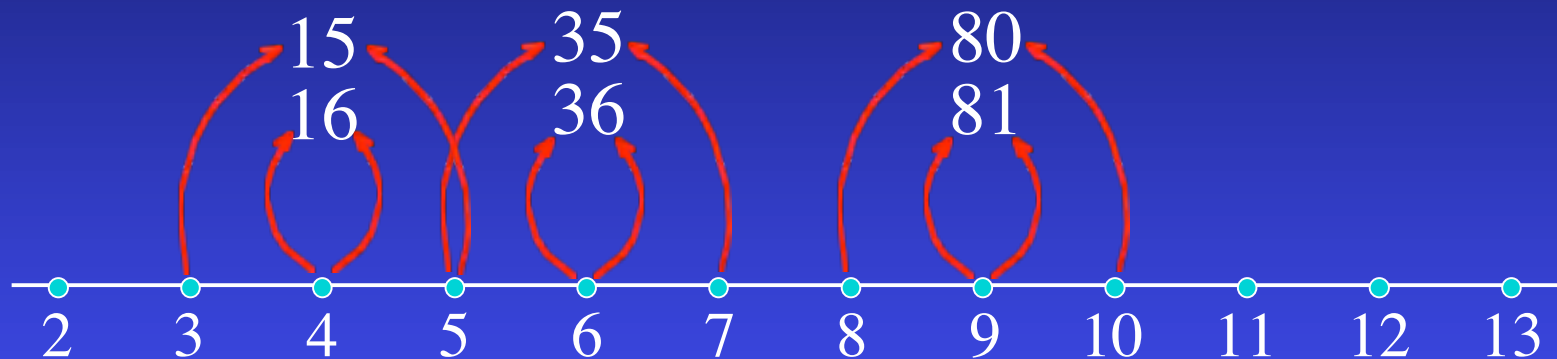
Is there anything less sexy than
memorizing multiplication facts?

What *helps* people memorize?

Something memorable!

Teaching without talking

Shhh... Students thinking!



Wow! Will it always work? Big numbers?



Take it a step further

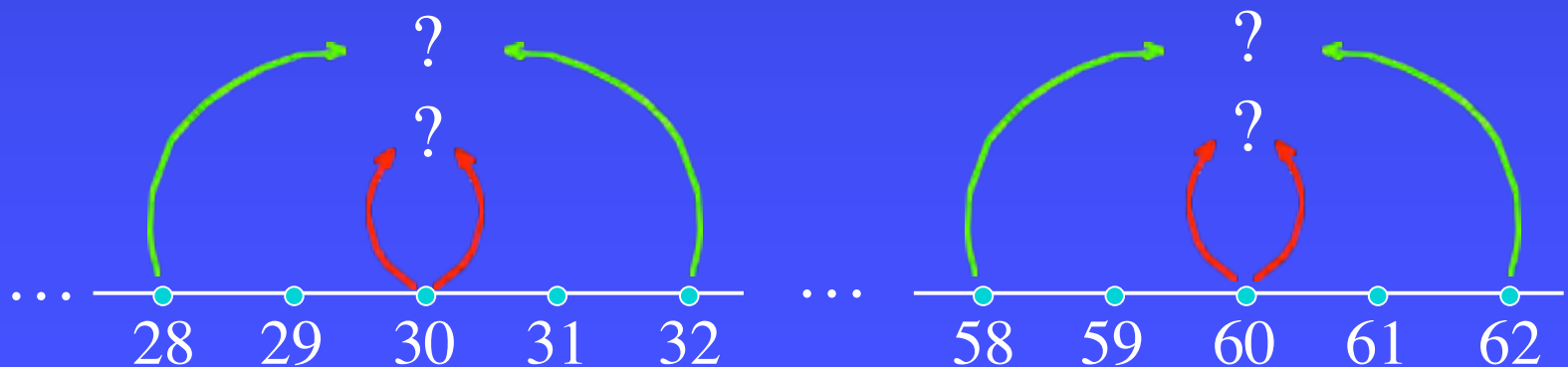
What about two steps out?

Teaching without talking

Shhh... Students thinking!



Again?! Always? Find some bigger examples.



Take it even further

What about three steps out?

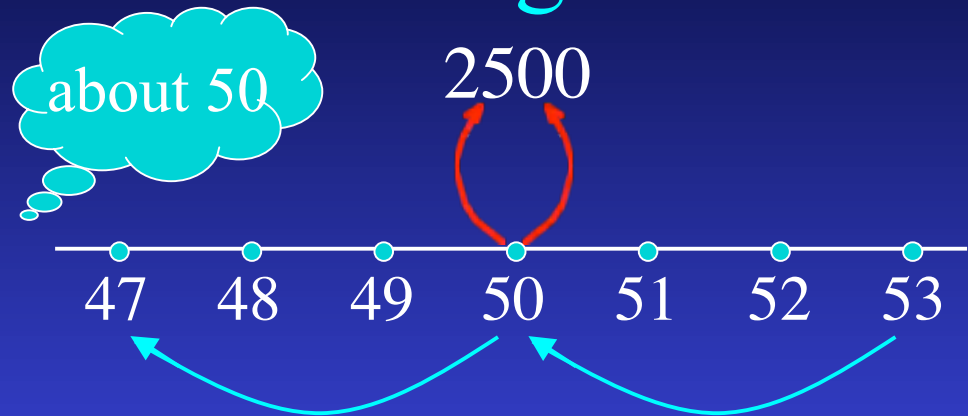
What about four?

What about five?

“Mommy! Give me a 2-digit number!”

■ “OK, um, 53”

■ “Hmm, well...”



■ ...OK, I’ll pick 47, and I can multiply those numbers faster than you can!”

To do...

$$\begin{array}{r} 53 \\ \times 47 \\ \hline \end{array}$$

I think...

$$\begin{array}{r} 50 \times 50 \text{ (well, } 5 \times 5 \text{ and ...)} \dots 2500 \\ \text{Minus } 3 \times 3 \\ \hline - 9 \\ 2491 \end{array}$$

Why bother?

- *Kids feel smart!*

- *Teachers feel smart!*

- *Practice.*

Gives practice. Helps me memorize, because it's memorable!

- *Something new.*

Foreshadows algebra. In fact, kids record it with algebraic language!

- *And something to wonder about:*

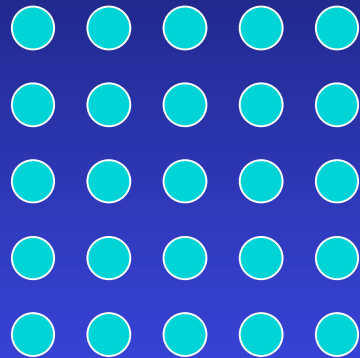
How does it work?

It matters!



One way to look at it

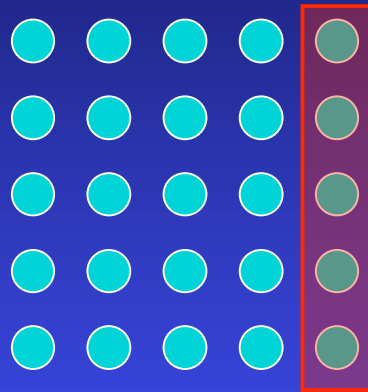
$$5 \times 5$$



One way to look at it

Removing a
column leaves

$$5 \times 4$$

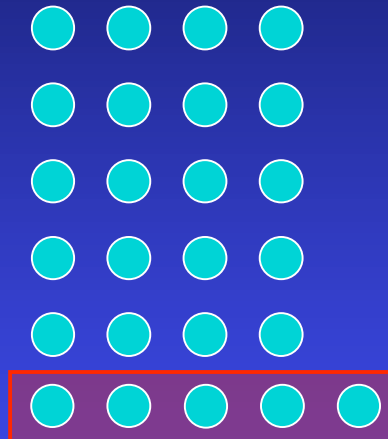


One way to look at it

Replacing as a
row leaves

$$6 \times 4$$

with one left
over.



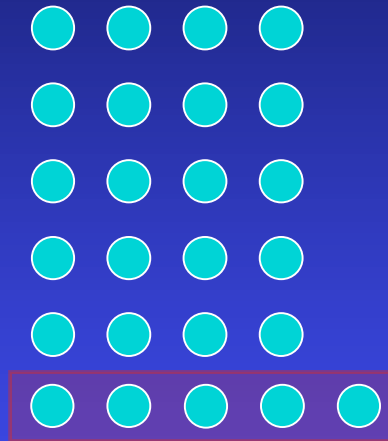
One way to look at it

Removing the
leftover leaves

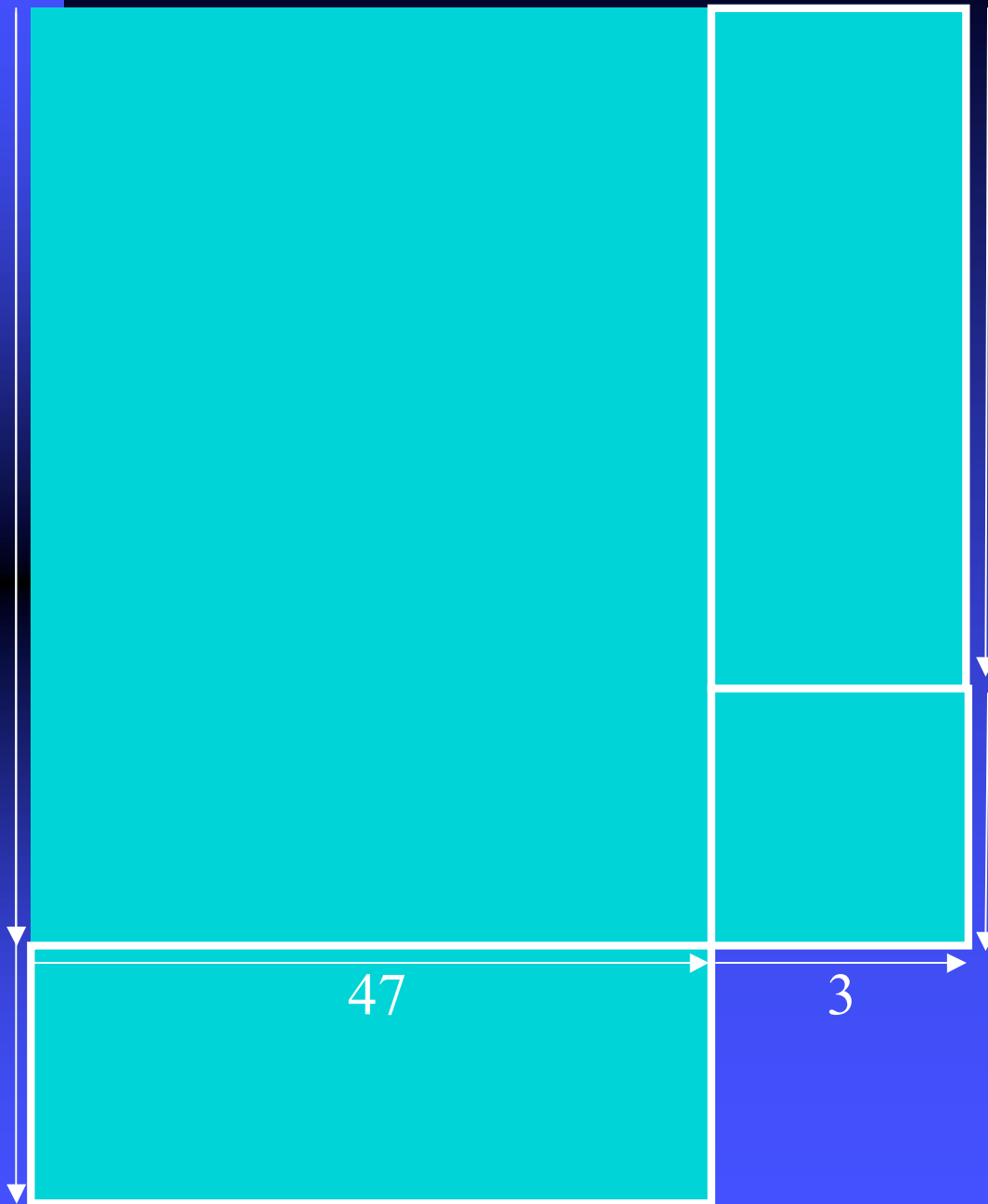
$$6 \times 4$$

showing that it
is one less than

$$5 \times 5.$$



50
53



How does
it work?

47

3

$$50 \times 50 - 3 \times 3 = 53 \times 47$$

An important propaganda break...

“Math talent” is *made*, not found

- *We all “know” that some people have...
musical ears,
mathematical minds,
a natural aptitude for languages....*
- *We gotta **stop believing it's all in the genes!***
- *And we are **equally** endowed with much of it*

A number trick

- *Think* of a number.
- Add 3.
- Double the result.
- Subtract 4.
- Divide the result by 2.
- Subtract the number you first thought of.
- Your answer is 1!

How did it work?

- *Think* of a number.
- Add 3.
- Double the result.
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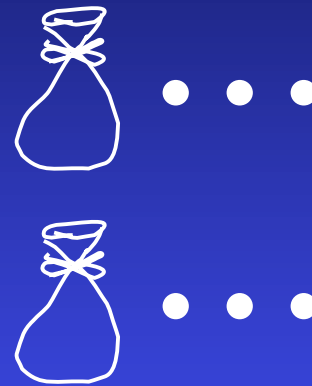
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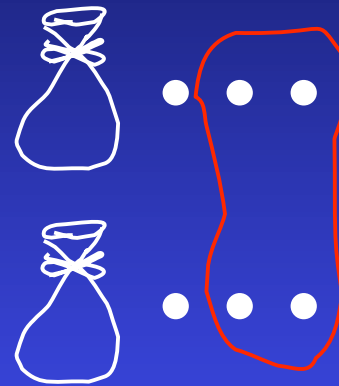
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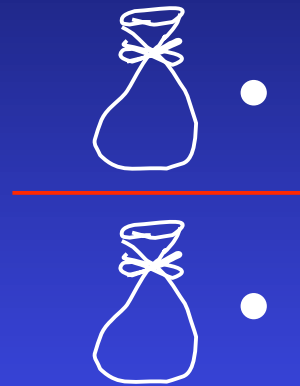
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How did it work?





- *Think* of a number.
- Add 3.
- Double the result.
- Subtract 4.
- Divide the result by 2.
- Subtract the number you first thought of.
- Your answer is 1!

Kids need to do it themselves...

Using notation: *following* steps

Words	Pictures	Dan	Cory	Sand	Chris
Think of a number.		5		y	
Double it.		10			
Add 6.		16			
Divide by 2. What did you get?		8	7	3	20





Using notation: *undoing* steps

Words	Pictures	Dan	Cory	Sand	Chris
Think of a number.		5	4	y	
Double it.		10	8		
Add 6.		16	14		
Divide by 2. What did you get?		8	7	3	20

Hard to undo using the words.

Much easier to undo using the notation.

Using notation: *simplifying steps*

Words	Pictures	Dan	Cory	Sand	Chris
Think of a		5	4	y	
Double it.		10			
Add 6.		16			
Divide by 2. What did you get?		8	7	3	20

Why a number trick? Why bags?

- *Computational practice, but much more*
- *Notation helps them understand the trick.*
- *Notation helps them invent new tricks.*
- *Notation helps them undo the trick.*
- *But most important, the idea that
notation/representation is powerful!*

Children are language learners...

- *They **are** pattern-finders, abstracters...*
- *...**natural** sponges for language **in context**.*

n	10	8	28	18	17			58	57
$n - 8$	2	0	20			3	4		

A game in grade 3

735 391 424

163 230

hundreds digit > 6

tens digit is
7, 8, or 9

ones digit < 5

the number is
a multiple of 5

the number
is even

the tens digit is
greater than the
hundreds digit

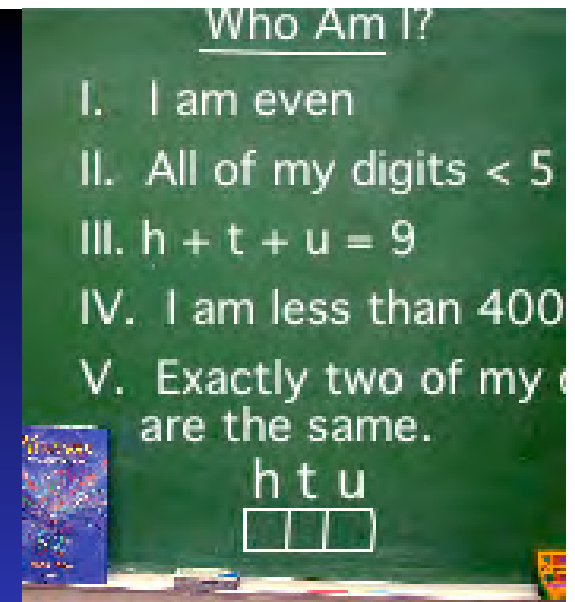
tens digit $<$ ones digit

the ones digit is
twice the tens digit

the number is
divisible by 3

3rd grade detectives!

- I. I am even.
- II. All of my digits < 5
- III. $h + t + u = 9$
- IV. I am less than 400.
- V. Exactly two of my digits are the same.



<i>h</i>	<i>t</i>	<i>u</i>	
1	4	4	432
	0	0	342
1	1	1	234
2	2	2	324
3	3	3	144
4	4	4	414
5	5	5	
6	6	6	
7	7	7	
8	8	8	
9	9	9	

Is it *all* puzzles and tricks?

- No. (And that's too bad, by the way!)
- Curiosity. How to start what we can't finish.
- Cats play/practice pouncing; sharpen claws.
- We play/practice, too. We've evolved fancy brains.

Representing *processes*

- *Bags and letters can represent numbers.*
- *We need also to represent...*
 - ◆ *ideas — multiplication*
 - ◆ *processes — the multiplication algorithm*

Representing multiplication, itself

Naming intersections, first grade

Put a red house at the intersection of A street and N avenue.

Where is the green house?

How do we go from the green house to the school?

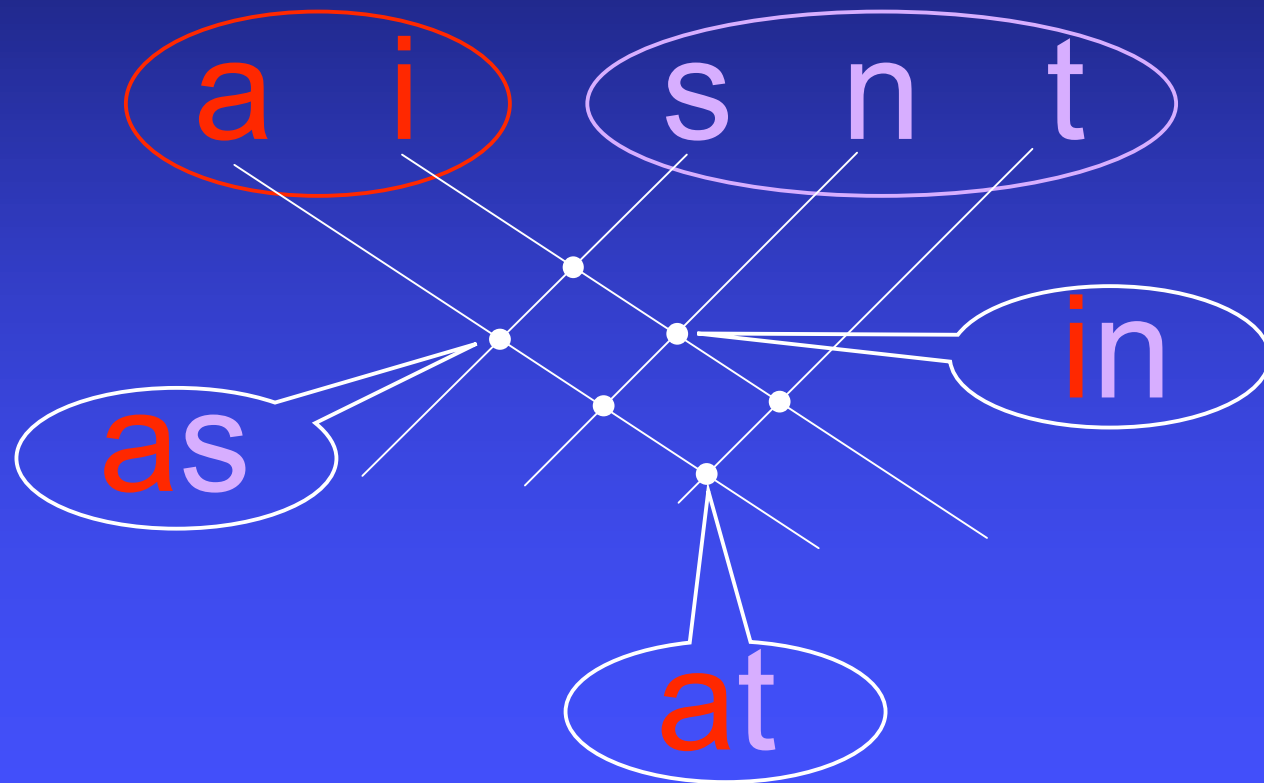


Combinatorics, beginning of 2nd

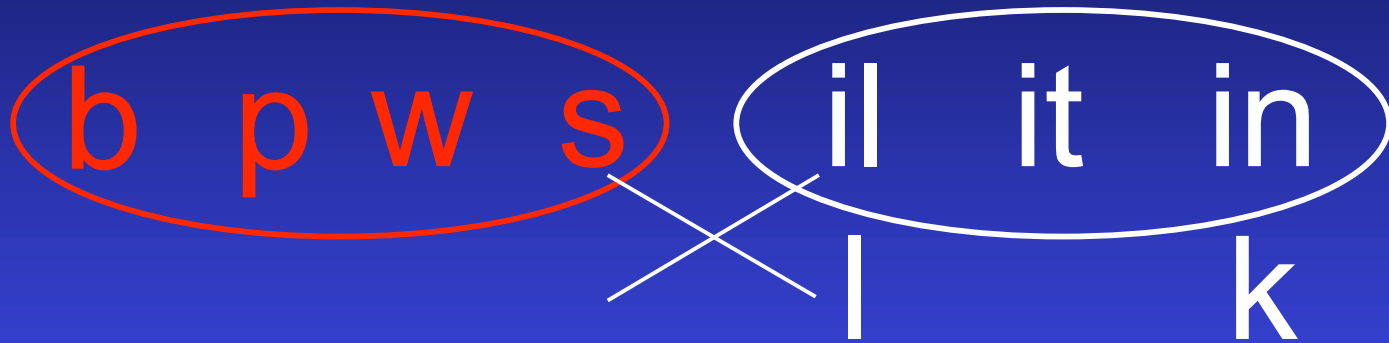
a i s n t

- How many two-letter words can you make, starting with a red letter and ending with a purple letter?

Multiplication, coordinates, phonics?



Multiplication, coordinates, phonics?



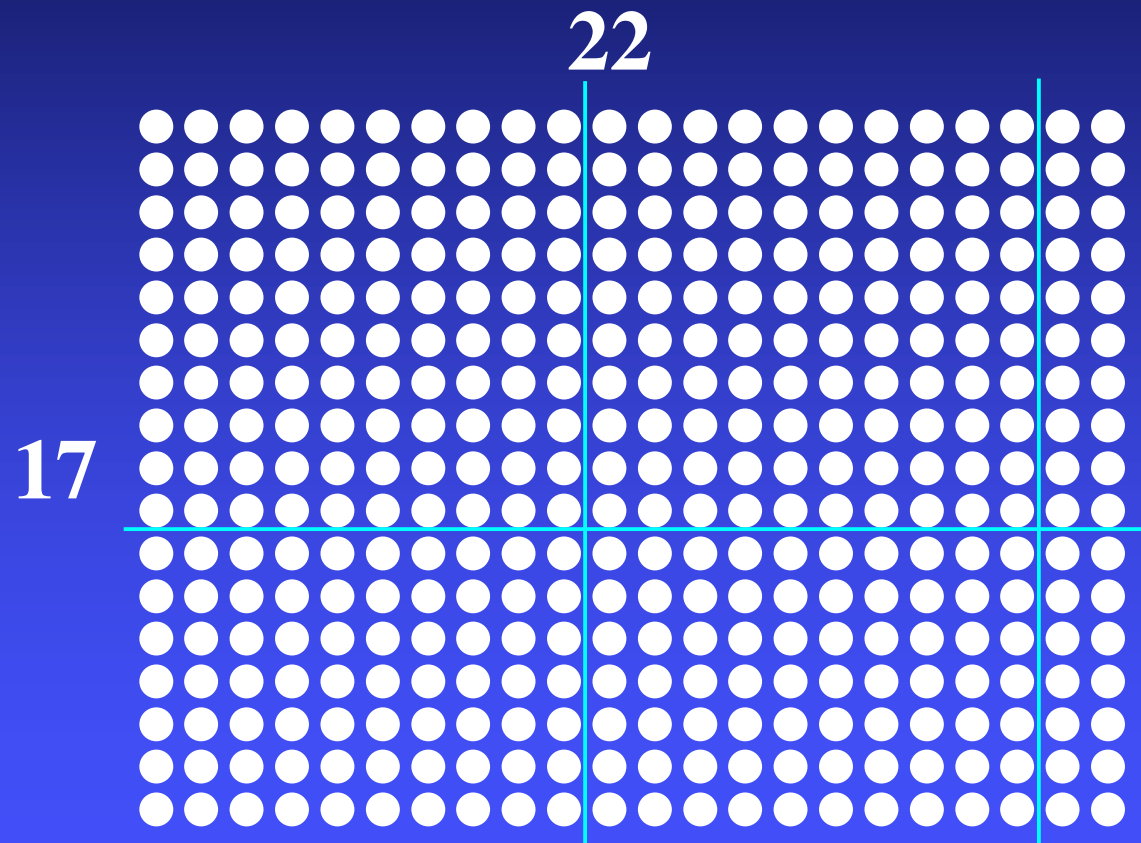
Similar questions, similar image

Four skirts and three shirts: how many outfits?

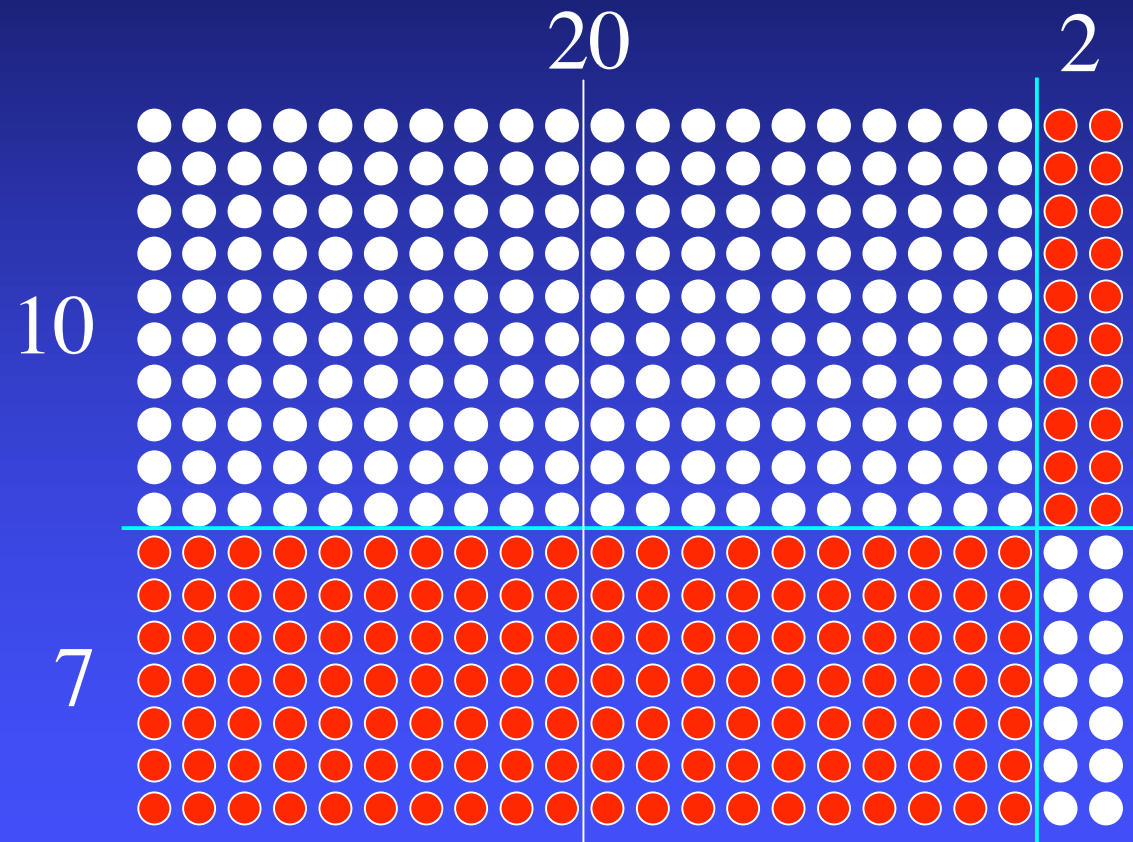
Five flavors of ice cream and four toppings:
how many sundaes? (one scoop, one
topping)

How many 2-block towers can you make from
four differently-colored Lego blocks?

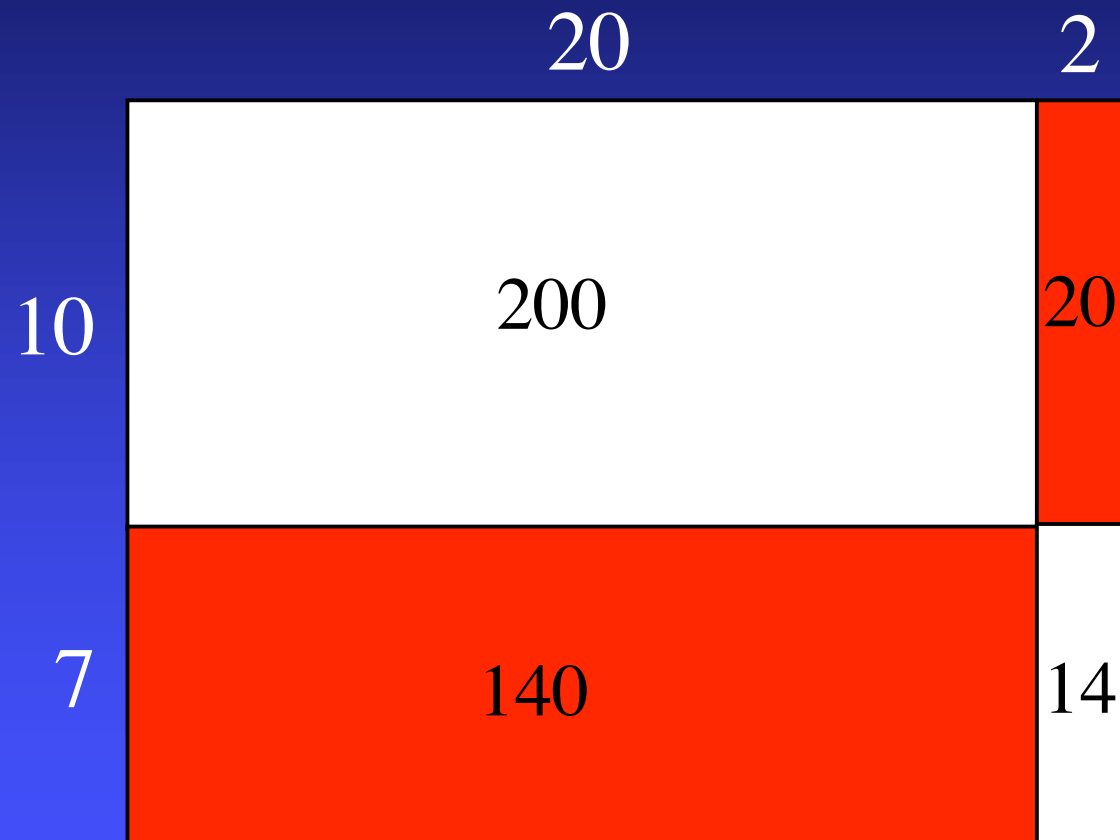
Representing 22×17



Representing the *algorithm*



Representing the *algorithm*



Representing the *algorithm*

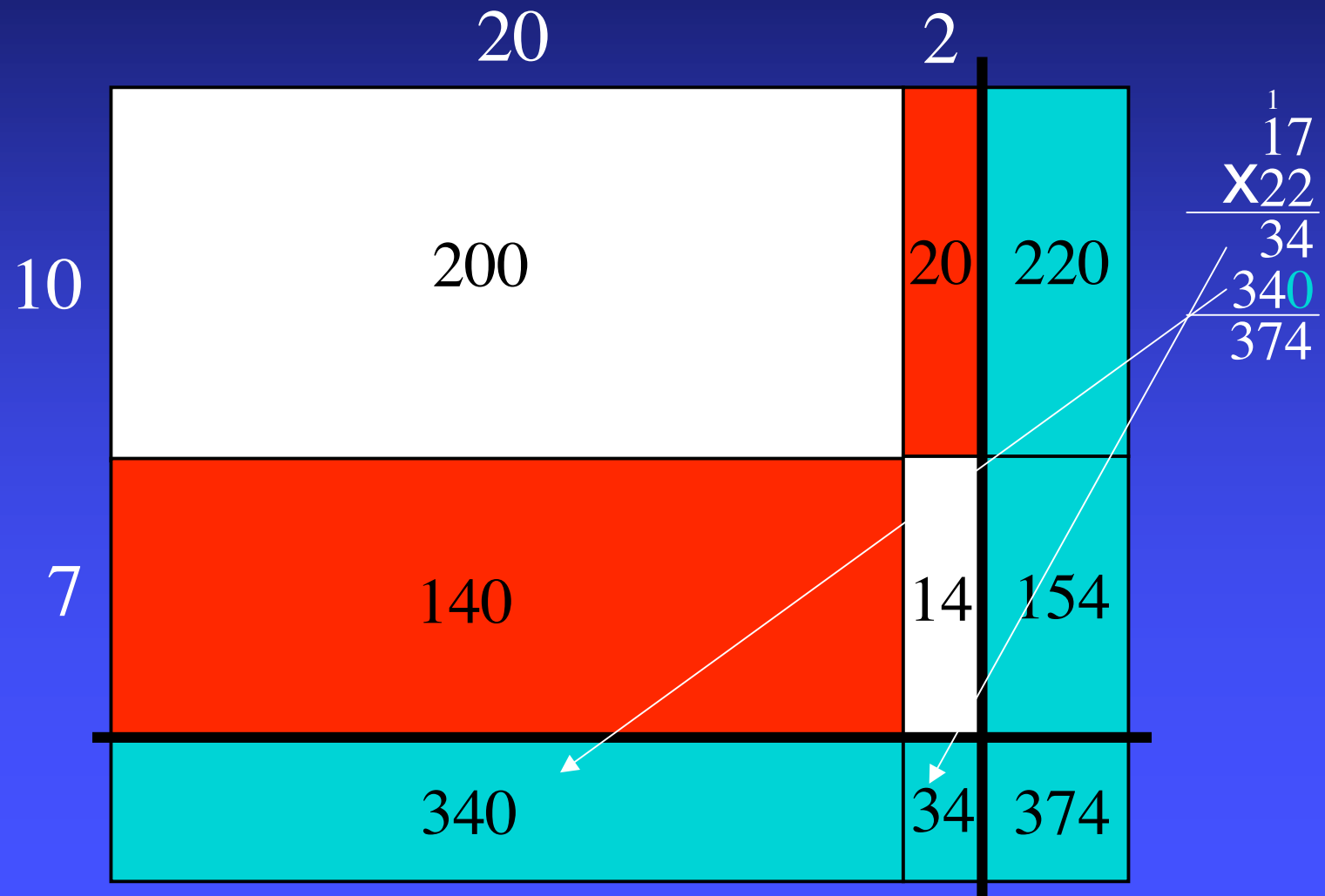
	20	2	
10	200	20	220
7	140	14	154
	340	34	374

Representing the *algorithm*

	20	2	
10	200	20	220
7	140	14	154
	340	34	374

$$\begin{array}{r} 1 \\ 22 \\ \times 17 \\ \hline 154 \\ 220 \\ \hline 374 \end{array}$$

Representing the *algorithm*

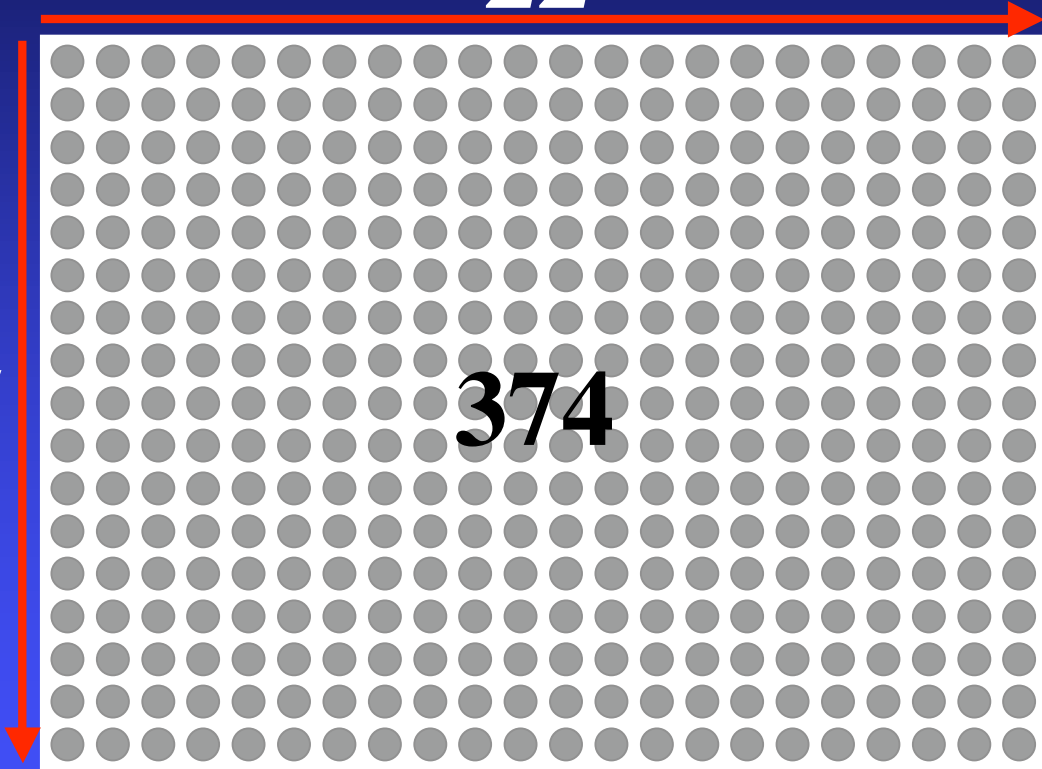


22

17

374

$$22 \times 17 = 374$$



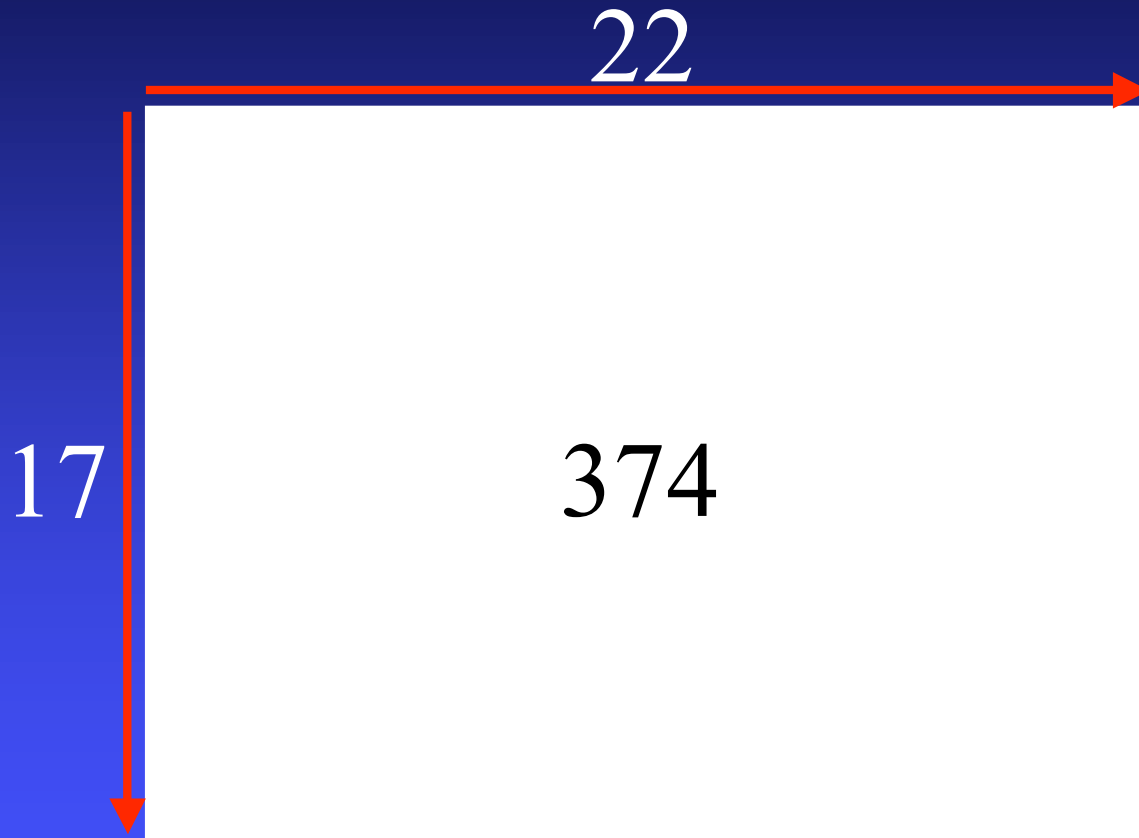
22

17

374

$$22 \times 17 = 374$$

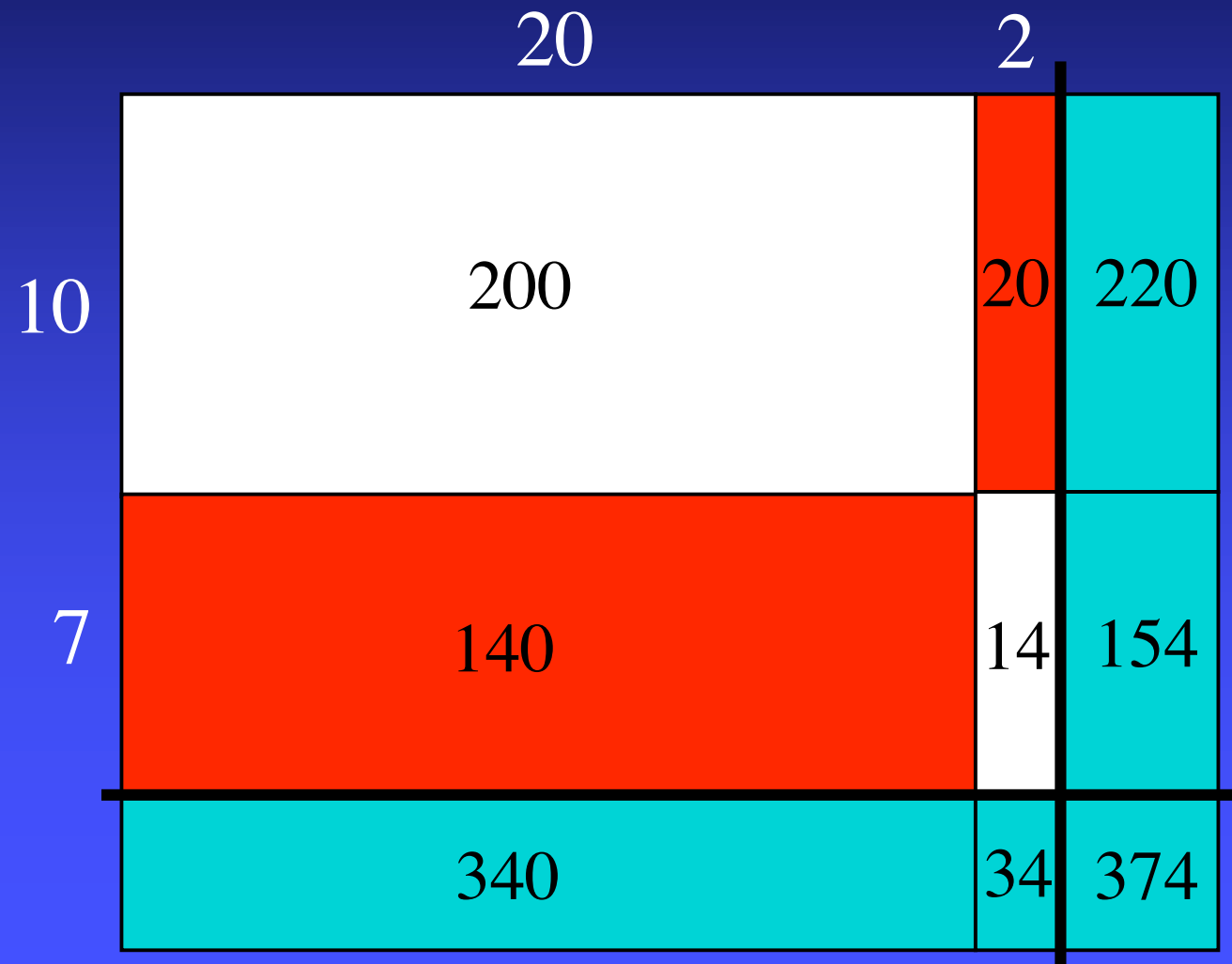
Representing division (not the algorithm)



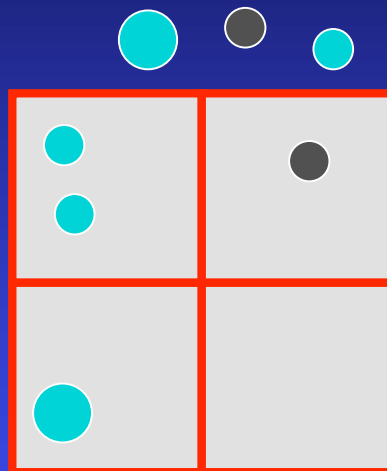
“Oh!
Division is
just
unmultipli-
cation!”

$$17 \overline{) 374} \quad 22 \quad \rightarrow \quad 374 \div 17 = 22$$

A kindergarten look at



Back to the very beginnings




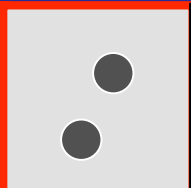
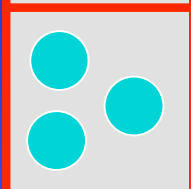
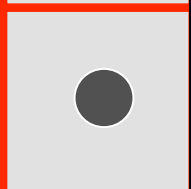
Picture a young child with a small pile of buttons.



Natural to sort.

We help children refine and extend what is already natural.

Back to the very beginnings





	blue	gray	
small			6
large			4
	7	3	10

Children can also summarize.

“Data” from the buttons.

Abstraction

If we substitute numbers for the original objects...

	blue	gray	
small			6
large			4
	7	3	10

4	2	6
3	1	4
7	3	10

Puzzling

Don't always start with the question!

7	6	13
5	3	8
12	9	21

Building the addition algorithm

Only multiples of 10 in yellow. Only less than 10 in blue.

20	5	25
30	8	38
50	13	63

Relating addition and subtraction

4	2	6
3	1	4
7	3	10

7	3	10
3	1	4
4	2	6

The subtraction algorithm

Only multiples of 10 in yellow. Only less than 10 in blue.

20	5	25
30	8	38
50	13	63

$$25 + 38 = 63$$

60	3	63
30	8	38
30	-5	25

$$63 - 38 = 25$$

The subtraction algorithm

Only multiples of 10 in yellow. Only less than 10 in blue.

20	5	25
30	8	38
50	13	63

$$25 + 38 = 63$$

50	13	63
30	8	38
20	5	25

$$63 - 38 = 25$$

The algebra connection: adding

4	2	6
3	1	4
7	3	10

$$\begin{array}{ccc} 4 + 2 = 6 \\ 3 + 1 = 4 \\ 7 + 3 = 10 \end{array}$$

The algebra connection: subtracting

7	3	10
3	1	4
4	2	6

$$\begin{array}{c} 7 + 3 = 10 \\ 3 + 1 = 4 \\ 4 + 2 = 6 \end{array}$$

The algebra connection: algebra!

$5x$	$3y$	23
$2x$	$3y$	11
$3x$	0	12

$$\begin{array}{l} 5x + 3y = 23 \\ 2x + 3y = 11 \\ 3x + 0 = 12 \\ x = 4 \end{array}$$

All from sorting buttons

$5x$	$3y$	23
$2x$	$3y$	11
$3x$	0	12

$$\begin{array}{l} 5x + 3y = 23 \\ 2x + 3y = 11 \\ 3x + 0 = 12 \\ x = 4 \end{array}$$

“Skill practice” in a second grade



V
i
d
e
o



Thank you!

- E. Paul Goldenberg
- <http://thinkmath.edc.org/>

Learning by doing, for teachers

- Professional development of 1.6M teachers
- To take advantage of time they already have, a curriculum must be...
 - ◆ Easy to start (well, as easy as it can be)
 - ◆ Appealing to *adult* minds (obviously to kids, too!)
 - ◆ Comforting (covering the bases, the tests)
 - ◆ Solid math, solid pedagogy (brain science, Montessori, Singapore, language)

Keeping things in one's head

