How the ideas and language of algebra K-5 set the stage for algebra 6–12

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EDC

2008

Before you scramble to take notes

http://thinkmath.edc.org

With downloadable PowerPoint

Ideas and approaches drawn from
Think Math!
a comprehensive K-5 program from
Houghton Mifflin Harcourt
School Publishers

Go to intersections
Go to Kindergarten sorting, CNPs
Go to "Guess my number" (mental buffer)
Go to marble bag trick
Go to 3rd grade detectives
Go to multiplication onions

Algebraic *language* & algebraic *thinking*

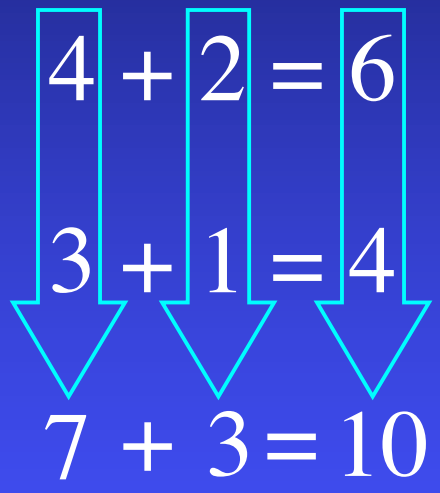
Algebraic thinking

Math could be spark curiosity!

Is there anything interesting about addition and subtraction sentences?

2nd grade

Write two number sentences…



To 2nd graders: see if you can find some that don't work!

How does this work?

Algebraic language

Math could be fascinating!

Is there anything less sexy than memorizing multiplication facts?

What *helps* people memorize?

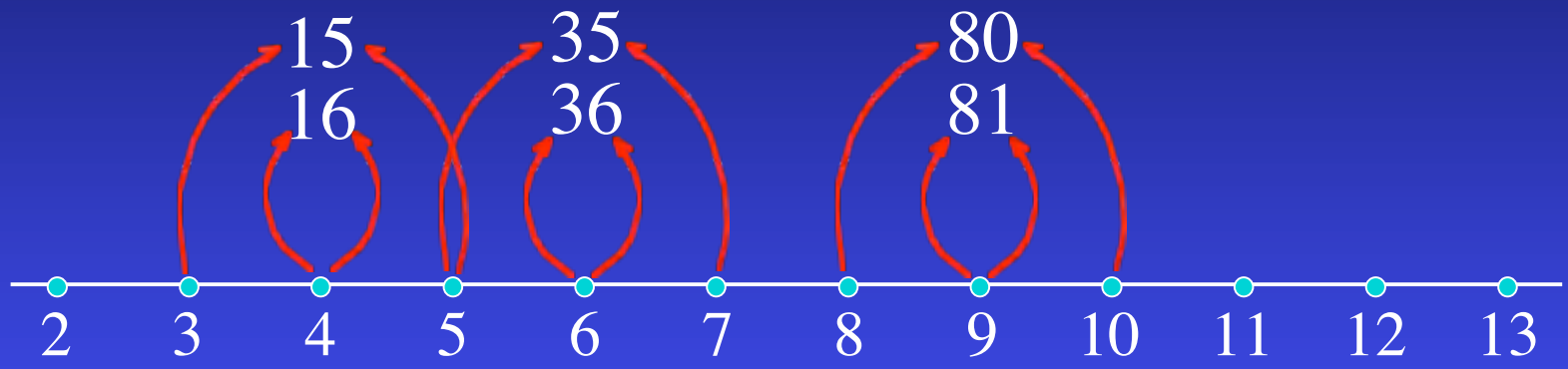
Something memorable!

4th grade

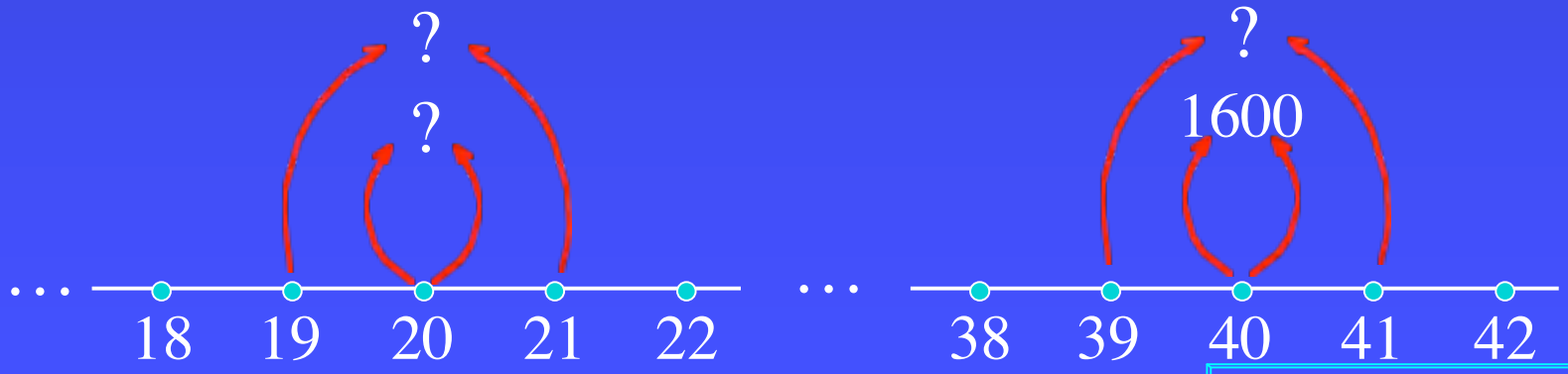
Go to "Mommy, give me…"
Go to visual way to understand
Go to index

Teaching without talking

Shhh… Students thinking!



Wow! Will it always work? Big numbers?



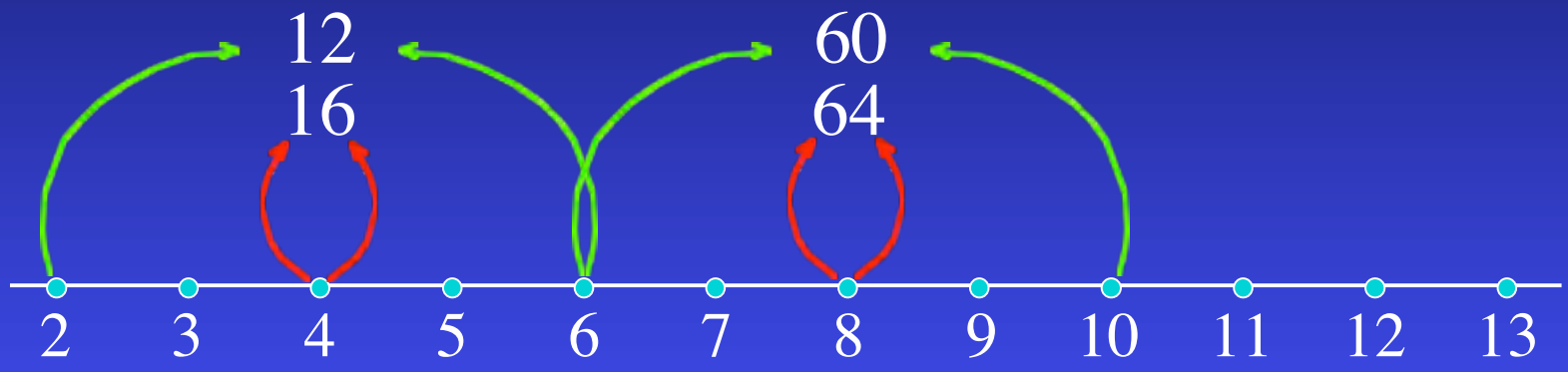
Go to visual way to understand

Take it a step further

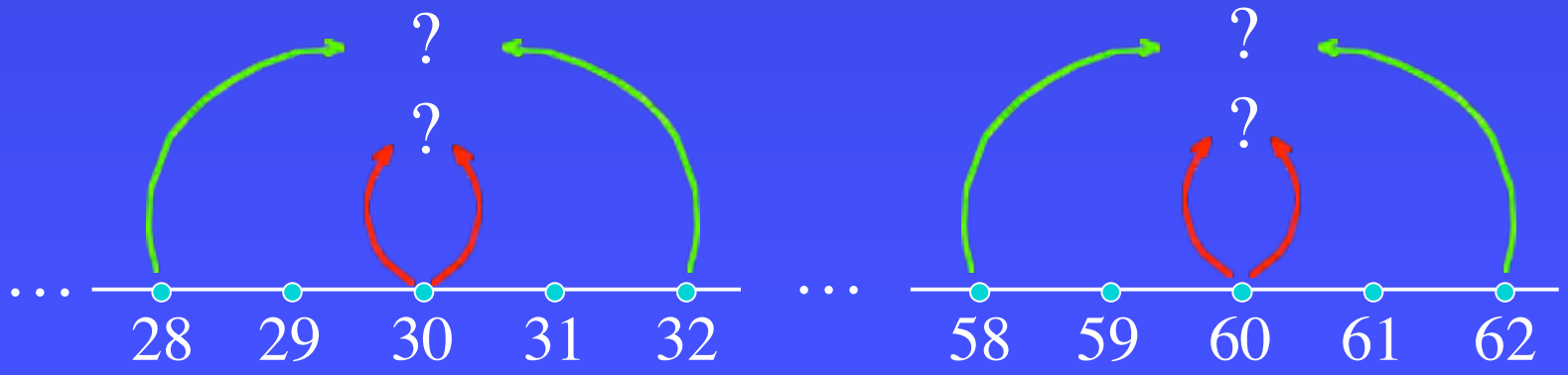
What about two steps out?

Teaching without talking

Shhh… Students thinking!



Again?! Always? Find some bigger examples.

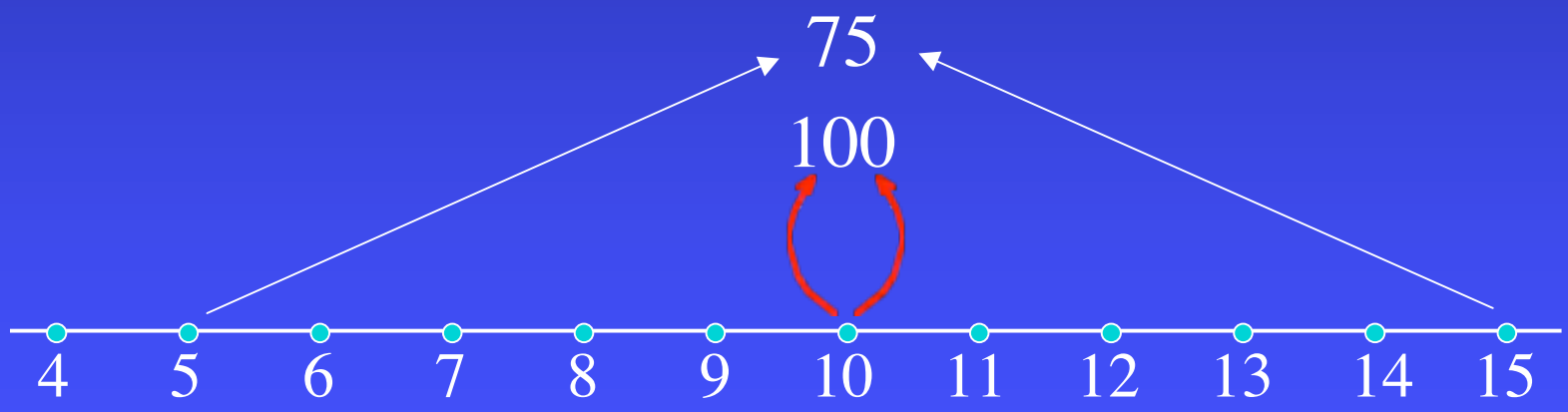


Take it even further

What about three steps out?

What about four?

What about five?



Take it even further

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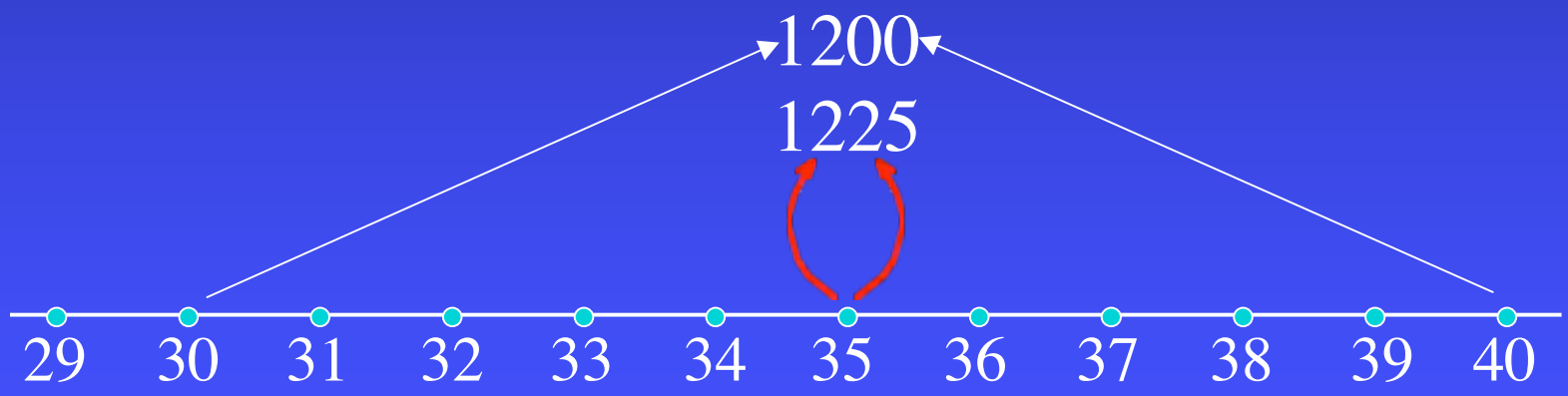


Take it even further

What about two steps out?

"Mommy! Give me a 2-digit number!"

- "OK, um, 53"
- "Hmm, well…

- …OK, I'll pick 47, and I can multiply those numbers faster than you can!"

To do…

$$\begin{array}{r} 53 \\ \times\ 47 \\ \hline \end{array}$$

I think…

50 × 50 (well, 5 × 5 and …)… 2500

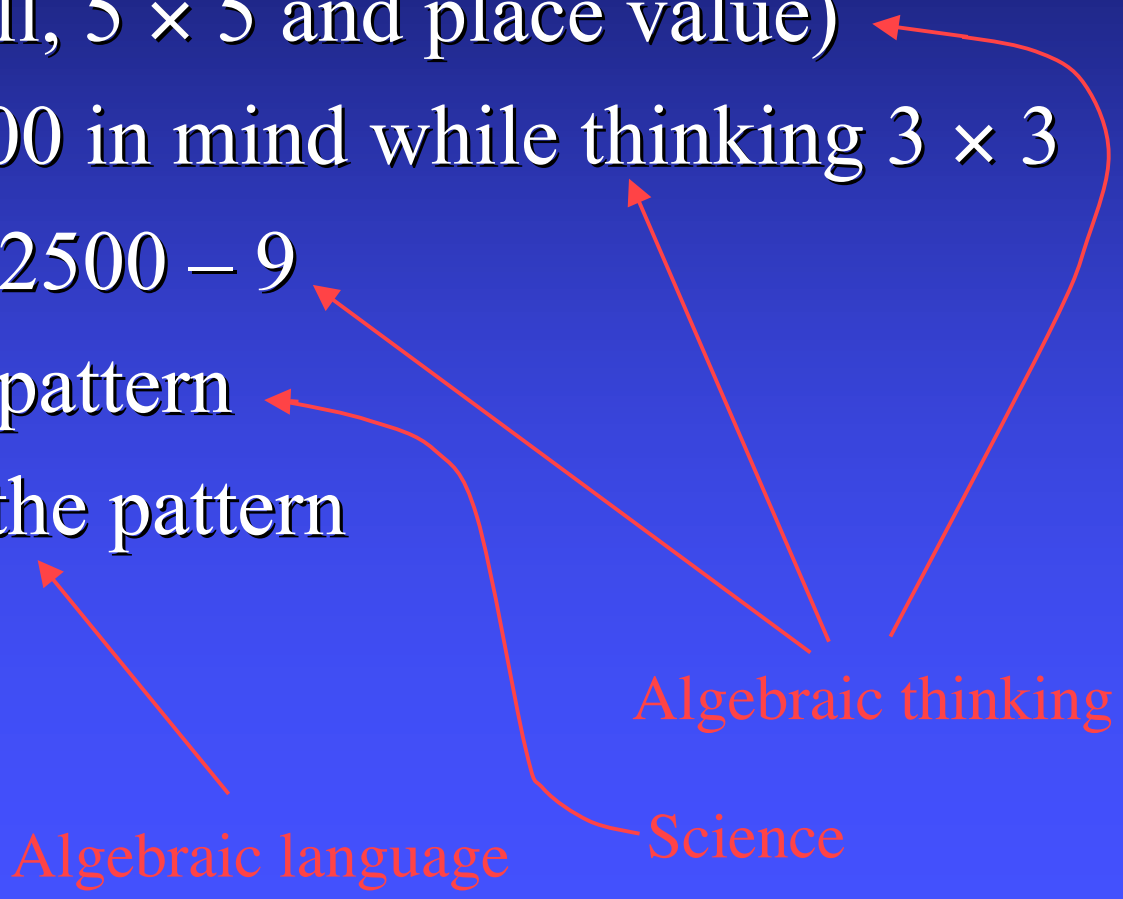
Minus 3 × 3 – 9

2491

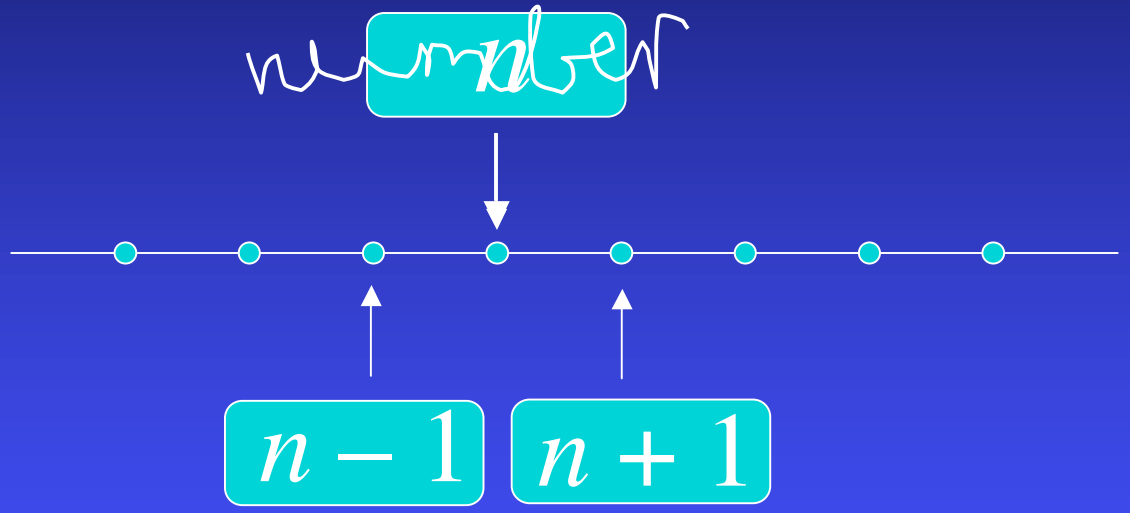
But *nobody cares* if kids can multiply 47×53 mentally!

What *do* we care about, then?

- 50 × 50 (well, 5 × 5 and place value)
- Keeping 2500 in mind while thinking 3 × 3
- Subtracting 2500 – 9
- Finding the pattern
- *Describing* the pattern

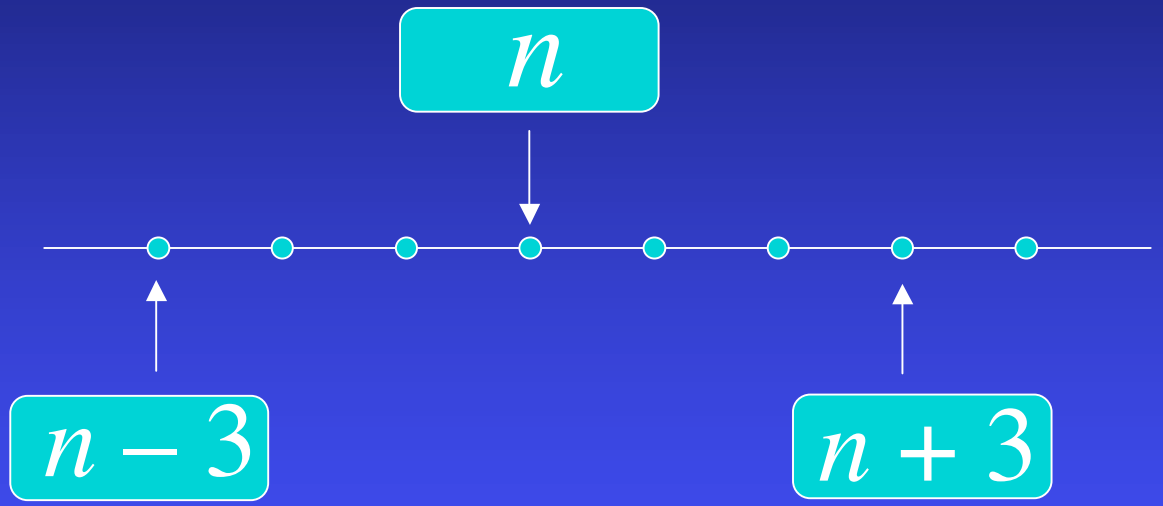


$$(7-1)\times(7+1)=7\times 7-1$$



$$(n-1)\times(n+1)=n\times n-1$$

$$(7-3)\times(7+3)=7\times7-9$$

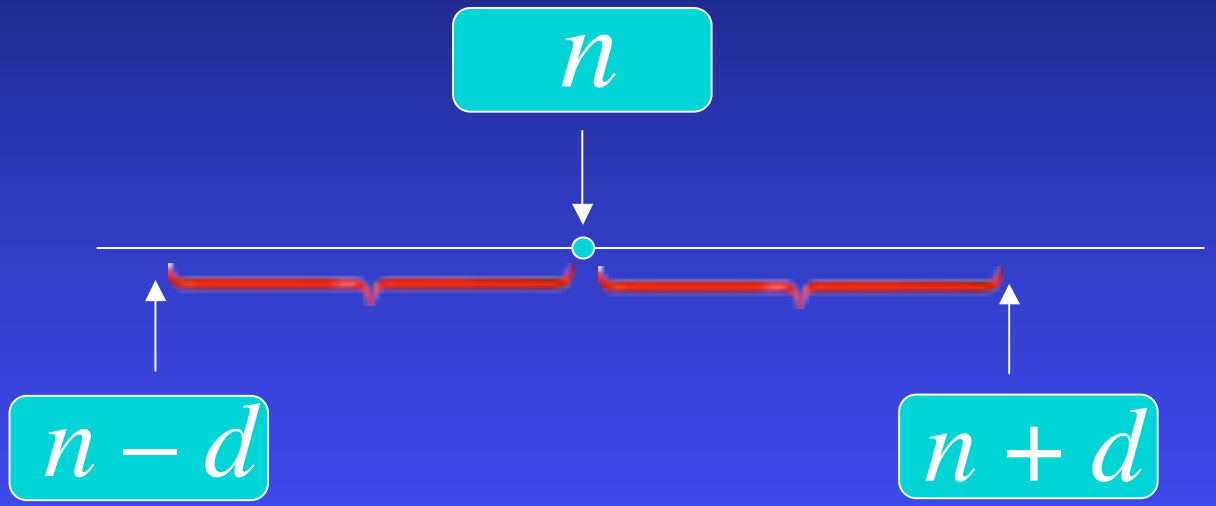


$$(n-3)\times(n+3)=n\times n-9$$

Make a table

Distance away	What to subtract
1	1
2	4
3	9
4	16
5	25
d	$d \times d$

$$(7-d)\times(7+d)=7\times 7-d\times d$$



$$(n-d)\times(n+d)=n\times n-d\times d$$

We also care about thinking!

- *Kids feel smart!*
  Why silent teaching?
- *Teachers feel smart!*
- *Practice.*
  Gives practice. Helps me memorize, because it's memorable*!*
- *Something new.*
  Foreshadows algebra. In fact, kids record it with *algebraic language!*

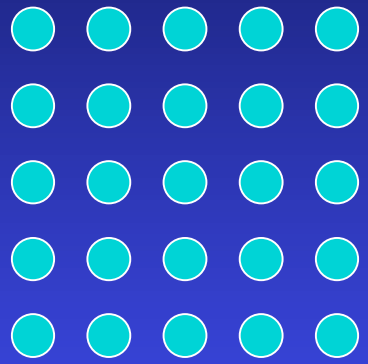
It matters!

- *And something to wonder about:*

How does it work?

One way to look at it

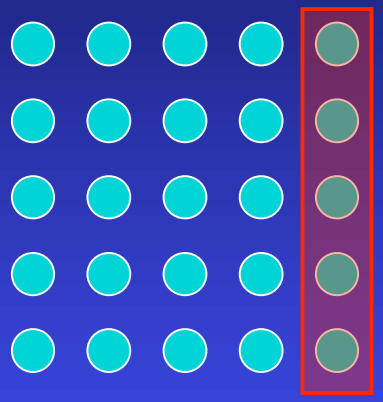
5×5



One way to look at it

Removing a column leaves

5×4

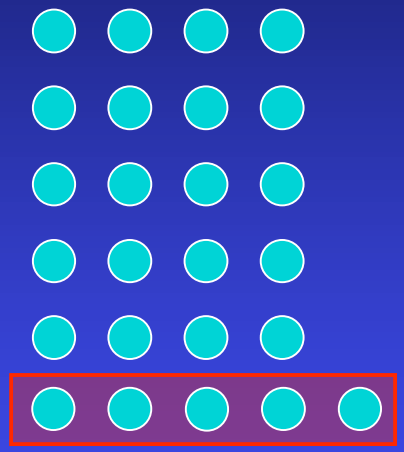


One way to look at it

Replacing as a row leaves

6×4

with one left over.



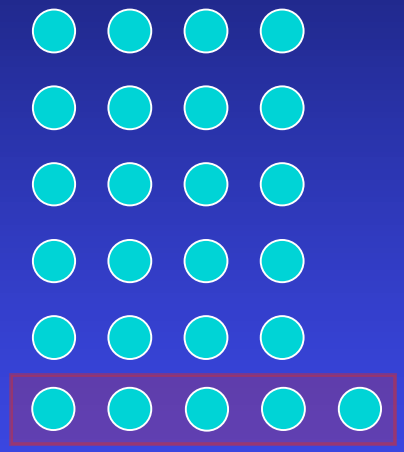
One way to look at it

Removing the leftover leaves

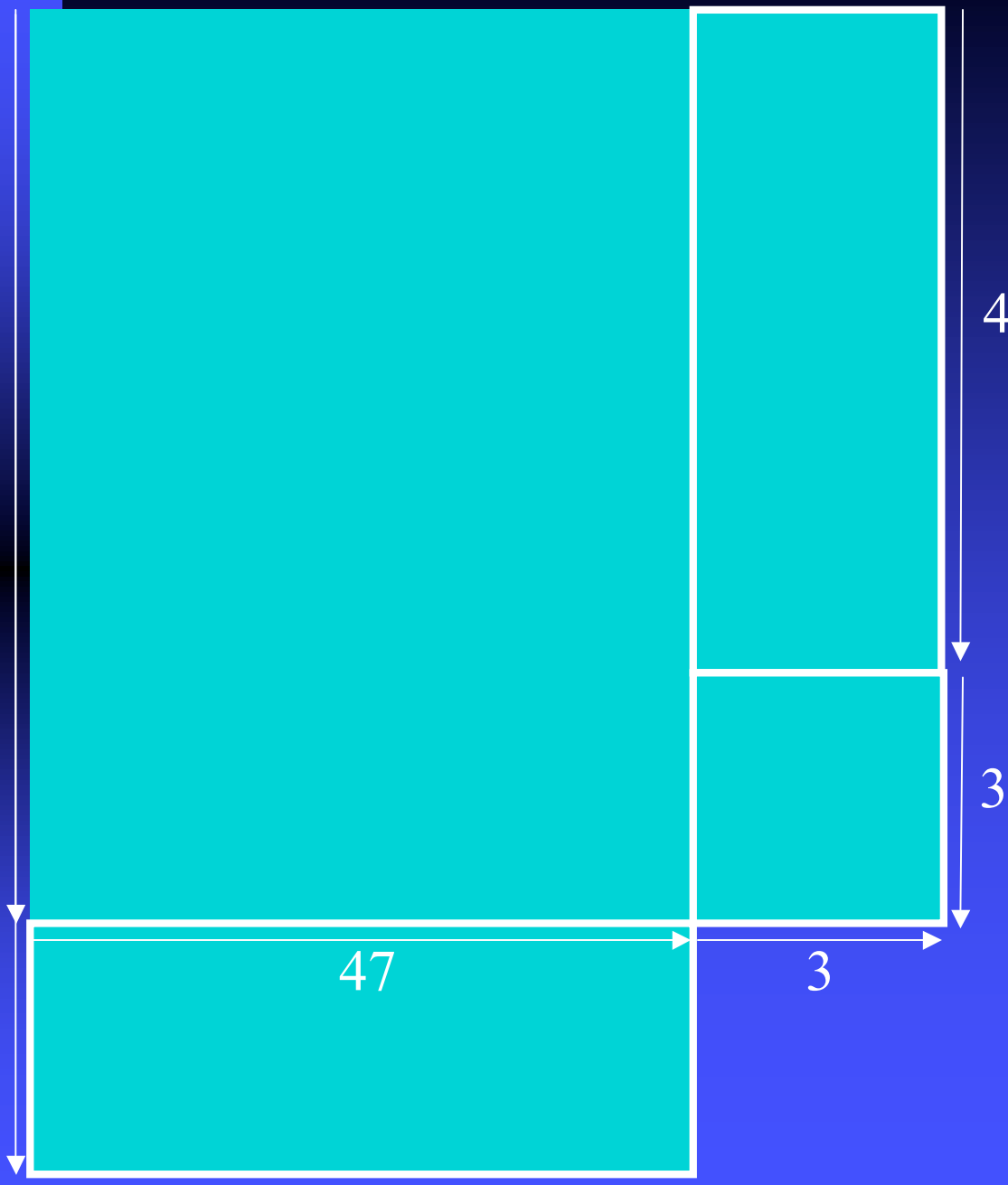
6×4

showing that it is one less than

5×5.



How does it work?



$$50 \times 50 - 3 \times 3$$
$$= 53 \times 47$$

An important propaganda break…

"Math talent" is *made*, not found

- *We all "know" that some people have…*

  musical ears,

  mathematical minds,

  a natural aptitude for languages….
- *Wrong! We gotta* stop believing it's all in the genes!
- *We are* equally *endowed with most of it*

Go to index

What could mathematics be like?

It could be surprising!

Surprise! You're good at algebra!

5th grade

Go to index

A number trick

- *Think* of a number.
- Add 3.
- Double the result.
- Subtract 4.
- Divide the result by 2.
- Subtract the number you first thought of.
- Your answer is 1!

How did it work?

- *Think* of a number.
- Add 3.
- Double the result.
- Subtract 4.
- Divide the result by 2.
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How did it work?

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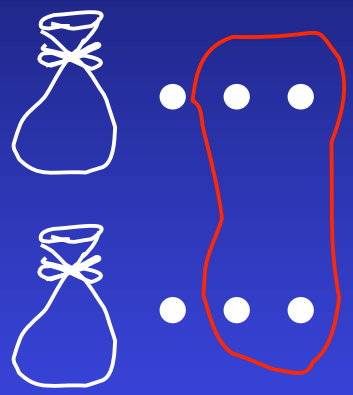

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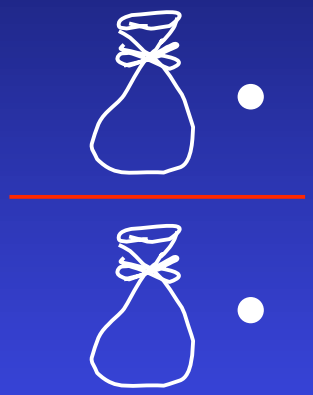

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Kids need to do it themselves…

Using notation: *following* steps

Words	Pictures	Dan	Cory	Sand	Chris
Think of a number.		5		y	
Double it.		10			
Add 6.		16			
Divide by 2. What did you get?		8	7	3	20

Using notation: *undoing* steps

Words	Pictures	Dan	Cory	Sand	Chris
Think of a number.		5	4	y	
Double it.		10	8		
Add 6.		16	14		
Divide by 2. What did you get?		8	7	3	20

Hard to undo using the words.

Much easier to undo using the notation.

Using notation: *simplifying* steps

Words	Pictures	Dan	Cory	Sand	Chris
Think of a number.		5	4	y	
Double it.		10			
Add 6.		16			
Divide by 2. What did you get?		8	7	3	20

Why a number trick? Why bags?

- *Computational practice, but* much *more*
- *Notation helps them* understand *the trick.*
- invent *new tricks.*
- undo *the trick.*
- *But most important, the idea that*

notation/representation is powerful!

Children are language learners…

- *They are pattern-finders, abstracters…*
- *…natural sponges for language in context.*

n	10	8	28	18	17			58	57
$n - 8$	2	0	20			3	4		

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3rd grade detectives!

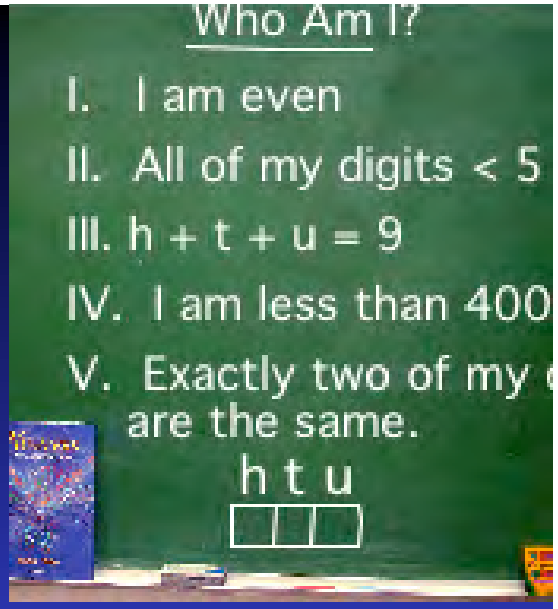
I. I am even.

II. All of my digits < 5

III. $h + t + u = 9$

IV. I am less than 400.

V. Exactly two of my digits are the same.



h	*t*	*u*
1	4	4

	t	*u*
	0	0
1	1	~~1~~
2	2	2
3	3	~~3~~
4	4	4
~~5~~	~~5~~	~~5~~
~~6~~	~~6~~	~~6~~
~~7~~	~~7~~	~~7~~
~~8~~	~~8~~	~~8~~
~~9~~	~~9~~	~~9~~

~~432~~
342
234
324
144
~~414~~

Representing *ideas* and *processes*

- *Bags and letters can represent numbers.*
- *We need also to represent…*
  - *ideas — multiplication*
  - *processes — the multiplication algorithm*

Representing multiplication, itself

Naming intersections, first grade

Put a red house at the intersection of
A street
and N avenue.

Where is the green house?

How do we go from the green house to the school?

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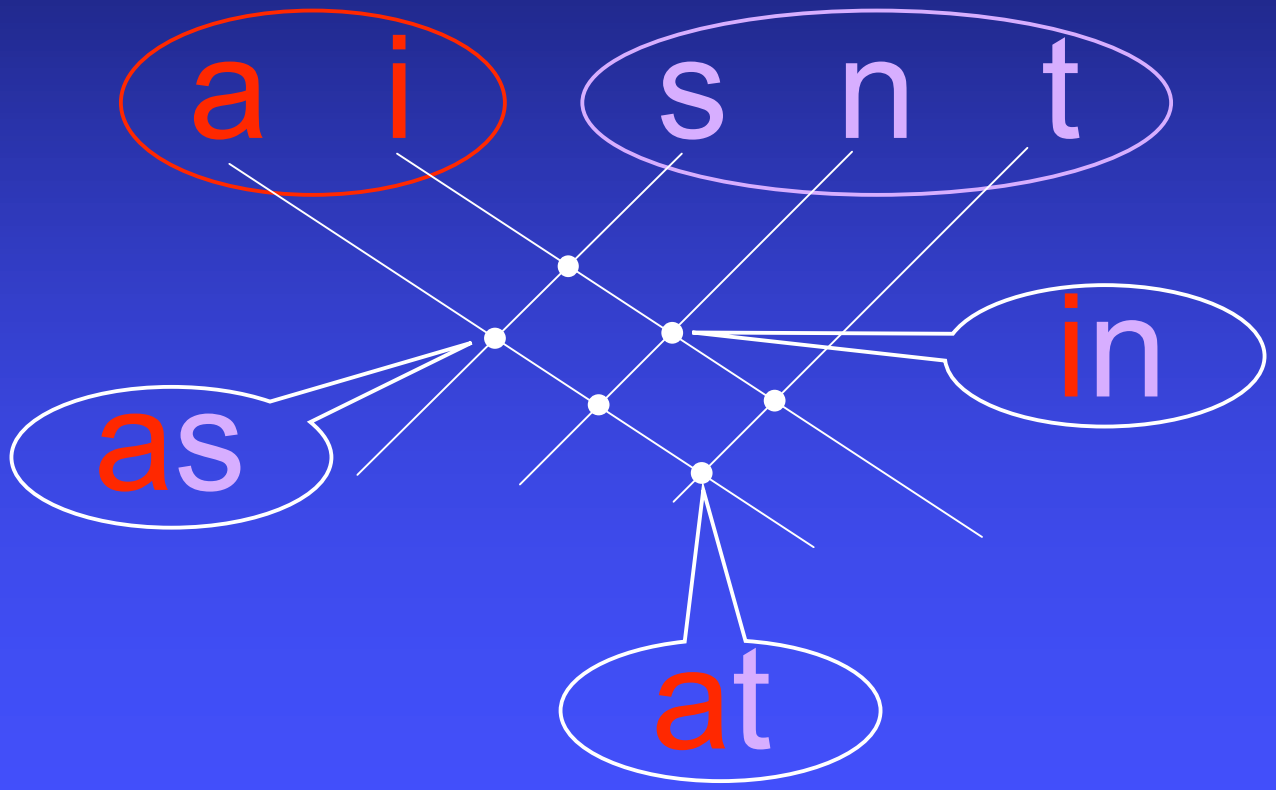
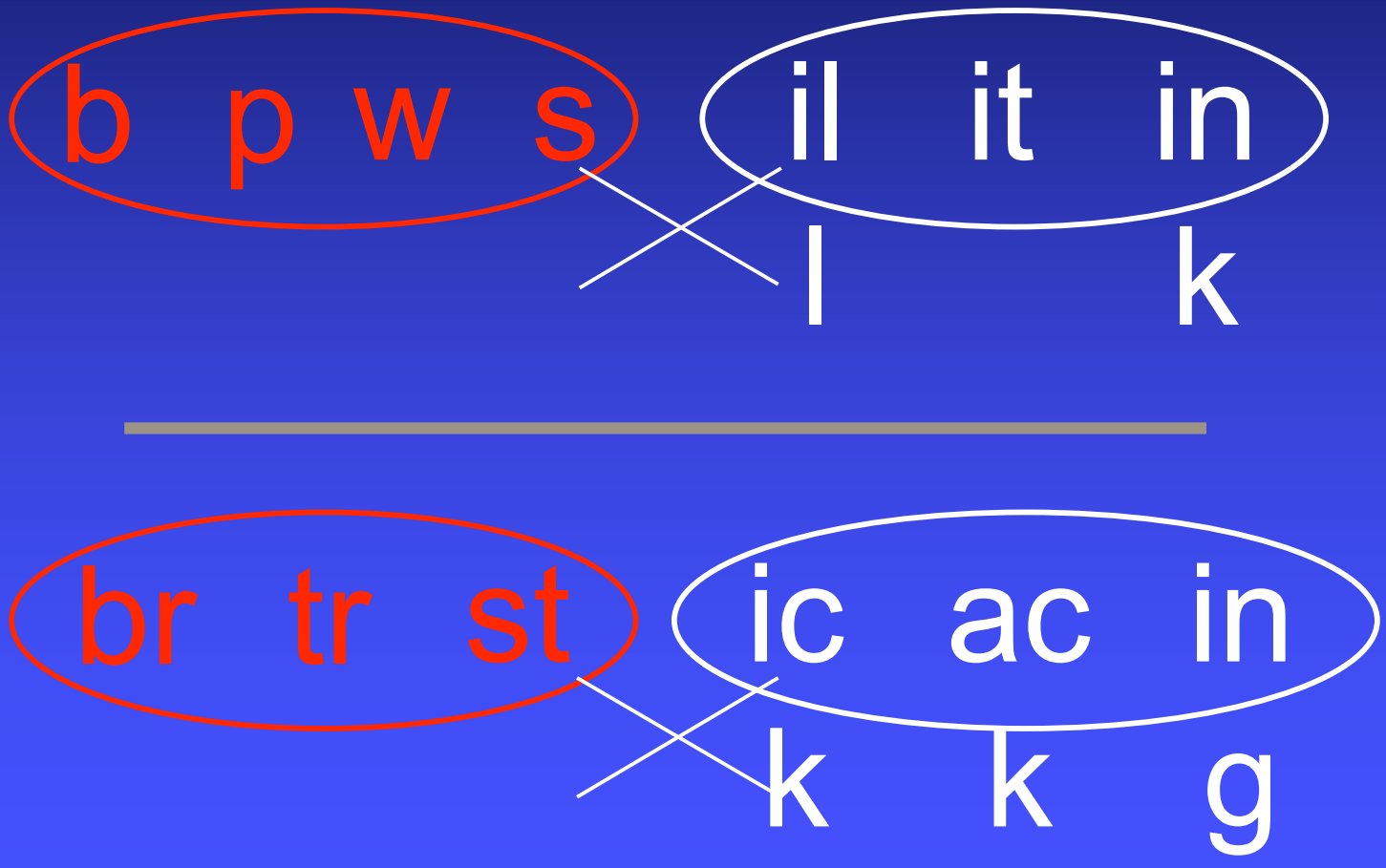
Combinatorics, beginning of 2nd

- How many two-letter words can you make, starting with a red letter and ending with a purple letter?

Multiplication, coordinates, phonics?



Multiplication, coordinates, phonics?

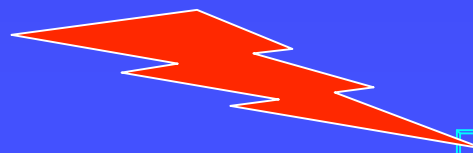


Similar questions, similar image

Four skirts and three shirts: how many outfits?

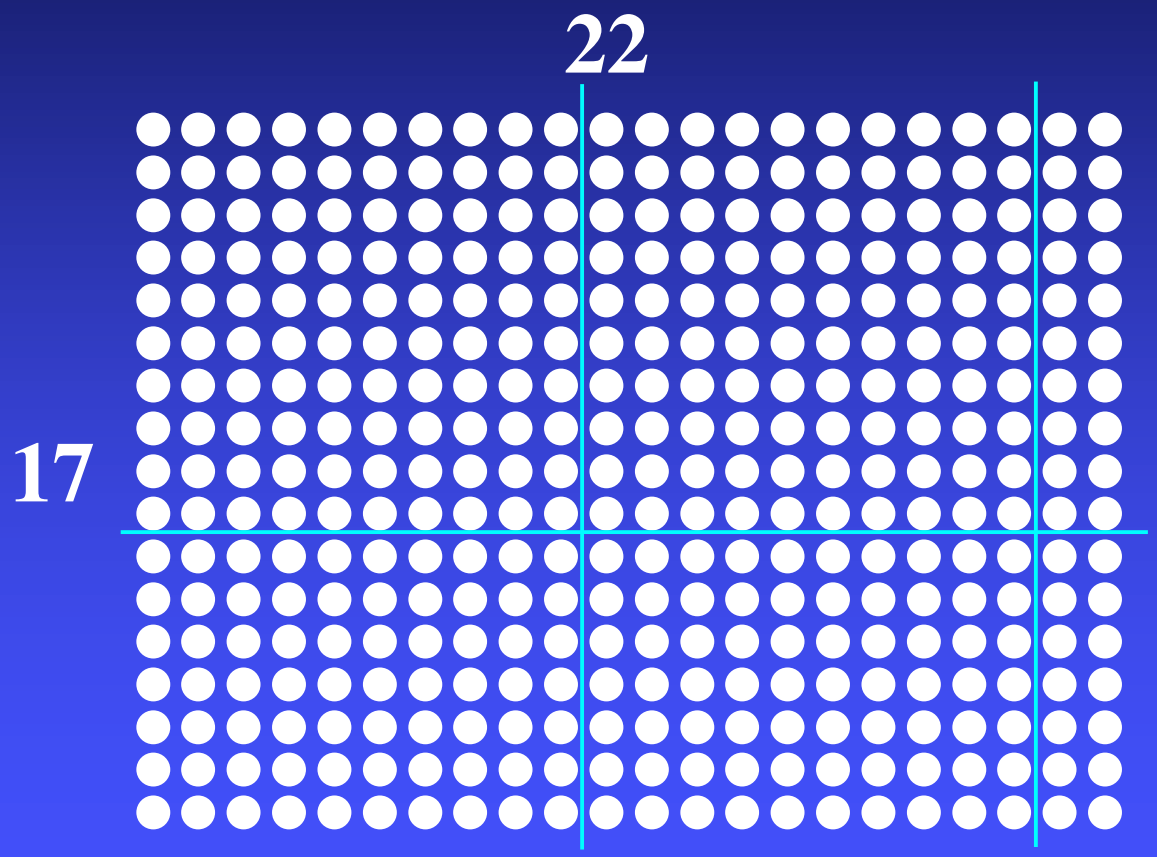
Five flavors of ice cream and four toppings: how many sundaes? (one scoop, one topping)

How many 2-block towers can you make from four differently-colored Lego blocks?

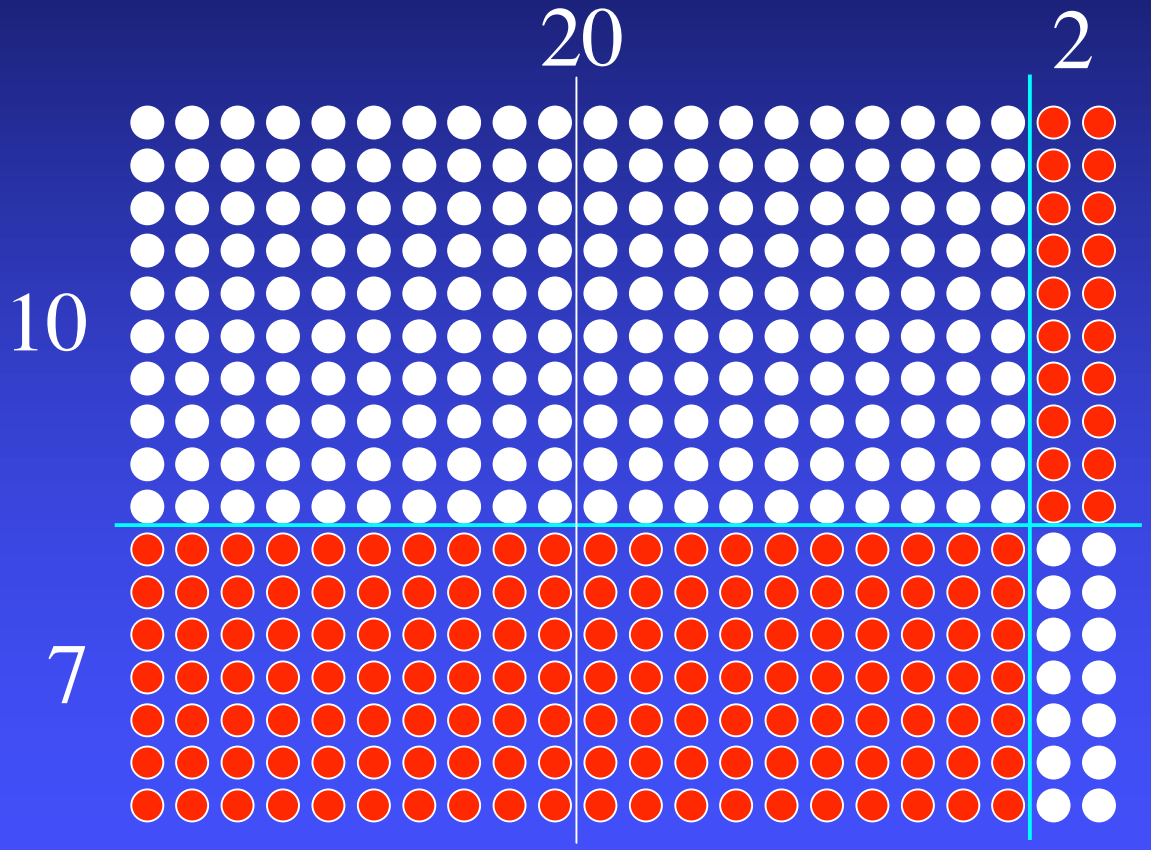


Go to Kindergarten sorting, CNPs | Go to index

Representing 22 × 17



Representing the *algorithm*



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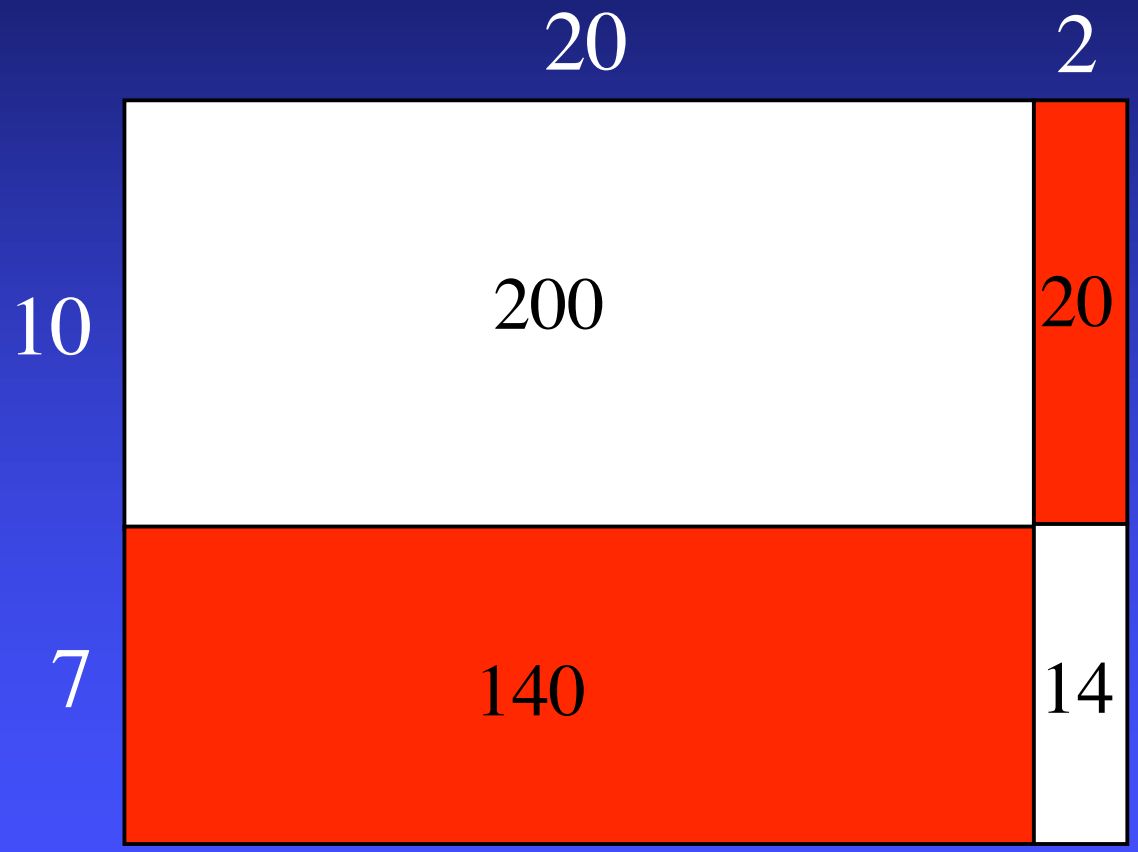

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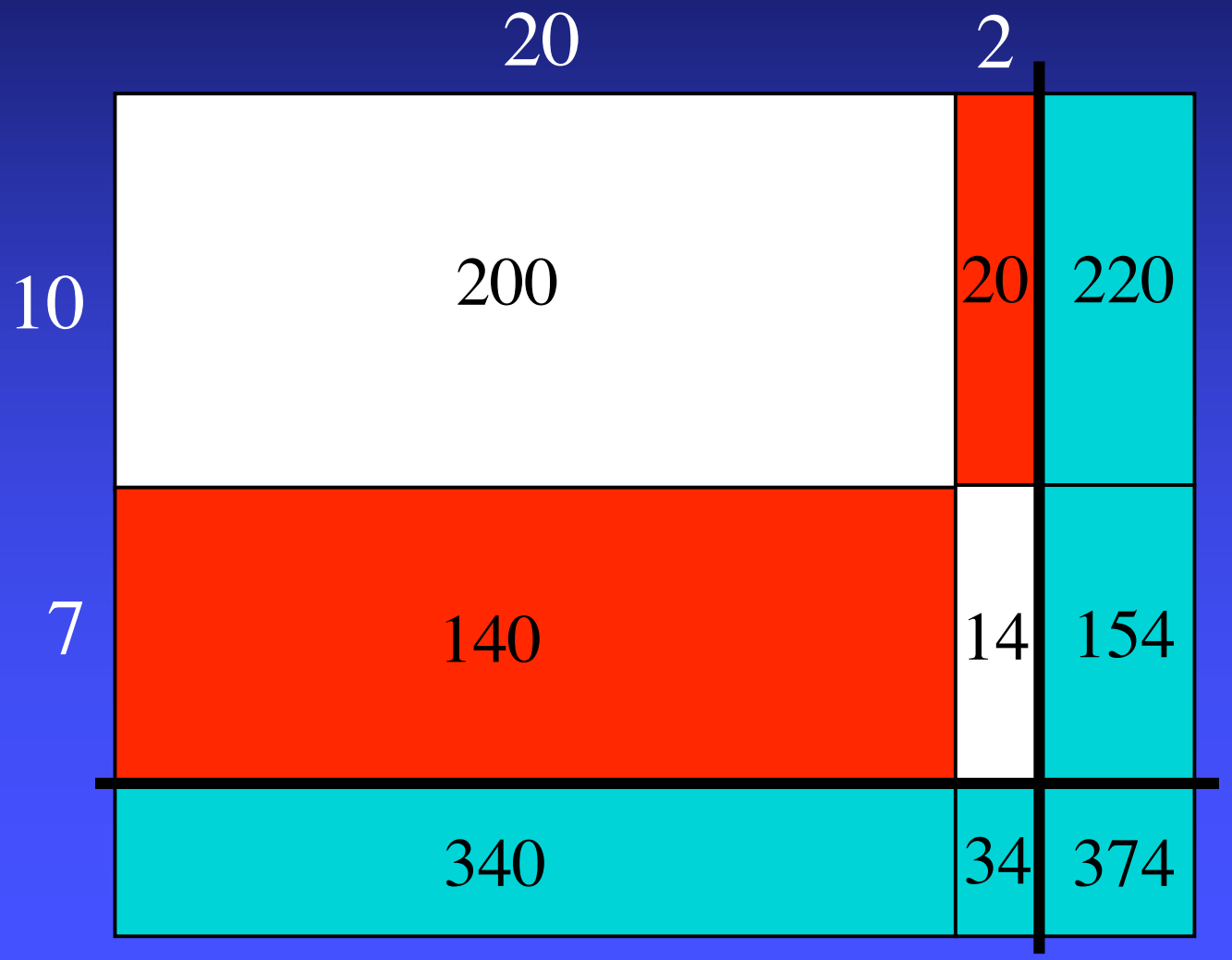

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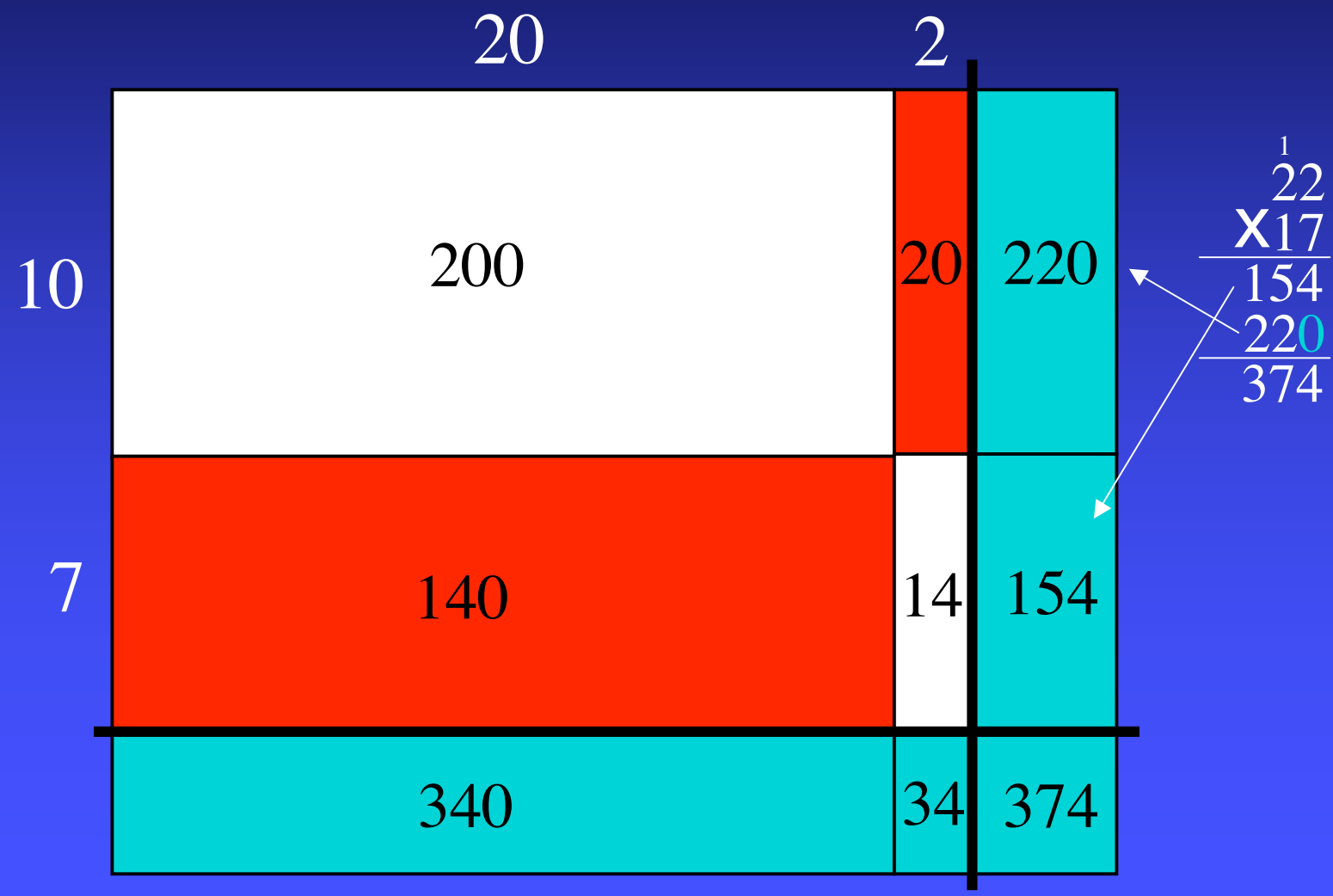

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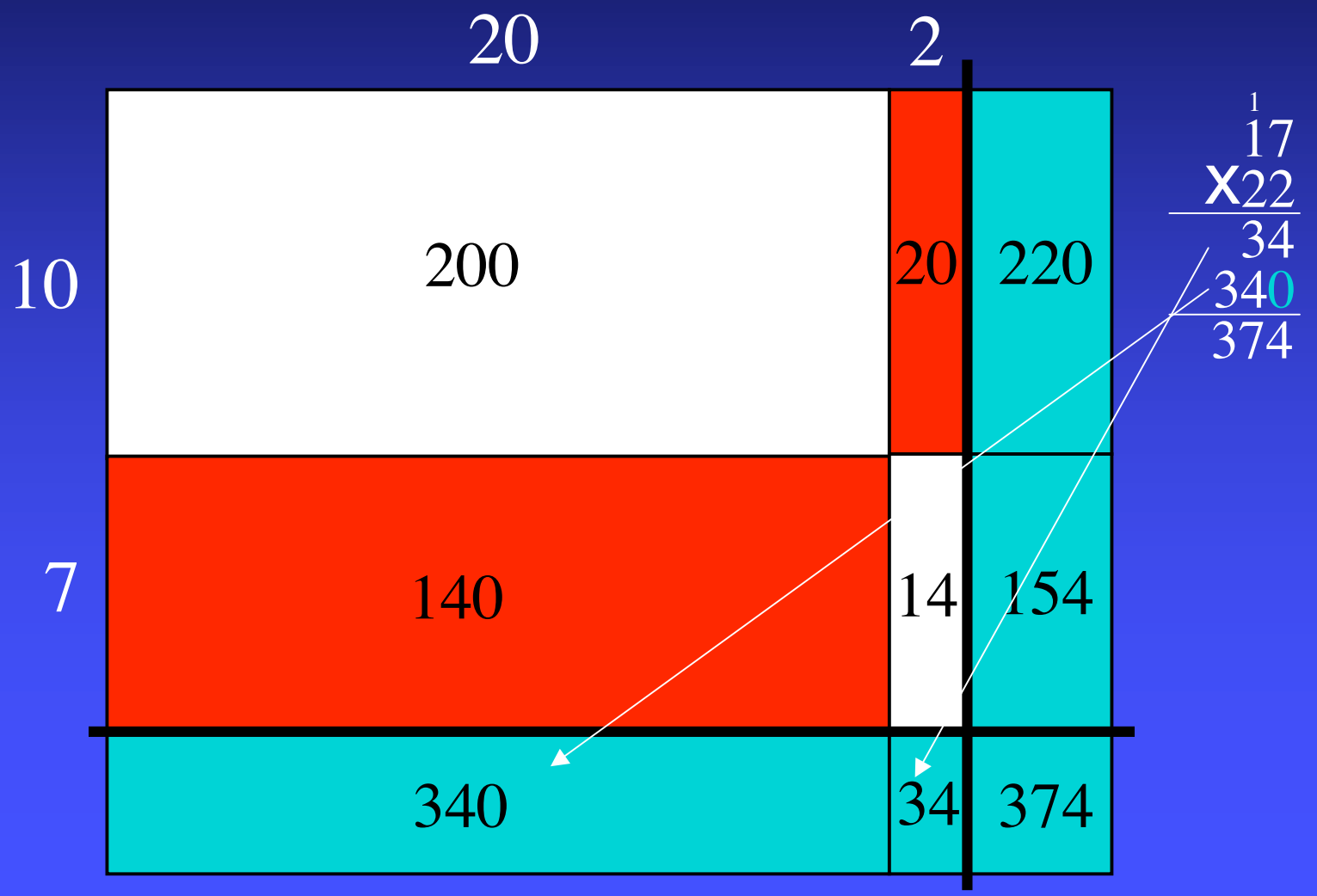
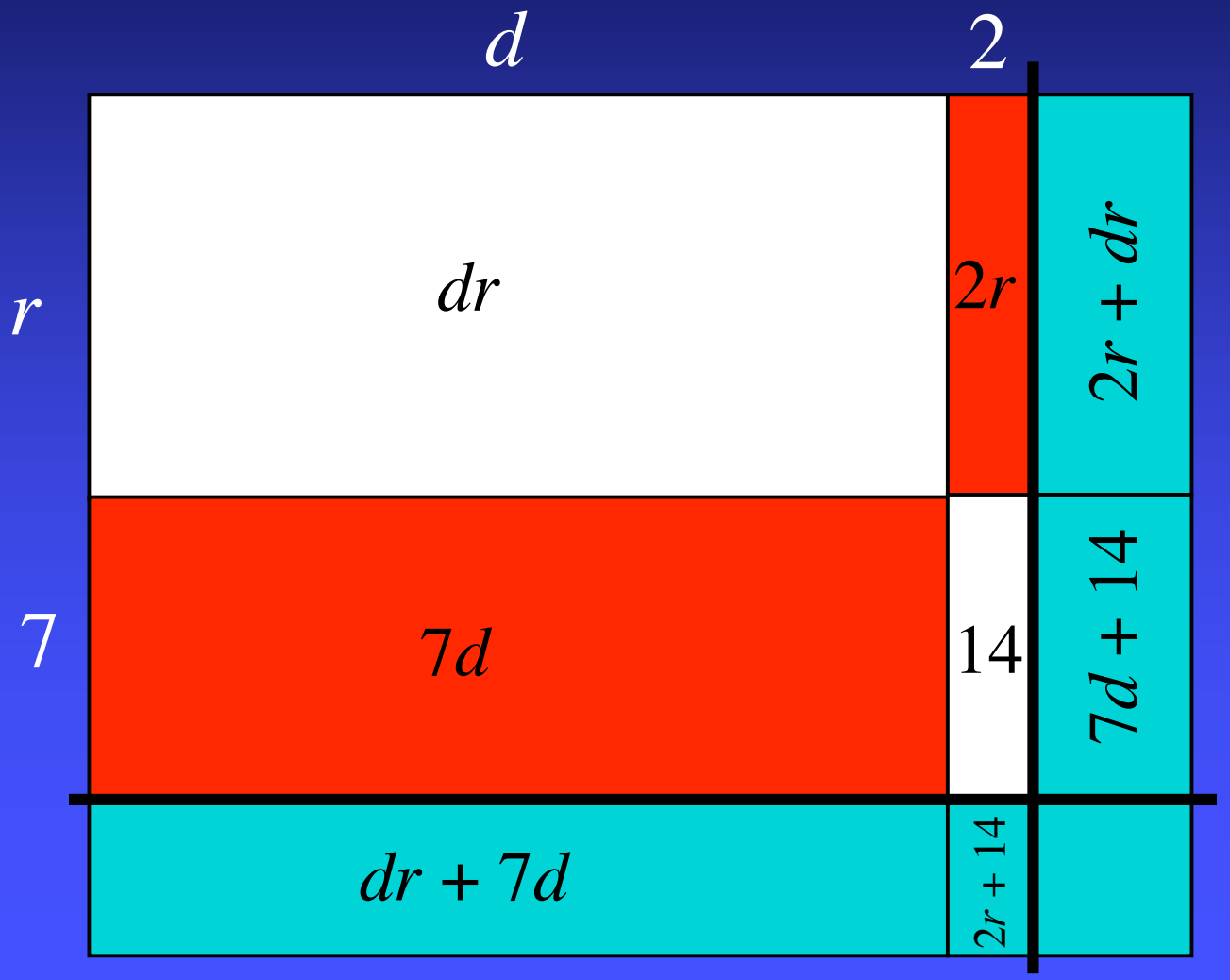


More generally, $(d+2)(r+7) =$



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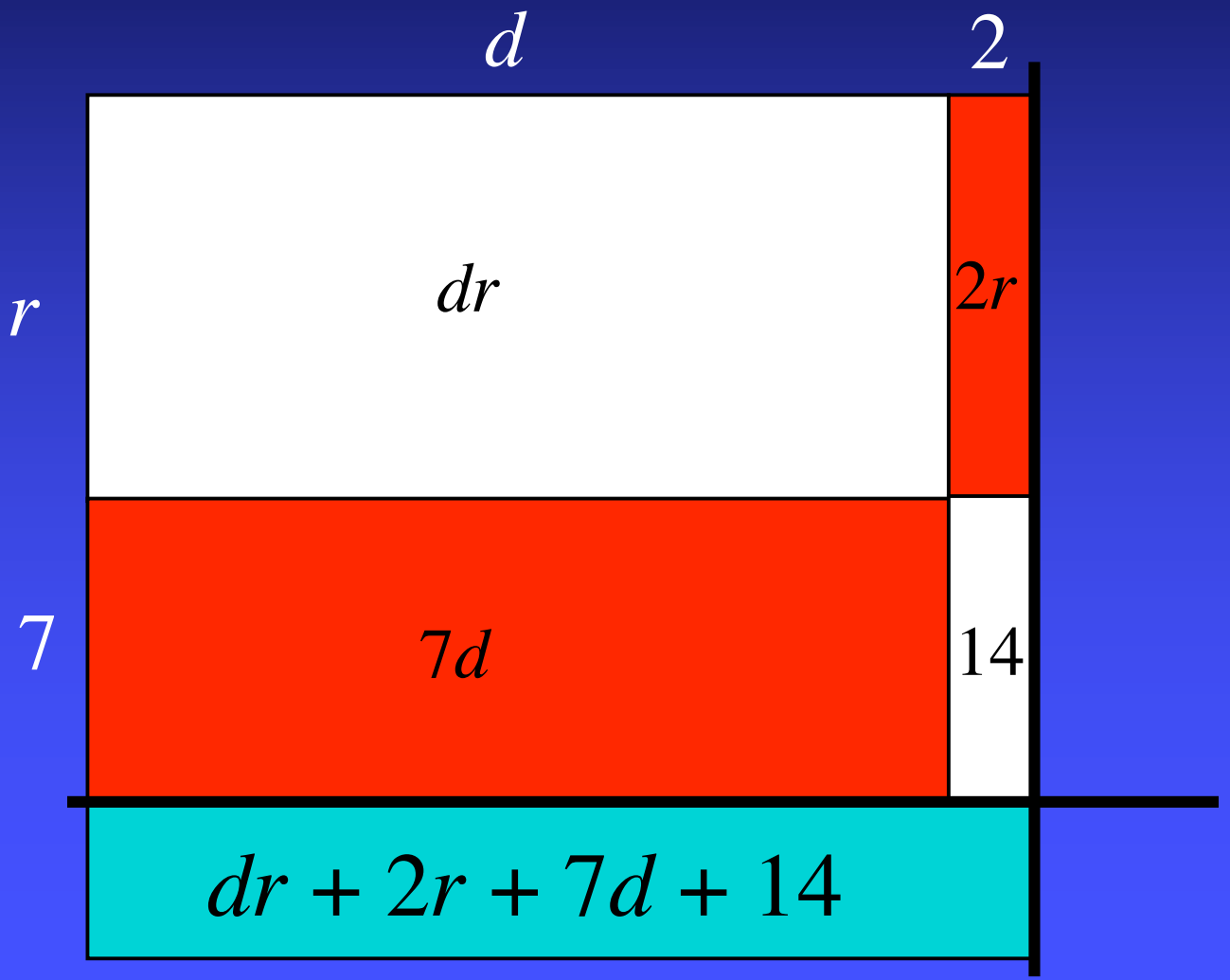
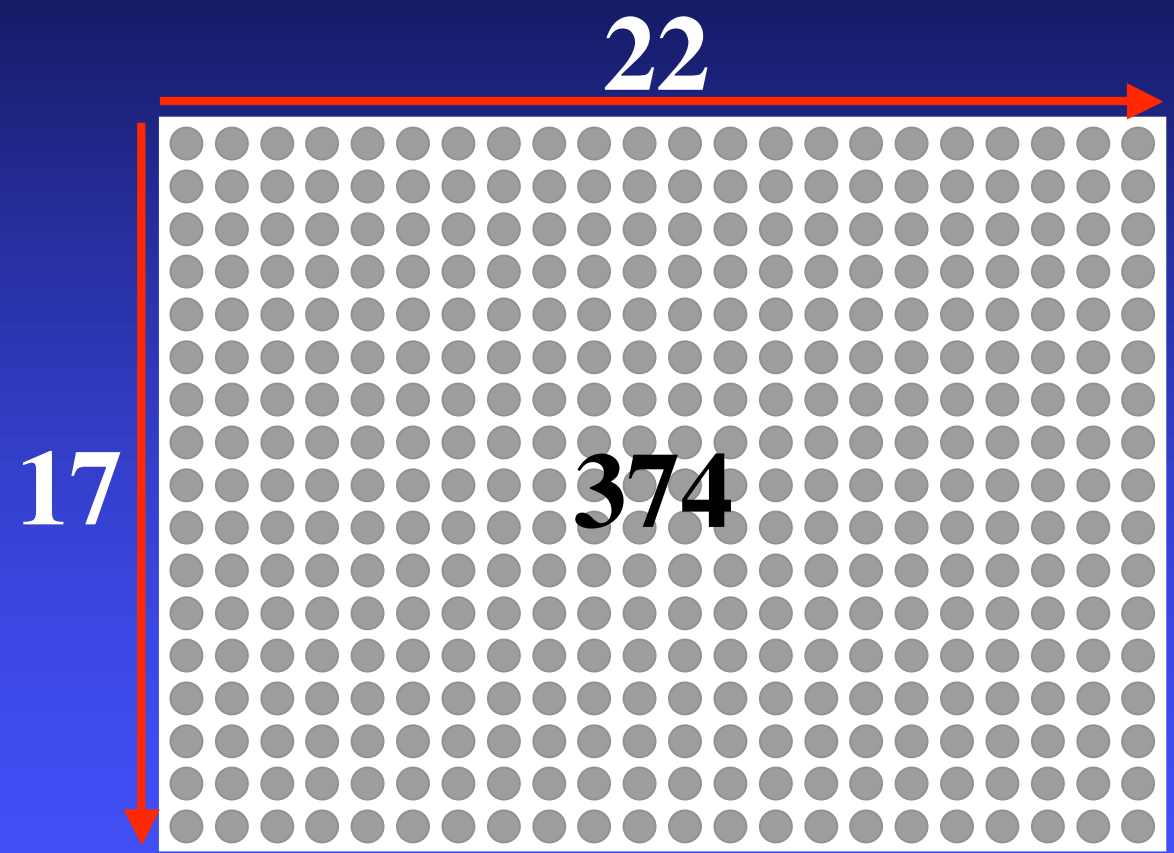
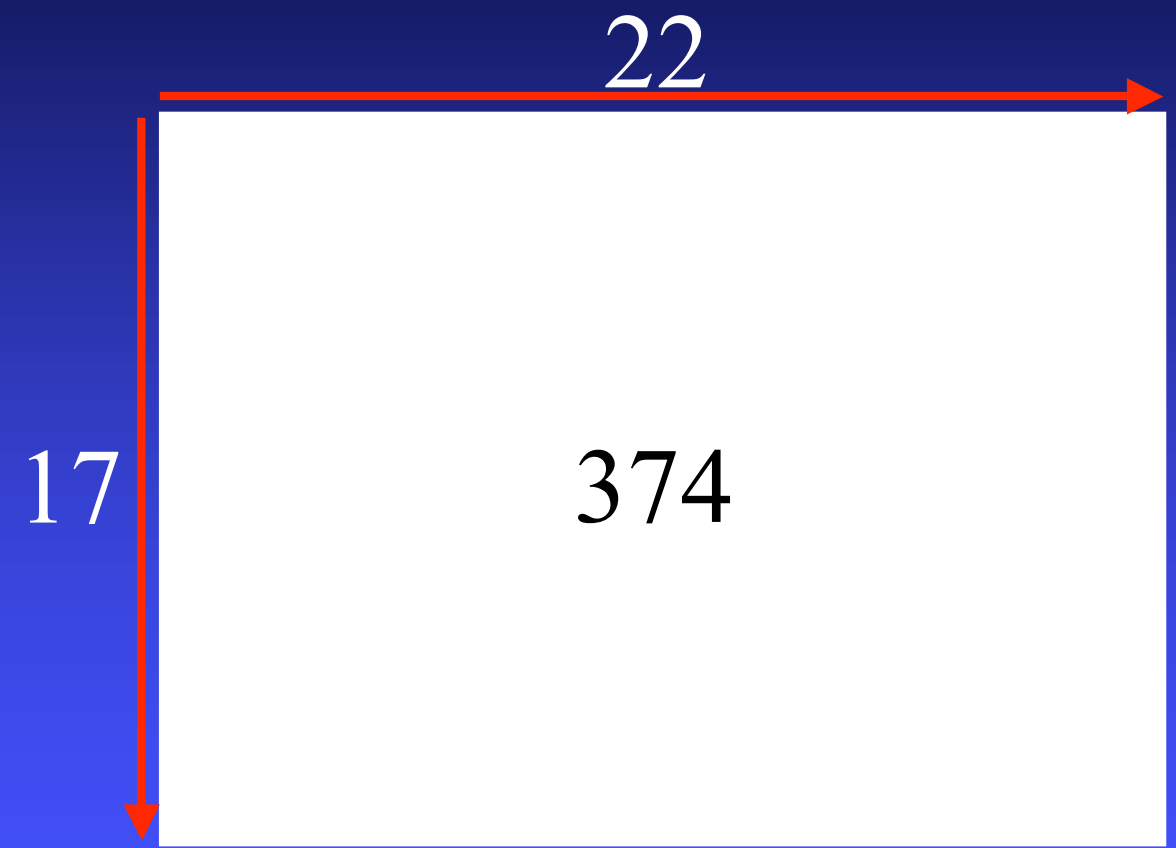


$$\begin{array}{r} 37 \\ \times\ 25 \\ \hline 600 \\ 140 \\ 150 \\ 35 \\ \hline 925 \end{array}$$

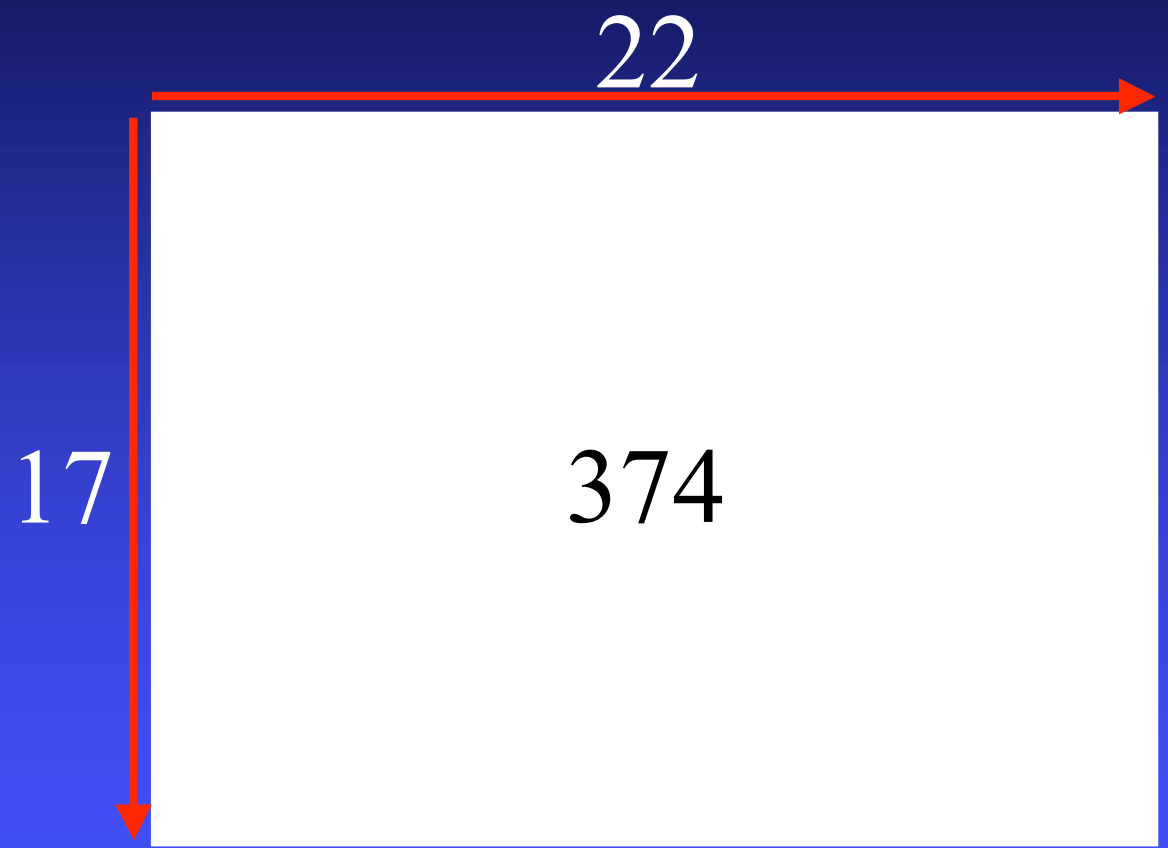


$22 \times 17 = 374$



$22 \times 17 = 374$

Representing division (not the algorithm)



"Oh! Division is just unmultipli-cation!"

$$\begin{array}{r} 22 \\ 17\overline{)374} \end{array}$$

$374 \div 17 = 22$

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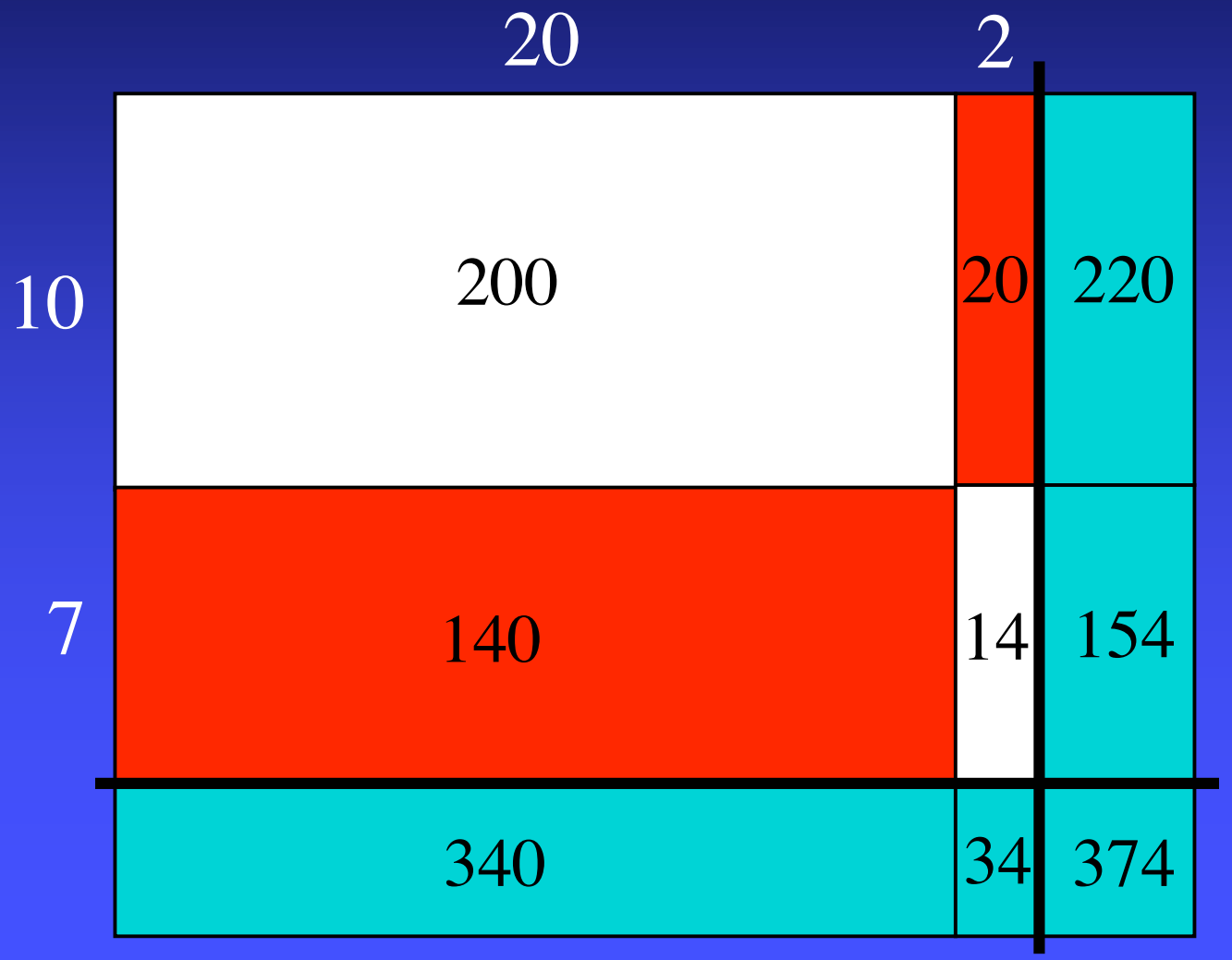
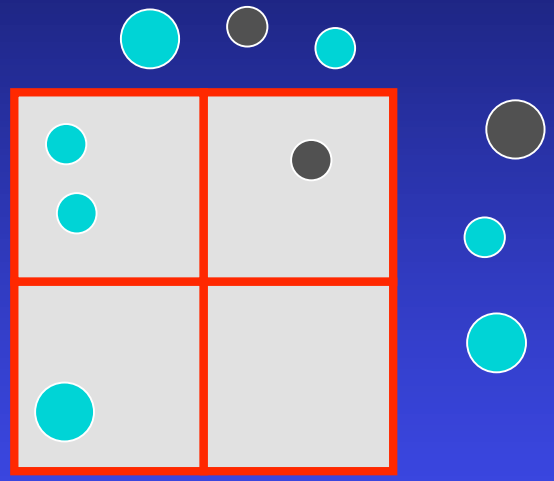
A kindergarten look at



Back to the very beginnings



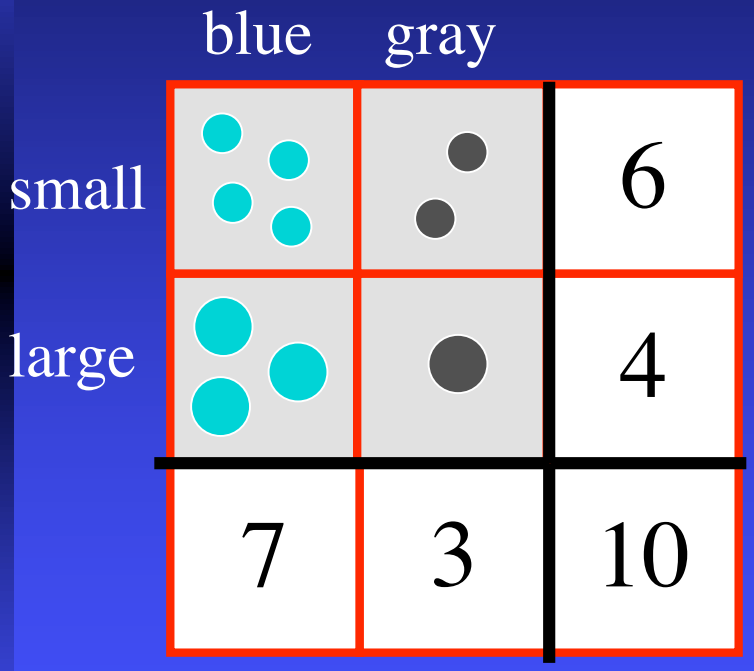
Picture a young child with a small pile of buttons.

Natural to sort.

We help children refine and extend what is already natural.

Go to number adding sentences | Go to Multiplication algorithm | Go to index

Back to the very beginnings

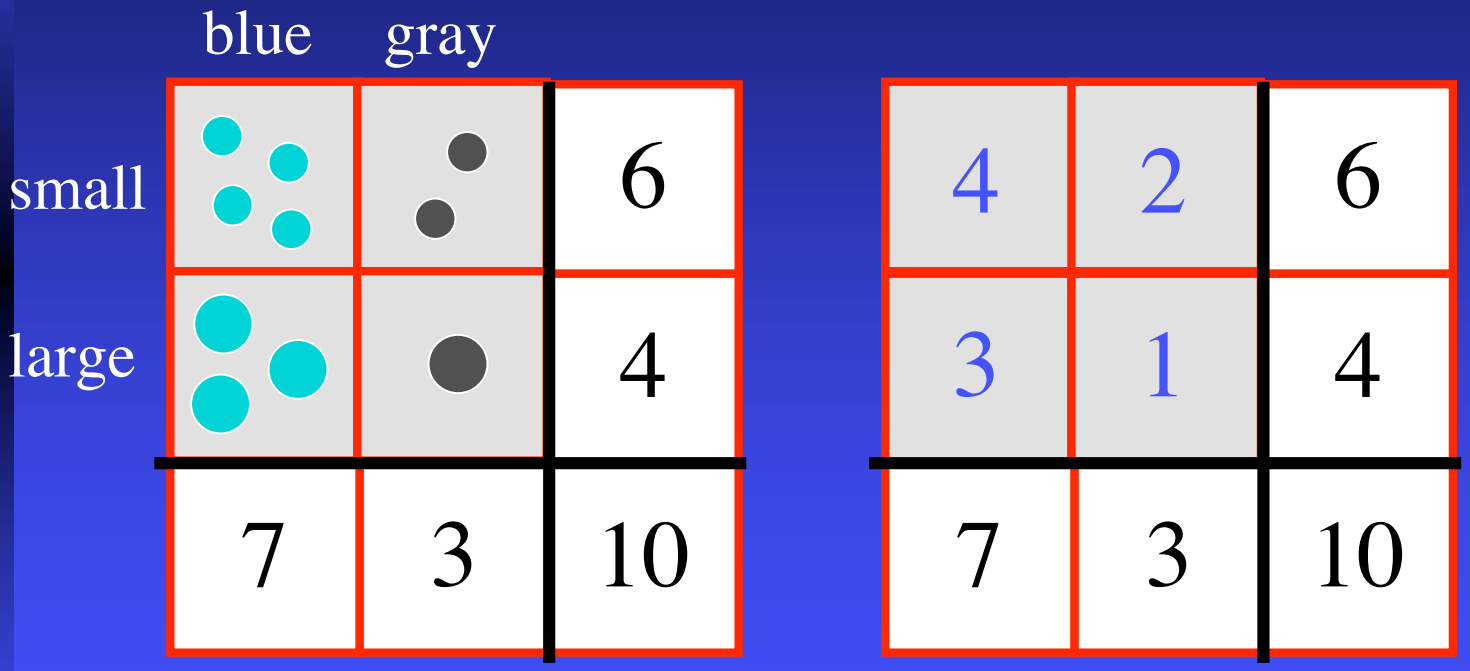


Children can also summarize.

"Data" from the buttons.

Abstraction

If we substitute numbers for the original objects…



A Cross Number Puzzle

Don’t always start with the question!

7	6	13
5	3	8
12	9	21

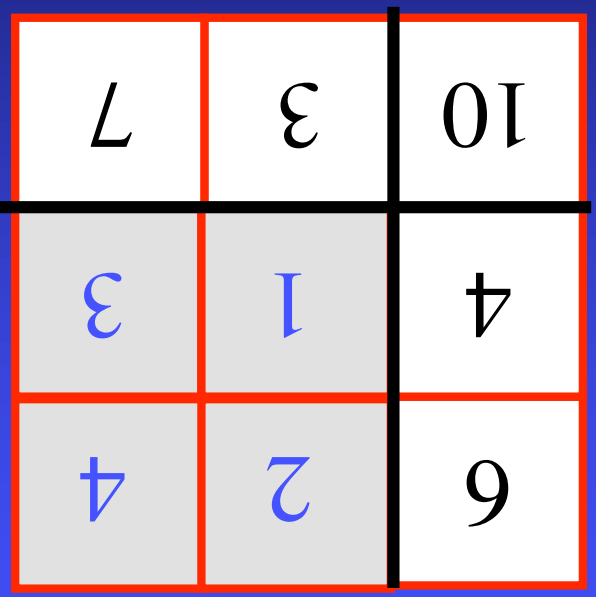
Building the addition algorithm

Only multiples of 10 in yellow. Only less than 10 in blue.

20	5	25
30	8	38
50	13	63

Relating addition and subtraction

4	2	6
3	1	4
7	3	10



The subtraction algorithm

Only multiples of 10 in yellow. Only less than 10 in blue.

20	5	25
30	8	38
50	13	63

25 + 38 = 63

60	3	63
30	8	38
30	-5	25

63 – 38 = 25

The subtraction algorithm

Only multiples of 10 in yellow. Only less than 10 in blue.

20	5	25
30	8	38
50	13	63

25 + 38 = 63

50	13	63
30	8	38
20	5	25

63 – 38 = 25

The algebra connection: adding

4	2	6
3	1	4
7	3	10

$$4 + 2 = 6$$

$$3 + 1 = 4$$

$$7 + 3 = 10$$

The algebra connection: subtracting

7	3	10
3	1	4
4	2	6

$7 + 3 = 10$

$3 + 1 = 4$

$4 + 2 = 6$

The algebra connection: algebra!

$5x$	$3y$	23
$2x$	$3y$	11
$3x$	0	12

$$5x + 3y = 23$$

$$2x + 3y = 11$$

$$3x + 0 = 12$$

$$x = 4$$

All from sorting buttons

$5x$	$3y$	23
$2x$	$3y$	11
$3x$	0	12

$5x + 3y = 23$

$2x + 3y = 11$

$3x + 0 = 12$

$x = 4$

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Thank you!

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Questions: Linguistics research in math?

Building the mental buffer? Counting what we don't see?

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Keeping things in one's head

“Skill practice” in a second grade

Video

fingers

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