Headline Stories

A Developmental Approach to Word Problems

excerpted from the Teacher's Guide front matter

To prepare children to be creative problem solvers and good deductive thinkers, *Think Math* regularly presents situations about which it possible to make many true statements. This unique feature, the "Headline Stories," provides open-ended situations in which specific mathematical ideas are embedded. It has three main purposes:

- to develop students' skills at deriving real-world meaning from mathematical statements and deriving mathematical meaning from real-world situations,
- to develop students' skills at using both natural language and mathematical language to describe ideas drawn from mathematics or the physical world, and
- to help students learn to solve word problems by understanding how they are built.

In different ways, both creative thinking and deductive thinking require people to look at a given situation and figure out what might follow from it: If I know *this* is true, then I can be sure *that* is also true. Like a newspaper headline, Headline Stories give clues about what might follow, but they leave out the details. Only rarely do they pose a specific problem. Envisioning a story and asking the right questions are left to the students. Part of the learning goal for students is to discover what mathematical questions are possible to ask, or answer, about a situation.

Brief but regular practice is the best way to develop skill at creating problems based on these headline situations. Setting aside a special time of day—5 to 10 minutes at morning meeting, snack, just after lunch, or some other regular time—for this one-a-day exercise helps ensure students get this practice.

An example Headline Story

Example 1: Grade 2

Liam earned \$1.00 on Monday, \$2.00 on Tuesday, \$4.00 on Wednesday, \$8.00 on Thursday, …

Sample responses: Each day he earned more than the day before. By Thursday, Liam earned \$15. 1 + 2 + 4 + 8 = 15. I think he'll earn \$16 on Friday. If this pattern keeps up, Liam is going to be quite rich! How much did he earn so far? If the pattern keeps up, how much will he earn next Monday? I don't think the pattern can continue.

M	T	W	T	F	S	S	M
\$1.00	\$2.00	\$4.00	\$8.00	\$16.00	\$32.00	\$64.00	\$128.00

Presenting this Headline Story

At the beginning of each year, there will be children who have never encountered this kind of problem before. Even those who are familiar with Headline Stories from earlier grades may need a reminder of what to do. The first challenge that students face in this example is that there doesn't seem to be a problem! This is just a statement; no question has been asked.

To present a Headline Story, you might read it to the class or (if appropriate) write it on the board for students to read themselves. Ask the students to think about the situation it presents and to share their thoughts. You may need to ask specific questions:

- What can you figure out from this information? What can you figure out about the situation?
- What questions can you ask?
- What problems can you pose?
- What additional information do you need?
- What information do you not need?

By themselves, these questions are quite vague, but you will see shortly how they acquire very specific meaning.

A variety of acceptable responses

With each headline story, the teacher guide presents some sample responses, kinds of statements that students might make that represent good mathematical responses to the headline. Of course, students say other things, too, some of which are as good as the sample responses given above and some of which are not. “I get an allowance” seems like a pretty sensible response to “What can you say?” in this situation, but it is not nearly as relevant, mathematically, as any of the other responses shown above. At first, you may need to accept relevant but not-very-mathematical ideas to help children feel free to express their thinking at all, but your goal is to sharpen, extend, and focus those ideas, helping your children develop (and recognize) mathematical ways of looking at the situations. This takes time.

In some cases, students might think of a new question to ask. Asking “How much did he earn so far?” is like inventing a word problem; the inference “by Thursday, Liam earned $15” is more like answering a word problem. Both are important.

The sample responses for this story included a table. Learning to read and write tables is an important general skill, and the underlying skills—gaining information from the position of some item in two dimensions (horizontal and vertical position on a grid, or row and column in an array)—are often used in mathematics. Sometimes, to let children know that a table is an acceptable kind of response, a Headline Story might explicitly request one. More often, the form of the response is left to the student.

As you look at the examples of good mathematical responses that we give with each Headline Story, notice that not all include numbers. In general, good responses

- describe some pattern implied by the information given (“It looks like he doubles each day”),
- describe a result that can be derived logically (“By Thursday, he earned $15”),
- pose a question or new problem around it (“How much will he earn next Monday?” or “Could this pattern continue?”),
- clarify or extend the situation (such as the table), or
- make a prediction that seems relevant and likely (“I think he’ll earn $16 on Friday”).

Try recording the most mathematically relevant responses so the class can see them (even if they are not yet reading well). In doing so, you can call special attention to them while accepting a wide range of other responses.

When you think your students are ready, you may even directly ask questions such as:

- Can you think of a good problem to pose?
- Can you make a prediction?

- What can you figure out from this situation?

Another example Headline Story

Example 2: Grade 3

In my hand I have 9¢.
Sample responses: Can it be shared evenly among three people? How can it be made? That's 2¢ more than I had yesterday. I used to have a dime, but I spent a penny.

This example seems to provide too little information to do anything with it, but as the sample responses show, even such a meager start can lead to good ideas and questions. Developing students' sense for word problems involves getting them to see the variety of questions that can be asked about a given situation. One creative strategy in situations like this is to add information (for example, "If I had one more penny…") or invent problems ("What coins might I have?" or "Can it be shared among two people?" or "If I bought gum for 5¢…").

Another approach is to see what can be logically deduced from the information given. To help children realize what they can deduce from so little information, you might introduce this story by concealing some coins in your hand, saying "In my hand I have 9¢," and asking "Can you be sure what coins I am holding?" (No; you could have 9 pennies or you could have a nickel and 4 pennies).

You might then ask, "Can you say *anything* for sure about the coins I'm holding?" The answer to this second question is "yes," but if your children have not played this kind of game before, you might help them get started by asking some more leading questions, like these:

- Can you say for sure whether or not I have any dimes? (Yes. If you have 9¢, then you can't have any dimes.)
- Can you say for sure that I have at least 5 coins? (Yes. A nickel and four pennies is the least number of coins you could have.)
- Can you say for sure that I do not have more than 9 coins? (Yes. If you had more than 9 coins, then even if they were pennies you'd have more than 9¢.) Note that this game of *If… then…* encourages precise expression! Children often have a hard time saying what they mean, and this is an opportunity for you to help them refine both their thinking and their verbal skills.
- Can you say for sure that I have at least one nickel? (No.)
- Can you say for sure that I have either 5 coins or 9 coins? (Yes.)

Change the coins in your hand: "Now, the coins I'm holding are worth 13¢ altogether. What can you say for sure about these coins?" Some acceptable answers are:

- You have at least 3 pennies.
- You have at least 4 coins.
- You do not have more than 13 coins.
- If you have 5 coins, they are 2 nickels and 3 pennies.
- If you have more than 9 coins then you have 13 pennies.
- If you have 4 pennies then you have at least 8 pennies.

There is another kind of deduction children can make in a situation like this. Without telling the children what coins you actually have in your hand, change the question: "Tell me some combinations of coins I could have." After a few have been suggested, ask: "How many different combinations of coins are there that are worth 13¢?" Students can make a chart, and conclude that there are exactly four possible combinations.

Further examples of Headline Stories

To be prepared to think mathematically about the variety of situations that come up in the real world, students must see similar variety in their instruction. Notes that accompany the following examples point out some of the features that distinguish one situation from others.

Example 3: Grade 2

My sister is two years older than me. When I was 4… When she was… I'll be 8 when she… When I was born, my sister was___.

Sample responses: When I was 3 she was 5. When she was just a baby, I wasn't anywhere. My sister is two years older than me, so I am two years younger then her!

In the beginning of the year, students need ideas of what kinds of things are relevant to say. In this case, the story guides them with some sentence starters for them to complete. They may, of course, always go beyond the fill-in-the-blank or complete-the-sentence suggestions, if they like. Later in the year, when students have more experience, the same headline story might be presented even more briefly as "My sister is two years older than me."

Example 4: Grade 1

□ + 3 = △

Sample responses: My sister is three years older than me. At our store, triangles cost three cents more than squares do. 4 + 3 = 7. △ - 3 = □.

□	4	2	7	10
△	7	5	10	13

Sometimes a Headline Story is presented in mathematical symbols, and students need to find a "real world" interpretation or derive new arithmetic ideas from the symbols.

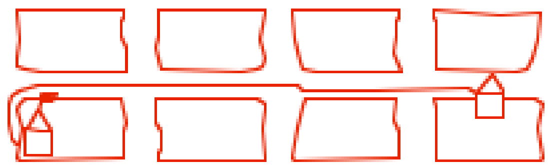
Again, the *kinds* of responses are quite varied. For some students, the open mathematical sentence inspires a single example of the relationship (4 + 3 = 7). Others may show the general relationship in a table. A few, even at this grade, can state the general relationship without examples as △ - 3 = □, a related sentence that is part of the "fact family" of □ + 3 = △, along with 3 + △ = □ and △ - □ = 3 .

Headline Stories can be pared down even further. For example, a Headline Story could say "60 + 40," which is only a phrase, not even a complete mathematical sentence (with a verb, like =, <, or >).

Example 5: Grade 2

Ethan's house is in the middle of the block. He walks out his front door, turns left, walks half a block to the corner and crosses the street. He walks three more blocks and then turns left again and walks almost one more block to school.

Sample responses: Ethan walks about four and a half blocks to school. I think he crosses three or four streets.



If children notice only the numbers in a story, you may need to help them think about the part that "paints a picture." If a picture, diagram, or map isn't part of the information given in the headline, you might encourage children to make it a part of their response.

How can I invent more open-ended problem solving experiences for my students?

Any word problem can become a headline story. Here's a simple problem from first grade.

> Shira had a bag with 18 candies in it. She took four candies out of her bag, and then decided to put two of them back. How many candies are in the bag now?

And here are some suggestions for modifying your favorite word problems.

1. Present all the information, but *drop out the question* and *ask kids what they can figure out from the given information*.

> Ex: Shira had a bag with 18 candies in it. She took four candies out of her bag, and then decided to put two of them back. What can you figure out from this information? (Teacher note: If children need help getting started, they might try acting this out with counters and a bag or box.) Sample responses: Two candies are still left out of the bag. Sixteen candies are in the bag. The bag has less than it had before. …

2. *Drop out one piece of information* and *ask kids what else they need to know to solve the problem*.

> Ex: Shira had a bag of candies. She took four candies out of her bag, and then decided to put two of them back. How many candies are in the bag now? (Teacher note: If children don't spontaneously bring it up, encourage them to think about what other information they would need to have to solve this problem.)

3. Present the problem with some *extraneous information* that looks like it might be relevant but is not, and *ask what they do not need to know* to solve the problem.

> Ex: Shira had a bag with 18 candies, and Leah had a bag with 6 candies. Shira took four candies out of her bag, but then decided to put two of them back. How many candies are in Shira's bag now? (Teacher note: Beginning problem solvers are sometimes distracted by extraneous information. If they try using all the information, for example, adding Leah's 6 candies, suggest that they review what the *question* is, and then say what information they need *and what information they don't need*, to answer that question.)

4. Variant on idea 1 (harder): Present all the information, *drop out the question* and *insert extra information that allows more than one thing to be figured out*. Then *ask kids what they can figure out from the given information*.

> Ex: Shira had a bag with 18 candies, and Leah had a bag with 6 candies. Shira took four candies out of her bag, but then decided to put two of them back. What can you figure out from this information? (Teacher note: There are many questions that children can answer. Encourage them to voice the questions as well as the answers.) Sample responses: How many candies does Shira have in her bag now? Who has more candy? How many more candies does she have?

5. Variant on idea 2 (harder): Present the problem without one or more of the numbers and ask children to describe *how they would solve the problem* if they had the numbers.

> Ex: Shira had a bag with some candies. She took 4 candies out of her bag, but then decided to put 2 of them back. If you knew how many were in the bag to start with, how could you figure out how many are in the bag now? (Teacher note: If children need help getting started, they might try acting this out with counters and a bag or box.) One sophisticated response: If she took 4 out and then put 2 back, the bag has two less now than it did before. So, if I knew how many it had to start with, I could subtract 2.

How can I help my students when they don't seem to know where to start? How can I help them find more than a superficial answer or approach?

One goal of Headline Stories is for students to learn to translate situations and information into problems that can be answered mathematically. The "sample responses" are deliberately broader than what a class is likely to produce in any session, just to suggest the kind of variety you *might* find. Over time, though, you want to help your students learn to give more varied responses and, eventually, produce this kind of variety. With each Headline Story, you might lead to *one* novel response by asking one new question explicitly, perhaps based on a "sample response" that you feel your students are ready for. Use the sample responses to give *you* ideas about approaches to the problem. Some headline stories will give you special suggestions, but here are some general ideas to use with the headline stories, if your children need help coming up with some of the ideas in the sample responses. You can use these for *Think Math* problems or other open-ended problems that you might make up.

1. Sometimes, if students are not sure where to start, it helps for them to review what they do know and what they don't know.

Ex: "Debbie drew four coins out of a jar that contained only nickels and quarters. What can you figure out?"

They know that Debbie took coins from a jar; they know how many; and they know that the coins can only be nickels and/or quarters. They don't know what coins Debbie actually took. What's possible? What's impossible?

2. After figuring out what information they have, it can be useful to see if information is missing. Is there information we *don't* need? Can we fill in missing information with reasonable guesses? How can we figure out what is a "reasonable" guess?

   Ex: Ra'anan is in charge of bringing pretzels, chips, and juice for the class party. The pretzels are $1.29, the chips are $2.49, and cookies are $3.19. He figured he'd need 3 quart-containers of orange juice. How much will he need to spend?

   We can ignore the cookies. Pretzels and chips, together, cost 2¢ less than $3.80, but we don't know how much a quart of orange juice costs. A quart of juice probably costs more than a bag of pretzels, but probably less than $5 (or we can't afford it!). Ra'anan will probably need more than $7 and certainly less than $20.

3. Pose additional *mathematical* questions that relate to the context, or encourage students to pose those additional questions. Questions that look at the limits—what's the most/least—are especially useful questions for opening up discussion.

   "What might Debbie's four coins be?" or "What is the greatest (smallest) amount Debbie could get?" or "Can we find all the possibilities?"

   What is the most that Ra'anan is likely to need? What is the least?

   Some questions are not mathematical: why might Debbie have wanted those coins? Why didn't Ra'anan buy the cookies, too? These may be important real-life questions in some situations, but they are not mathematical questions. Certain questions are especially mathematical. What makes a question sensible depends on the context.

4. *Set* limits—the most/least—that haven't been specified. Ask: How can I narrow down the problem?

   Ex: "The Ice Cream Counter has six flavors of ice cream today: chocolate, vanilla, mint chip, mocha, strawberry, and blueberry. They'll serve it in a cup, a sugar cone, or a cake cone. Sorry, no toppings today! What can you figure out?"

   Asking about most or least seems to make no sense in this context: it is not something one can "figure out" from the information. In fact it is hard to figure out *anything* without focusing the problem in some way: "suppose we bought just one scoop" or "suppose there is a limit of two scoops" or "let's start by assuming people buy only one flavor of ice-cream, but can get one, two, or three scoops." One reason that we present situations that require more constraints (like the number of scoops) and pose the general question "what can you figure out?" is that the *real* problem we want students to encounter is how to narrow, simplify, and focus a mathematical situation.

After setting some limit on the problem—like "one scoop per person, today"—here are some of the most common and important kinds of mathematical questions. They also make sense for the problem about Debbie and the coins.

- Is there *any* solution? What might a solution look like? (We can buy a mocha-chip cake-cone at the Ice Cream Counter. Debbie might have taken two quarters and two nickels.)
- Is this the only solution? (more than one possibility in the Ice Cream Counter; more than one possible collection of coins for Debbie)
- How many solutions are there? (How many different sets of four coins could Debbie take? How many different purchases at the Ice Cream Counter?)
- What is the maximum/minimum? (How much/little might Debbie have taken? This question makes no obvious sense in the ice cream example.)
- Could we solve this problem with less information? (This is best for puzzles with several clues. Are all the clues needed?)
- What if we change one number in the problem? (What if the Ice Cream Counter had only 5 flavors? What if Debbie had taken 5 coins? How would that change the problem?)

Over time, you will discover other ways to help students generate more focused mathematical questions and more varied and creative mathematical answers.

