

Learning by Doing, for Teachers

Algebraic ideas from arithmetic

NCSM — Anaheim, 2005

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To serve the teacher...

*To work, PD must improve life for the teacher. It must start where teachers are and acknowledge what they *do* know as well as what they might not know.*

- Connect directly with their practice
- Capture time they already have
- If it's curriculum, it must Serve the children
- Capture the adult's curiosity
- Meet the needs of the *job* of teaching tests
- Allow easy start without *independent* PD

...what could be
less sexy
than memorizing
4th grade
multiplication facts?

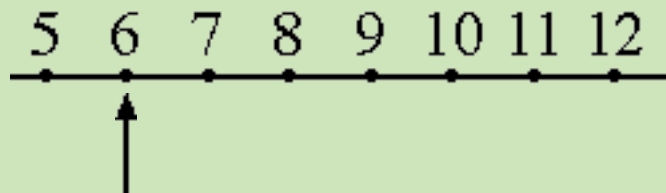
Just the facts

- Start by knowing 4×4 , 5×5 , 6×6 , 7×7 , ...
- Have most others and easily work out what they don't have memorized.
- Goal now is to consolidate!

What helps kids
memorize
multiplication facts?

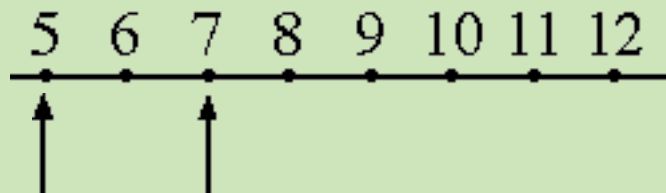
Something memorable!

Surprise



What is 6×6 ?

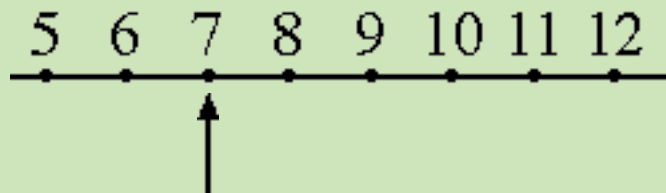
Surprise



What is 6×6 ? 36

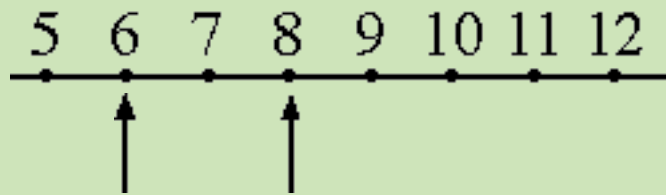
What is 5×7 ? 35

Surprise



What is 7×7 ?

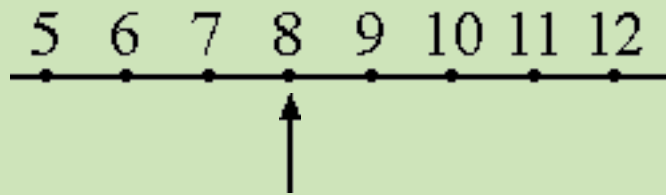
Surprise



49 What is 7×7 ?

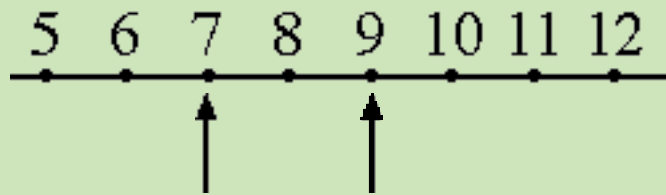
48 What is 6×8 ?

Surprise



What is 8×8 ?

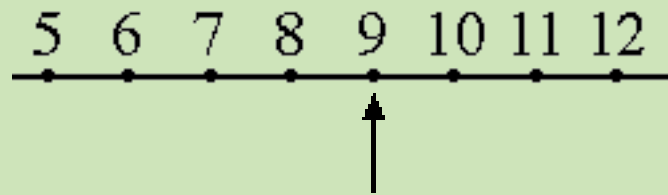
Surprise



What is 8×8 ? 64

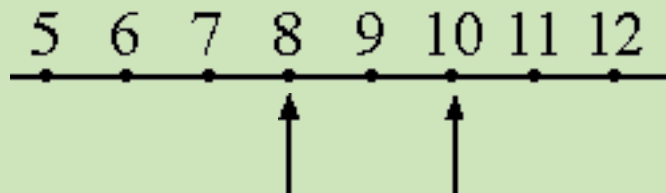
What is 7×9 ? 63

Surprise



What is 9×9 ?

Surprise



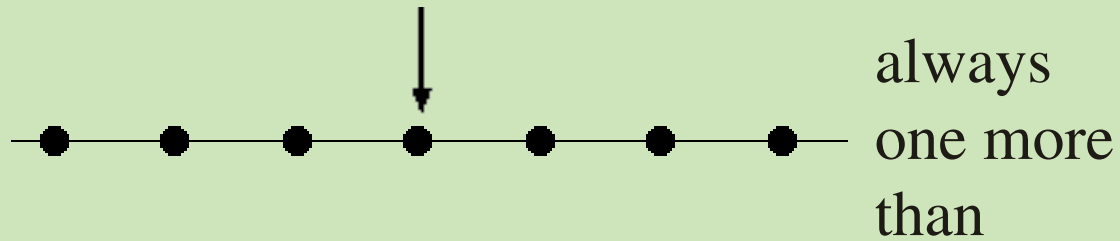
81 What is 9×9 ?

80 What is 8×10 ?

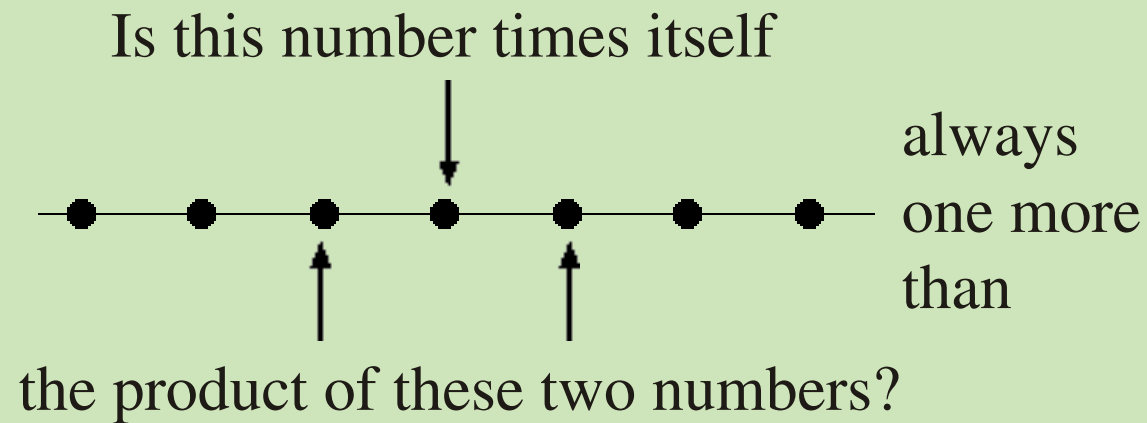
Is this always true?

Is this always true?

Is this number times itself



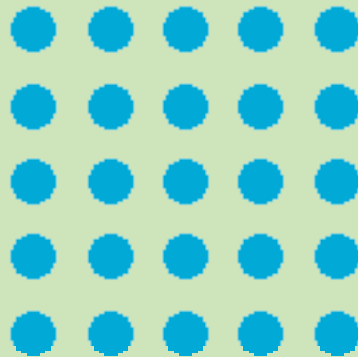
Is this always true?



But *why* does it work?!

One way to look at it

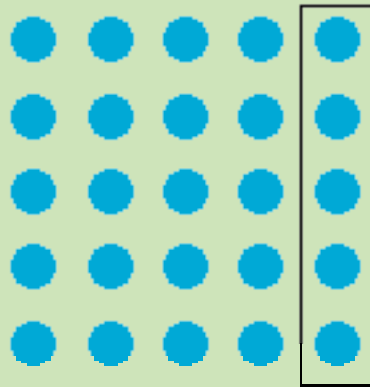
$$5 \times 5$$



One way to look at it

Removing a
column leaves

$$5 \times 4$$

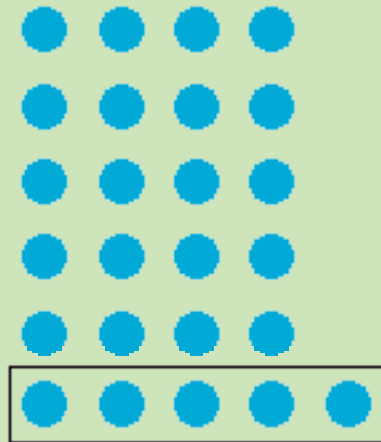


One way to look at it

Replacing as a
row leaves

$$6 \times 4$$

with one left
over.



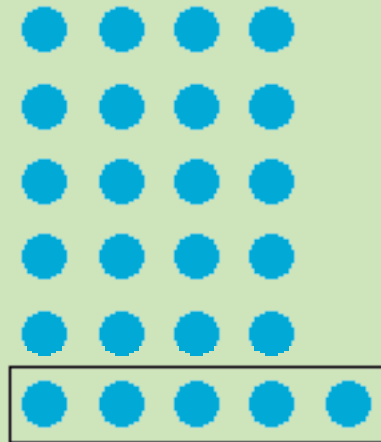
One way to look at it

Removing the
leftover leaves

$$6 \times 4$$

showing that it
is one less than

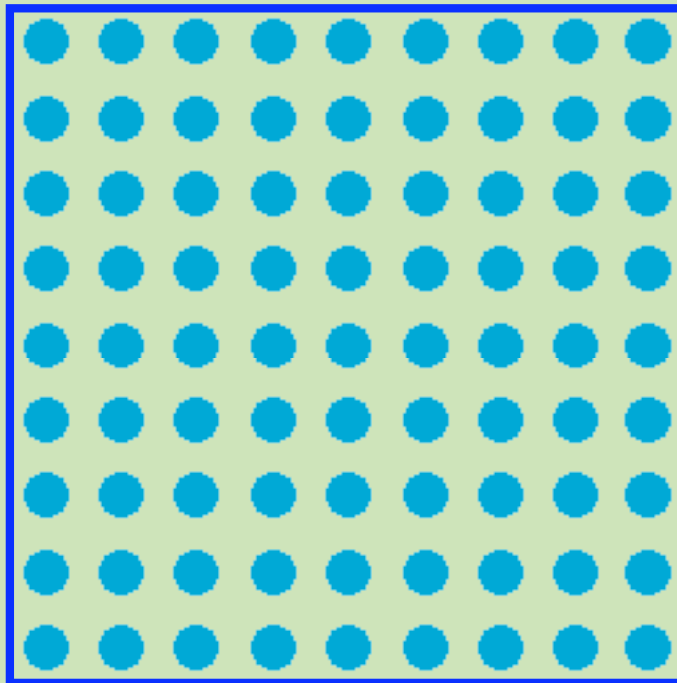
$$5 \times 5.$$



A second look

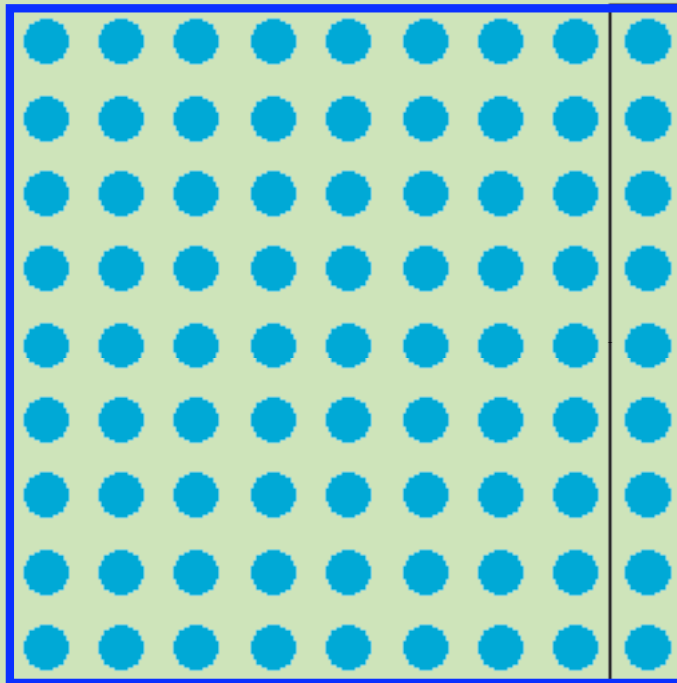
Don't
bother
counting!

A square array.



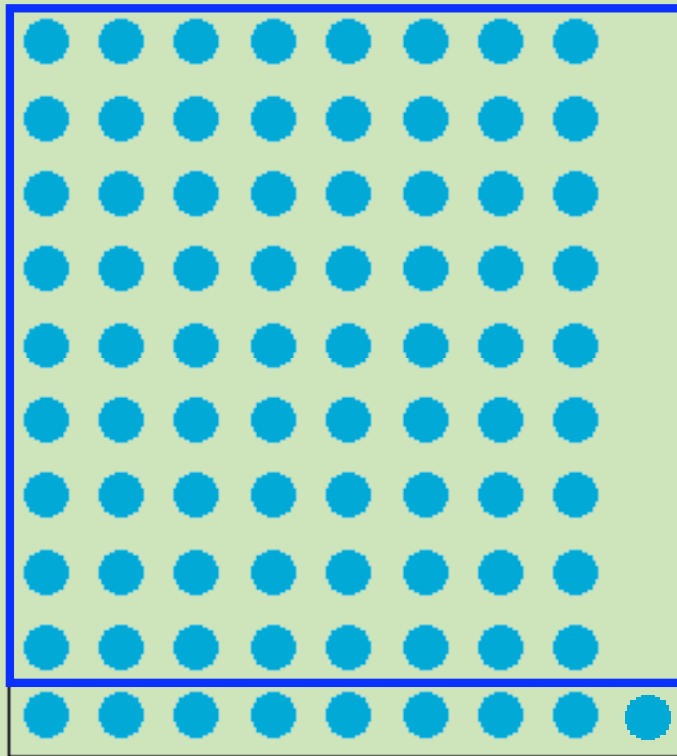
A second look

Removing a column leaves it narrower by 1.



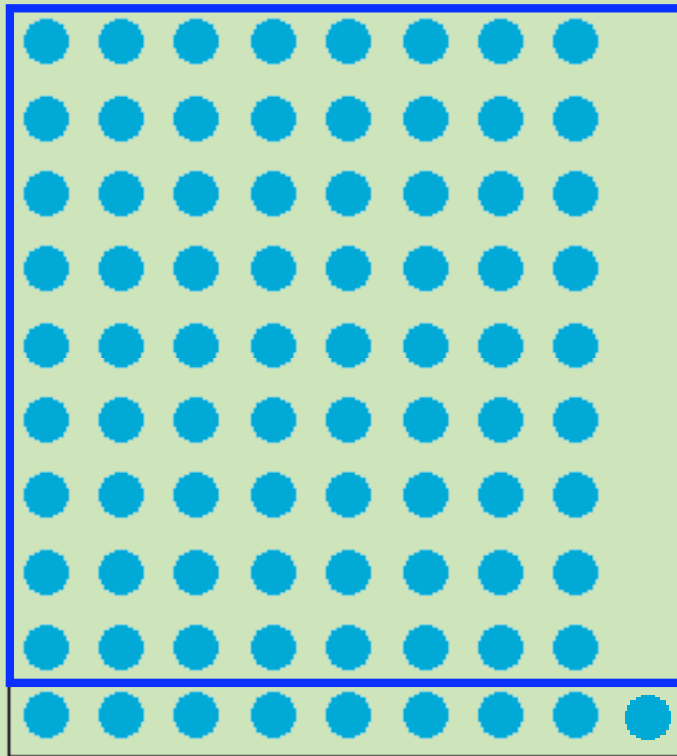
A second look

Replacing as a row leaves it narrower by 1 and taller by 1 (with 1 left over).



A second look

Removing the leftover shows that the new array contains one less dot than the square.



What's the gain?

- An aid for remembering 6×8 or 7×9

➤ $7 \times 7 = 49$

➤ $6 \times 8 = 48$

➤ $(6 \times 8) = (7 \times 7) - 1$



Direct
benefit!

What's the gain?

- An aid for remembering 6×8 or 7×9

- $7 \times 7 = 49$

- $6 \times 8 = 48$

- $(6 \times 8) = (7 \times 7) - 1$

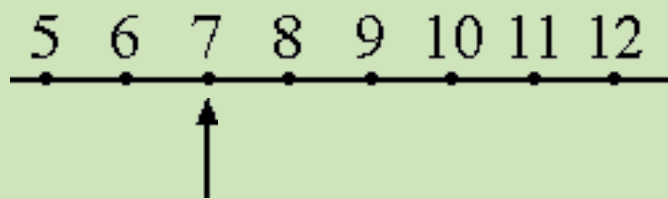
- A practical tool for (some) calculations
- A hint at a BIG IDEA lurking



Investment in
the future!

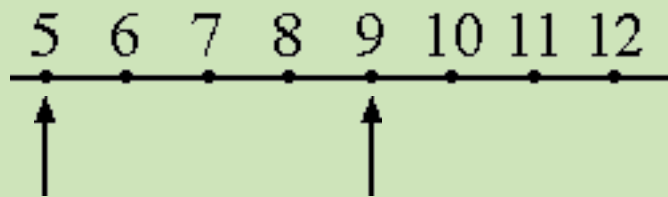
Further Investigation

- In the process of taking this idea further, the children get more multiplication practice.
- Is there a pattern that lets us use $7 \times 7 \dots$



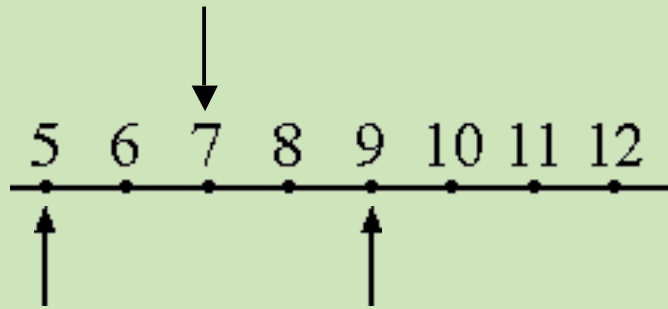
Further Investigation

- In the process of taking this idea further, the children get more multiplication practice.
- Is there a pattern that lets us use 7×7 to derive 5×9 ?



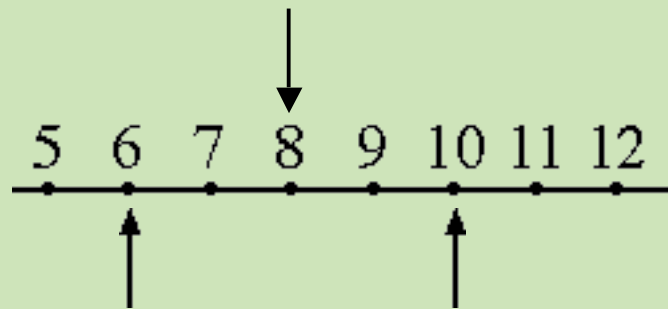
Experiment a moment

Find a pattern that
shows how 7×7 relates to $5 \times 9 \dots$



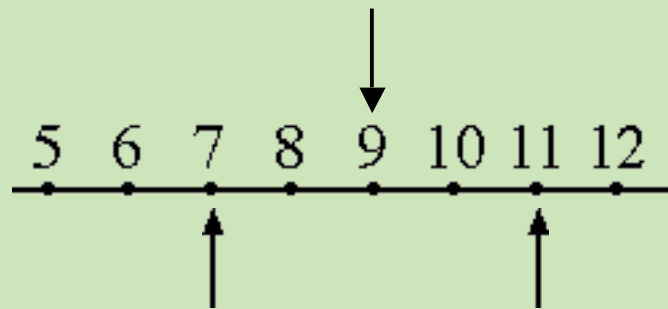
Experiment a moment

...or how 8×8 relates to 6×10 ...

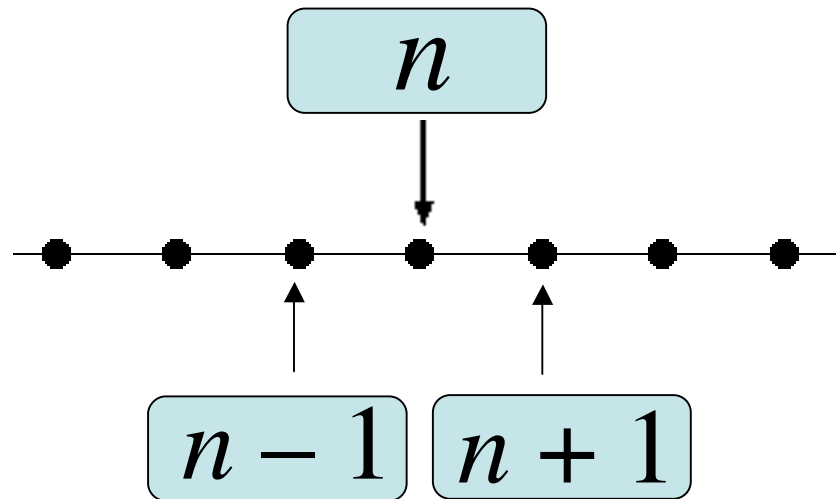


Experiment a moment

...or how 9×9 relates to 7×11 ...

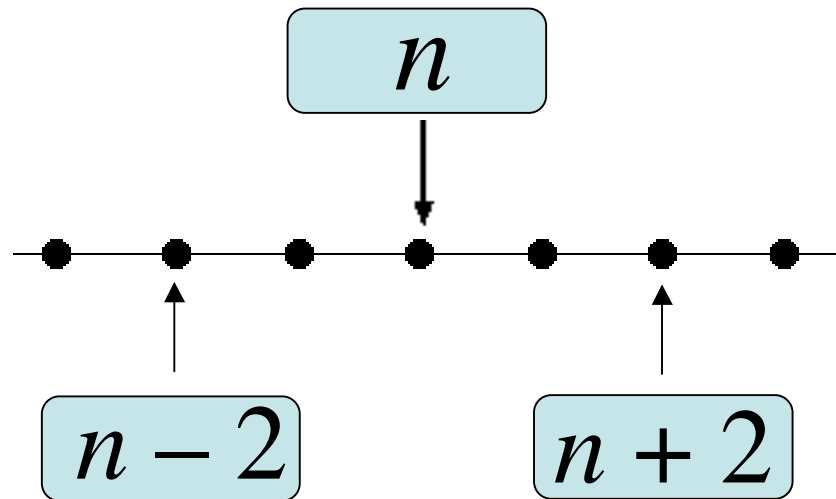


$$(7 - 1)(7 + 1) = 7 \times 7 - 1$$



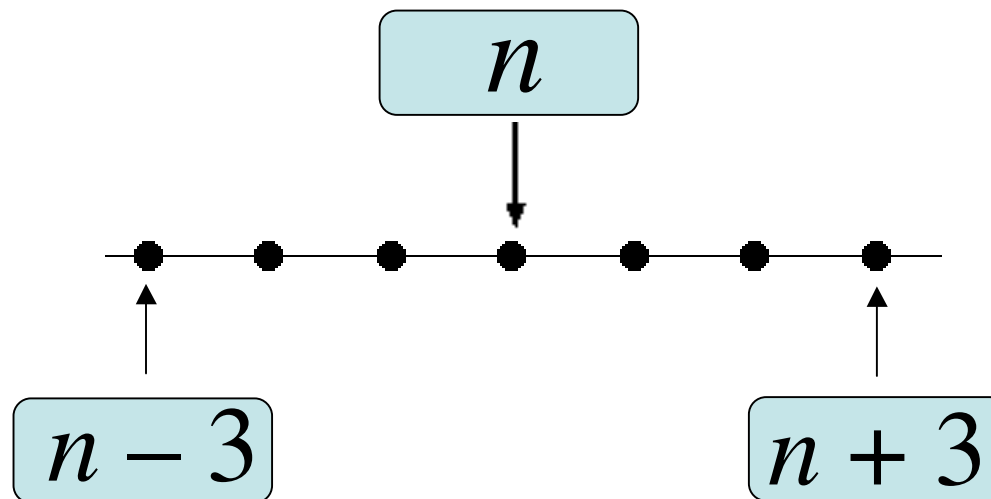
- Or use 9 as an example
- $(9 - 1) \times (9 + 1) = 9 \times 9 - 1$
- $8 \times 10 = 81 - 1$

$$(7 - 2)(7 + 2) = 7 \times 7 - 4$$



- Or use 8 as an example
- $(8 - 2) \times (8 + 2) = 8 \times 8 - 4$
- $6 \times 10 = 64 - 4$

$$(7 - 3)(7 + 3) = 7 \times 7 - 9$$



- Or use 10 as an example
- $(10 - 3) \times (10 + 3) = 10 \times 10 - 9$
- $7 \times 13 = 100 - 9$

Where does this lead?

Where does this lead?

To do...

$$\begin{array}{r} 53 \\ \times 47 \\ \hline \end{array}$$

Where does this lead?

To do...

$$\begin{array}{r} 53 \\ \times 47 \\ \hline \end{array}$$

...I think...

← 3 more than 50

Where does this lead?

To do...

...I think...

53 ← 3 more than 50

× 47 ← 3 less than 50

• 50×50 (well, 5×5 and ...) ... 2500

• Minus 3×3 - 9

Where does this lead?

To do...

...I think...

$$\begin{array}{r} 53 \\ \times 47 \\ \hline \end{array}$$

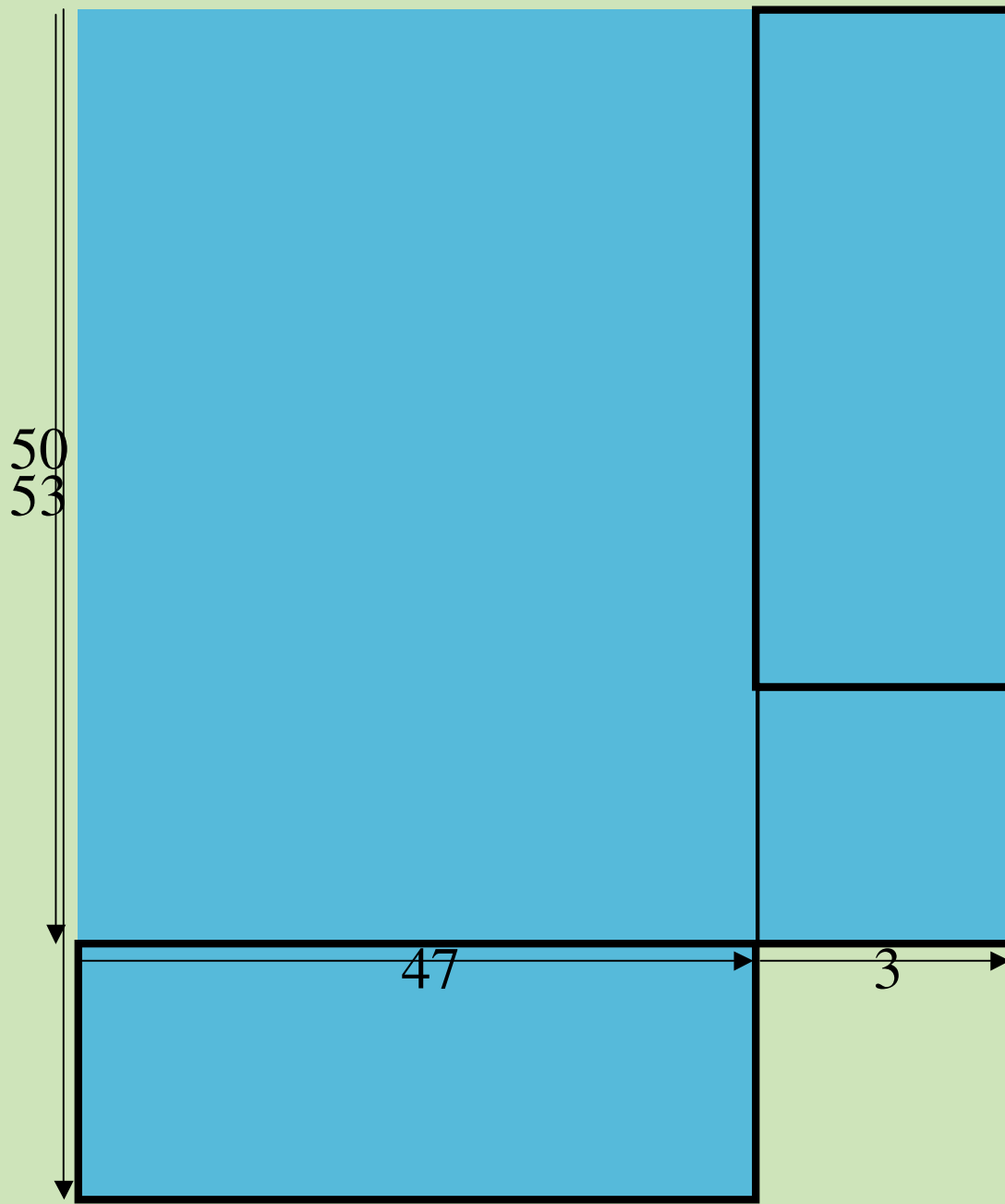
← 3 more than 50
← 3 less than 50

• 50×50 (well, 5×5 and ...) ...

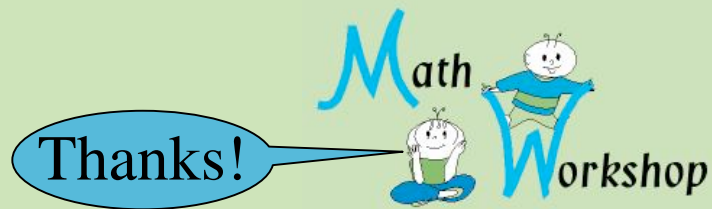
2500

• Minus 3×3

$$\begin{array}{r} 2500 \\ - 9 \\ \hline 2491 \end{array}$$



Why does
it work?



Contact Information

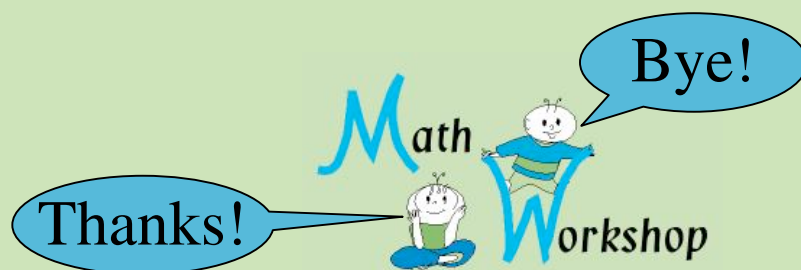
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