Using Classroom Practice for Teachers' Professional Learning– **Teachers *Learning by Doing***

Math Workshop

A project of EDC

With support from National Science Foundation

www2.edc.org/mathworkshop

The problem

- 1.6 million elementary school teachers
- Weak math knowledge?
- Yes, but perhaps even more issues with confidence and interest baby picture books Kdg teachers can count and read numbers!
- What *external* professional development program can build knowledge, confidence, and interest for 1,600,000 people?

The idea:

What if we could capitalize, even slightly, on time spent *teaching*?

- Not a complete solution, but…
- A *lot* of time: ~150 available hours
- Alert, engaged time
- Capturing *interest* could be a springboard for more PD

Capitalizing on time spent *teaching*…

- …depends on events in the classroom.
- In other words, some curricular material

Liping Ma

What would it take?

- Excellent *comprehensive* curriculum for kids
- Easy to use without extensive prior PD
- Appealing to the district, teachers, parents
- Something developmentally appropriate for children…

… that still holds surprises for the *adult* mind

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The examples are from

- Fully tested “vintage” American curriculum that anticipated current international best-practice standards: Wirtz, Botel, Beberman, Sawyer
- Comprehensive K-6, *Math Workshop*
- Skills, understanding, algebraic focus from K
- NSF support for EDC to redevelop this classic…
- …to capitalize on classroom time for teacher PD
- Learning by doing!

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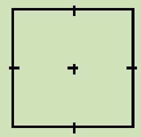
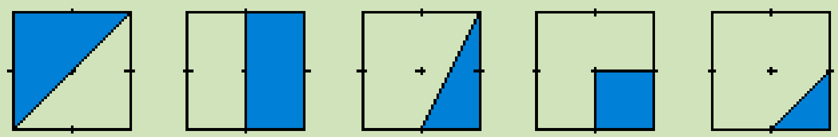
Surprises for the adult mind

Fractions in 3rd grade

If we agree that  = 1 unit,

a b c d e



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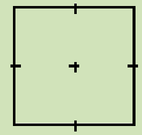
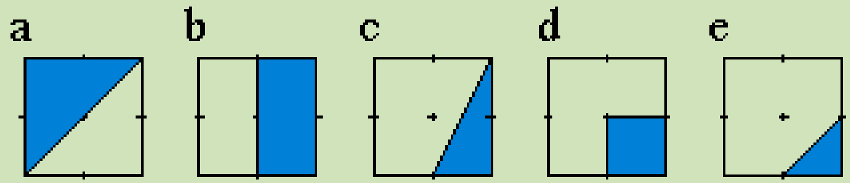
Shaded	$\frac{1}{2}$				
Unshaded	$\frac{1}{2}$				

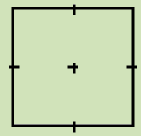
Fractions in 3rd grade

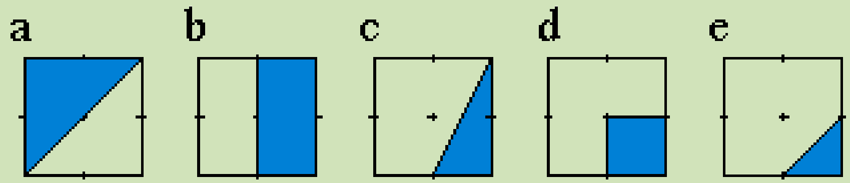
If we agree that  = 1 unit,



	a	b	c	d	e
Shaded	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$
Unshaded	$\frac{1}{2}$				

Fractions in 3rd grade

If we agree that  = 1 unit,

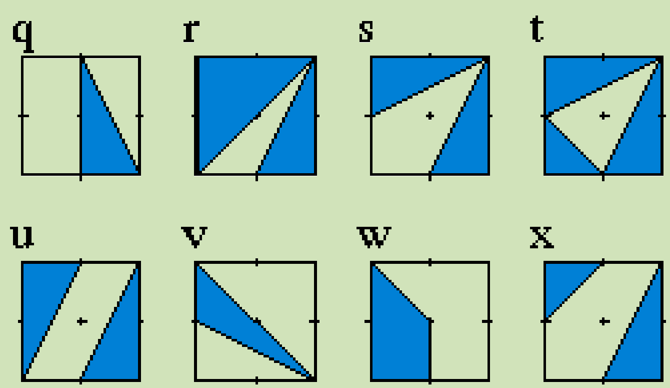


	a	b	c	d	e
Shaded	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$
Unshaded	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$		

Surprises

sometimes come from students' "naïve" answers.

Hard work below!



	q	r	s	t	u	v	w	x
Shaded	$\frac{1}{4}$							
Unshaded								

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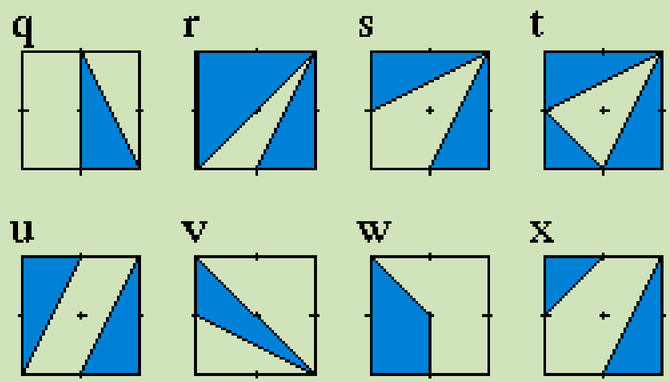

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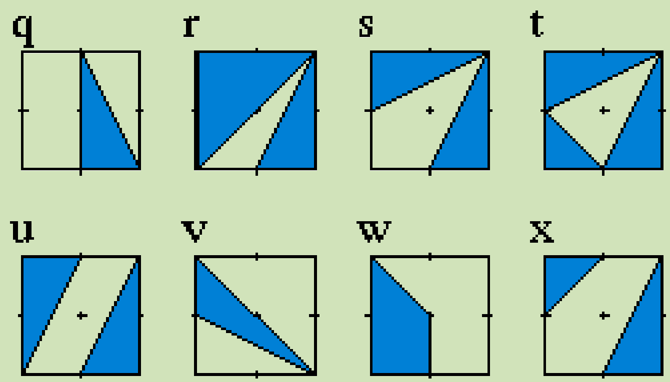

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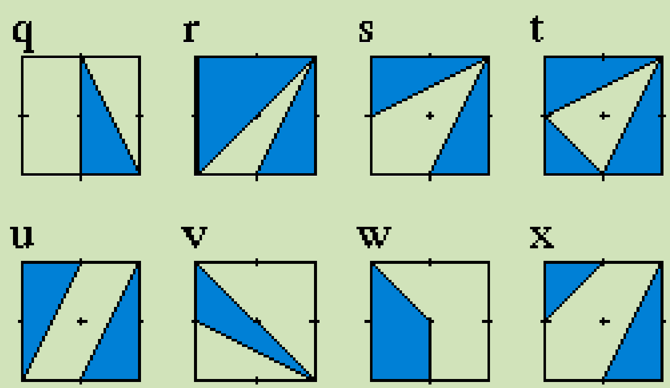

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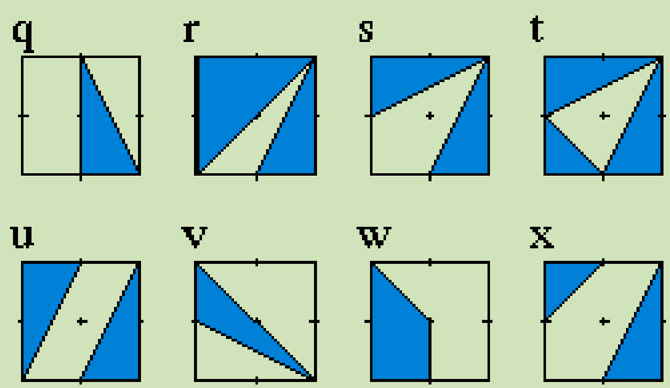

	q	r	s	t	u	v	w	x
Shaded	$\frac{1}{4}$	$\frac{3}{4}$				$\frac{1}{4}$		
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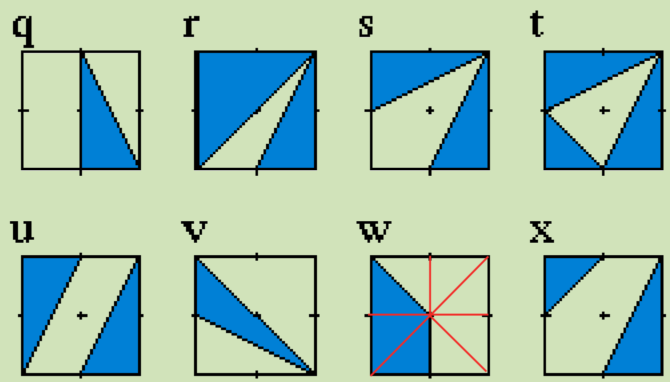

	q	r	s	t	u	v	w	x
Shaded	$\frac{1}{4}$	$\frac{3}{4}$				$\frac{1}{4}$	$\frac{3}{8}$	
Unshaded		$\frac{1}{4}$				$\frac{3}{4}$		

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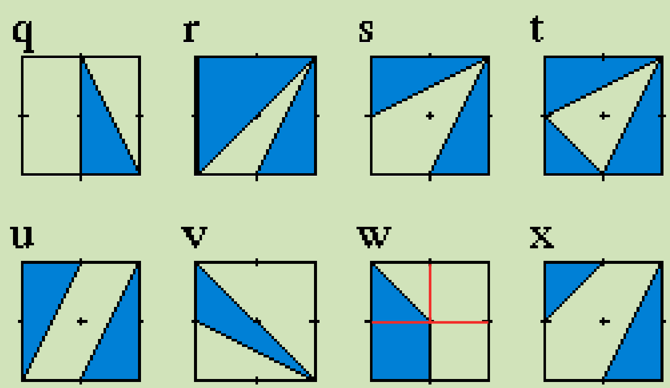

	q	r	s	t	u	v	w	x
Shaded	$\frac{1}{4}$	$\frac{3}{4}$				$\frac{1}{4}$	$\frac{1\frac{1}{2}}{4}$	
Unshaded		$\frac{1}{4}$				$\frac{3}{4}$		

$\frac{3}{8}$

A teacher's response

- "I thought of fractions mostly in terms of equal parts of a whole. I never thought about proportionality, measurement, or geometry as ways to understand fractions."
- "The [problems], which often looked dry to me and sometimes too difficult for third graders, excited the students and got them thinking."
- "Watching the children examine problems, make up rules, test their rules, talk about them, and often say 'I get it,' was incredible."

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4th grade multiplication facts

- MW kids already "know" 4×4, 5×5, 6×6, 7×7, …
- Have scattered others and easily work out rest
- Goal now is to consolidate!

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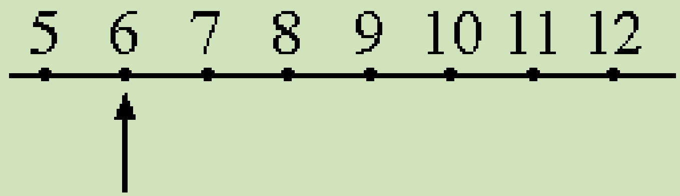
What helps kids memorize multiplication facts?

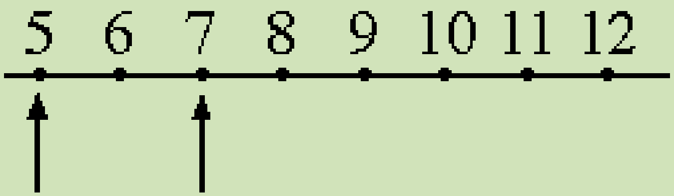
Something memorable!

Surprise



What is 6×6?

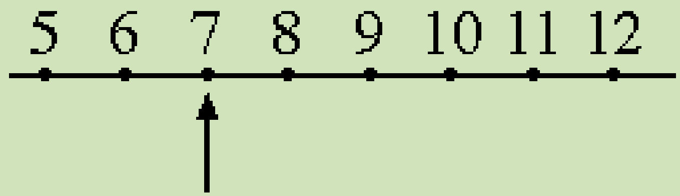
Surprise



What is 6 × 6? 36

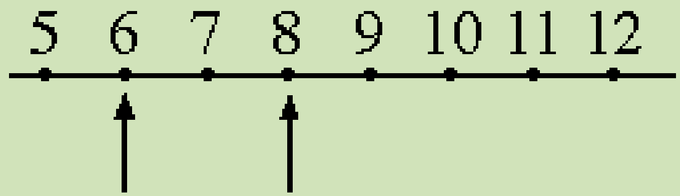
What is 5 × 7? 35

Surprise



What is 7×7?

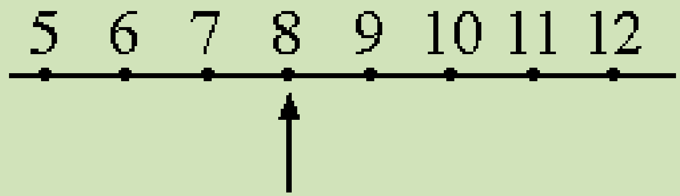
Surprise



49 What is 7 × 7?

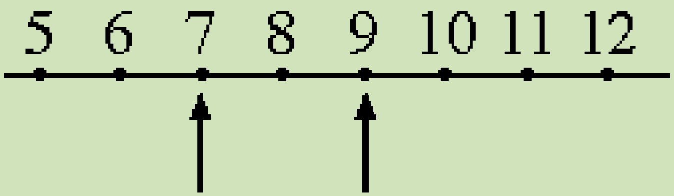
48 What is 6 × 8?

Surprise



What is 8 × 8?

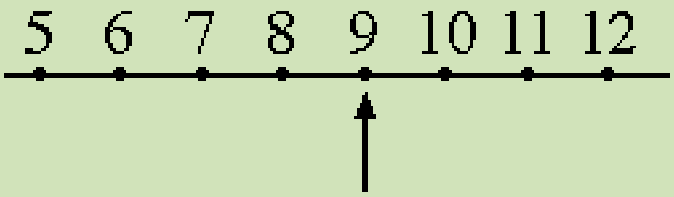
Surprise



What is 8 × 8? 64

What is 7 × 9? 63

Surprise



What is 9×9?

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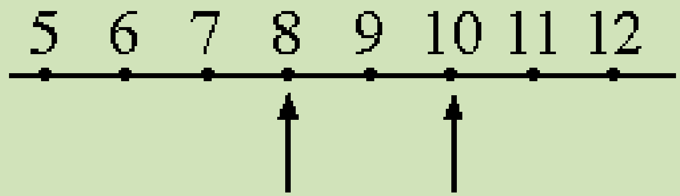
Surprise!



81 What is 9 × 9?

80 What is 8 × 10?

Is this always true?

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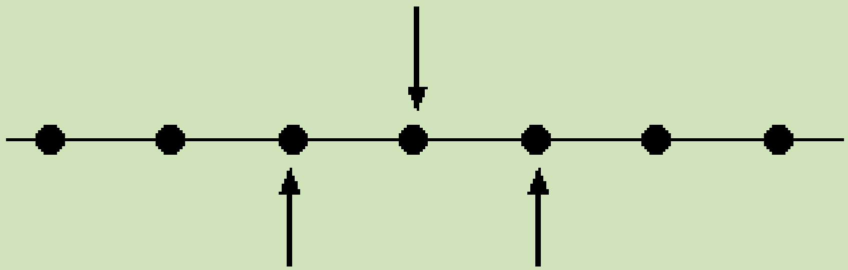
Is this always true?

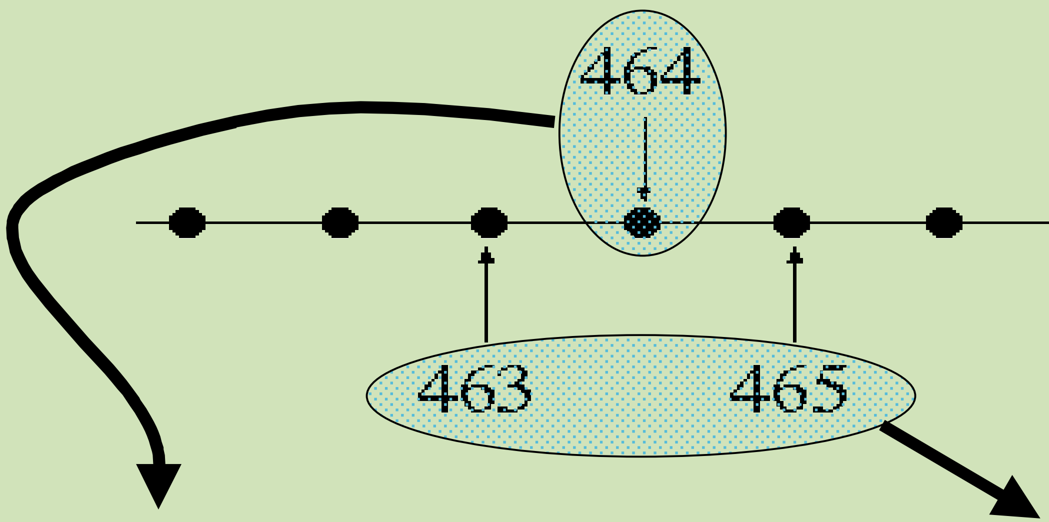
Is this number times itself



always
one more
than

the product of these two numbers?

Try a wild case



$464 \times 464 = 215{,}216$

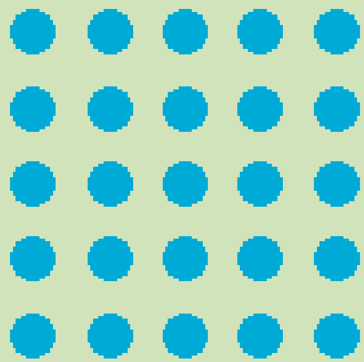
$463 \times 465 = 215{,}295$

$436 \times 465 = (464 \times 464) - 1$

Can we be sure it will work for *all* numbers?

One way to look at it

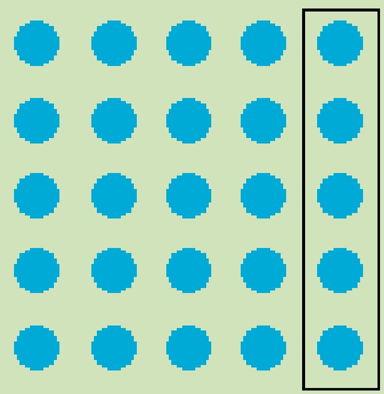
5×5



One way to look at it

Removing a column leaves

5×4



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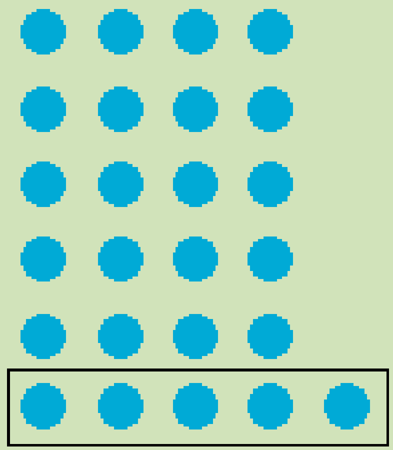
One way to look at it

Replacing as a row leaves

$$6 \times 4$$

with one left over.

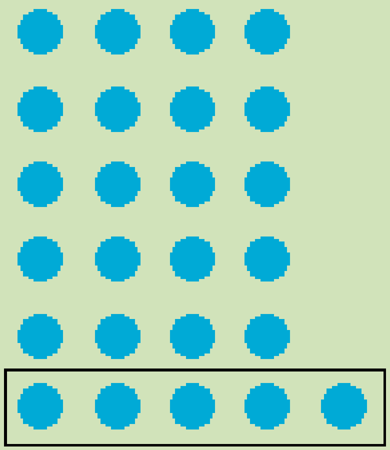


One way to look at it

Removing the leftover leaves

6×4

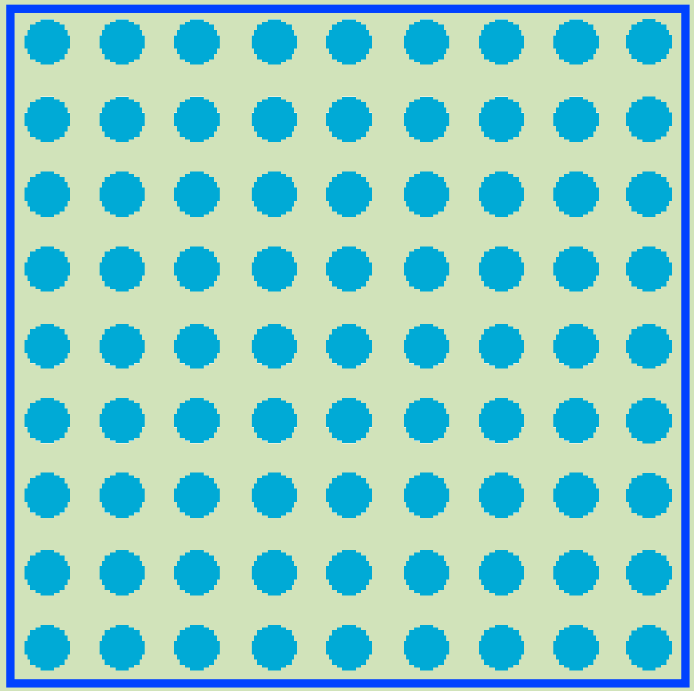
showing that it is one less than 5×5.



A second look

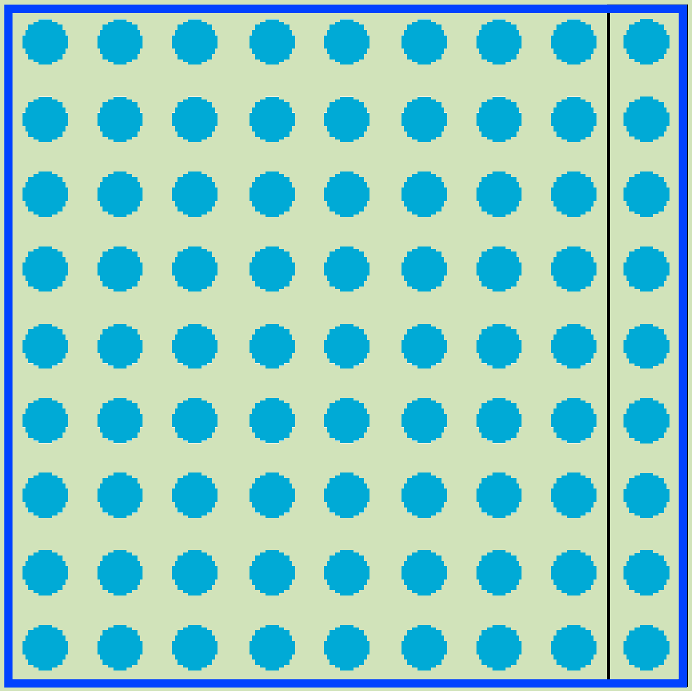
Don’t bother counting!

A square array.



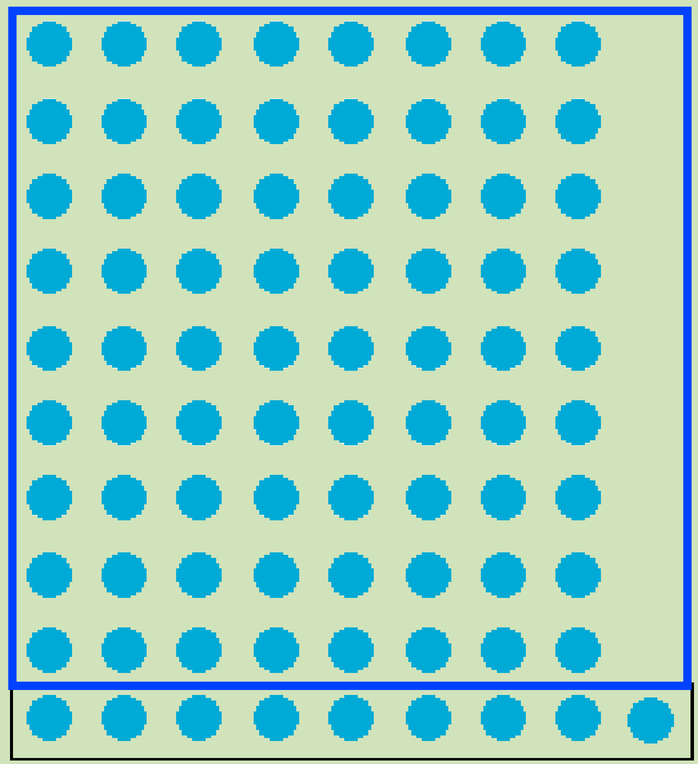
A second look

Removing a column leaves it narrower by 1.



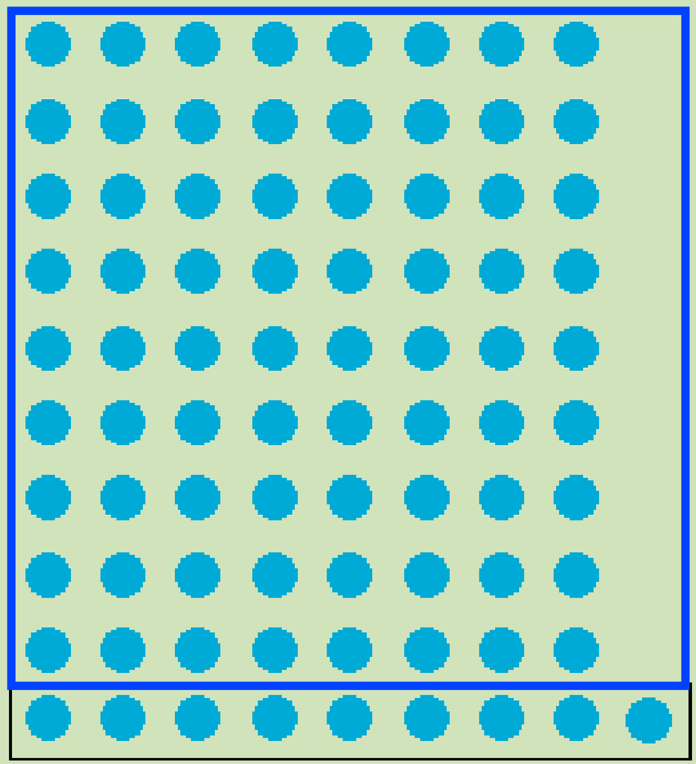
A second look

Replacing as a row leaves it narrower by 1 and taller by 1 (with 1 left over).



A second look

Removing the leftover shows that the new array contains one less dot than the square.



What's the gain?

For kids...

- An aid for remembering 6×8 or 7×9
  - $7 \times 7 = 49$
  - $6 \times 8 = 48$
  - $(6 \times 8) = (7 \times 7) - 1$

Direct benefit!

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For kids...

- An aid for remembering 6×8 or 7×9
  - $7 \times 7 = 49$
  - $6 \times 8 = 48$
  - $(6 \times 8) = (7 \times 7) - 1$
- A hint at a BIG IDEA lurking

Investment in the future!

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For teachers...

- A practical teaching tool
- A surprise connection with their algebraic past

$$(n + 1)(n - 1) = n^2 - 1$$

  but without the (often uncomfortable) baggage
- A connection with the kids' algebraic future
- A glimpse of the order and generality within arithmetic

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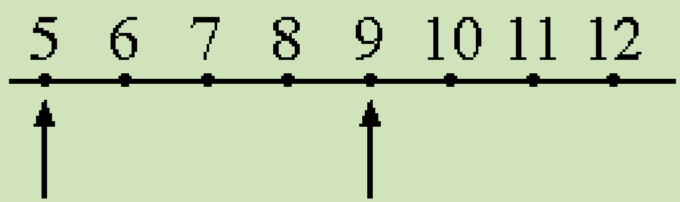
Further Investigation

- Are there other shortcuts like this?
- For example, is there a pattern that would let us use knowledge of 7 × 7 to derive 5 × 9?



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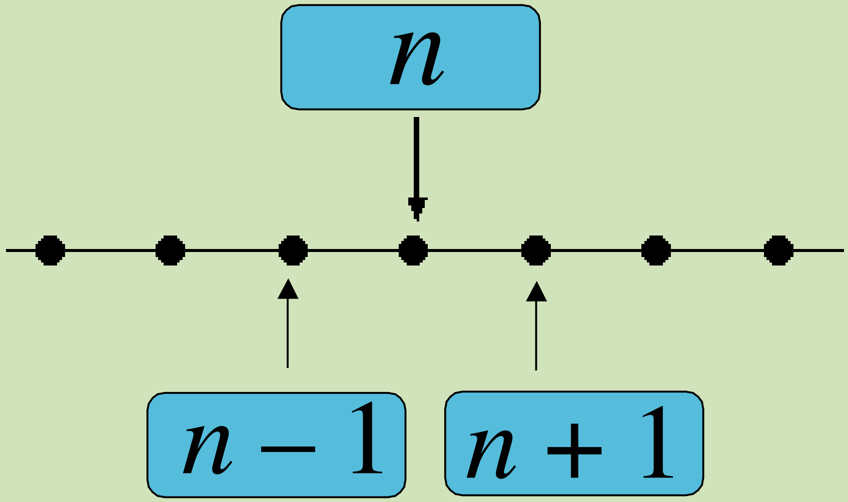
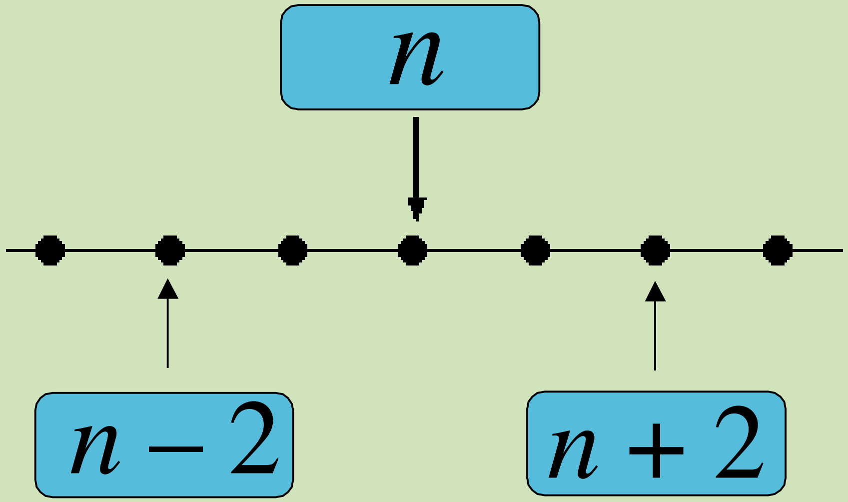
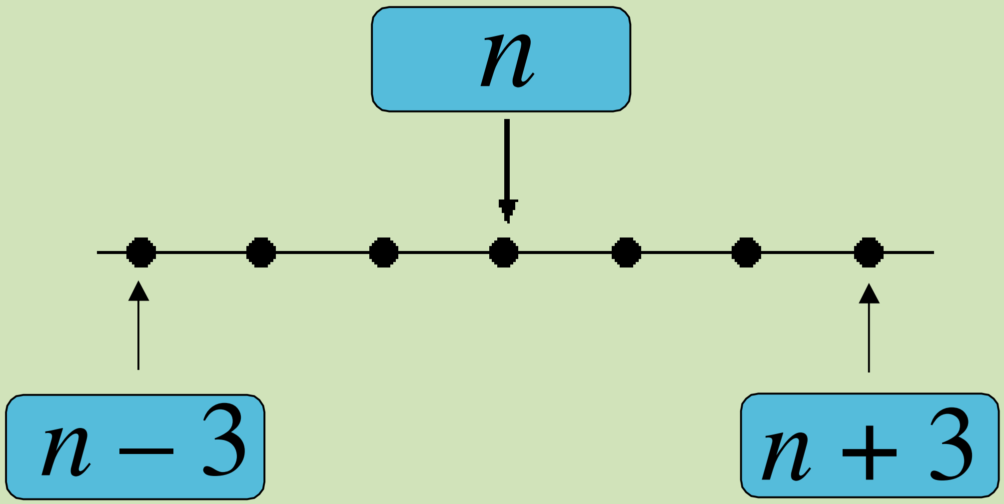
$$(n - 1)\,(n + 1) = (n \times n) - 1$$



$$(n-2)\,(n+2) = (n \times n) - 4$$



$$(n-3)\,(n+3) = (n \times n) - 9$$



Where does this lead?

To do…		…I think…
53	←	3 more than 50
$\times$ 47	←	3 less than 50

- 50 × 50 (well, 5 × 5 and …) … 2500
- Minus 3 × 3 − 9

$$\begin{array}{r} 2500 \\ -\ 9 \\ \hline 2491 \end{array}$$

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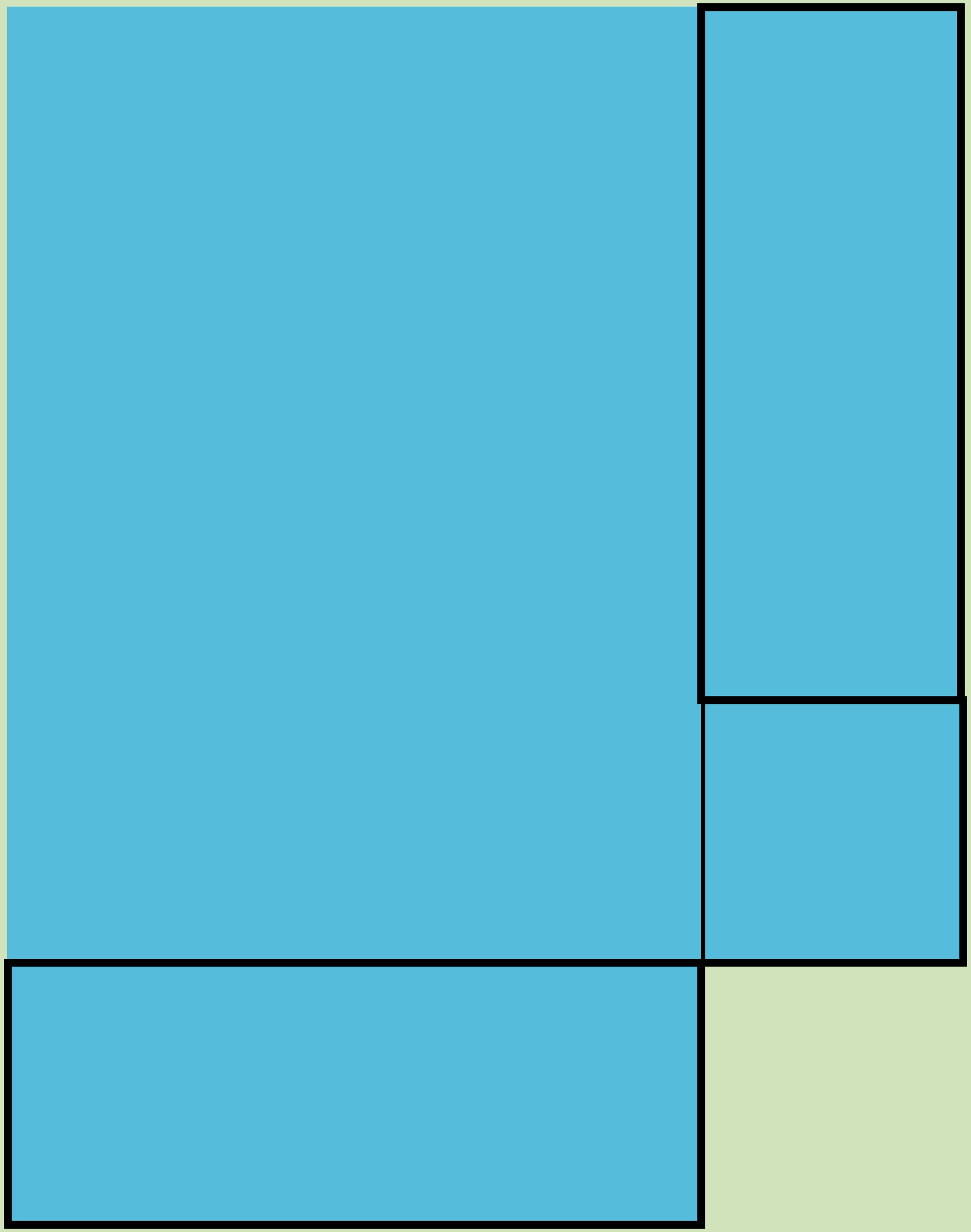

The generic case!

A Number Trick

- Think of a number.
- Add 3.
- Double the result.
- Subtract 4.
- Divide the result by 2.
- Subtract the number you first thought of.
- Your answer is 1!

How did it work?

How did it work?

- Think of a number.

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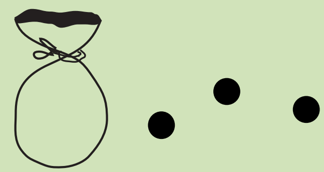

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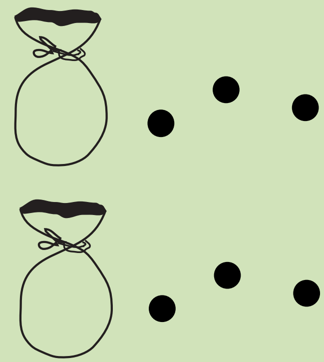

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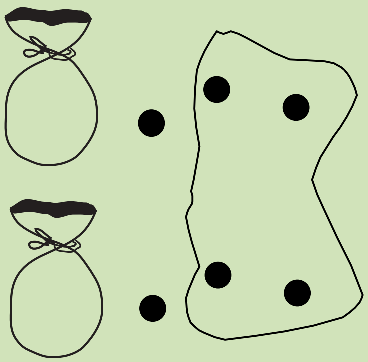

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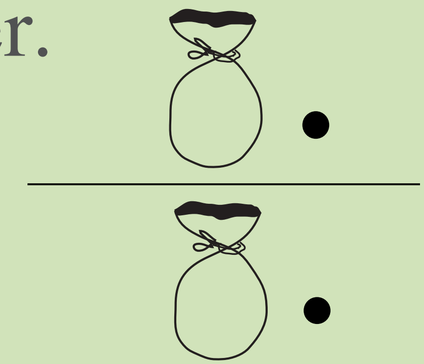

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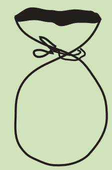

How did it work?

- Think of a number.
- Add 3.
- Double the result.
- Subtract 4.
- Divide the result by 2.
- Subtract the number you first thought of.
- Aha! Your answer *is* 1!

-

A bit corny, but...

 soon becomes x

A surprise in 2^{nd} grade

a i

s n t

How many two-letter combinations can you make starting with one of these two vowels, and ending with one of these three consonants?

as in at ...

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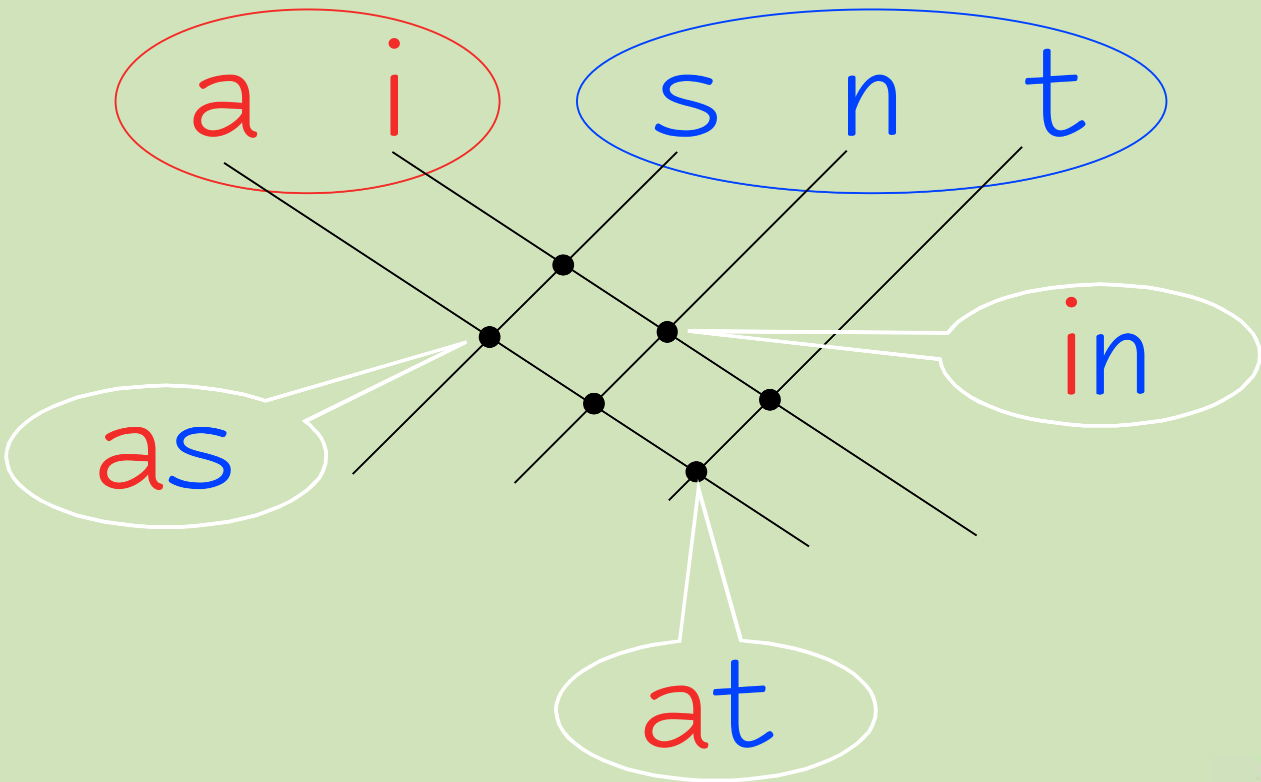
A surprise in 2nd grade



Seeing Multiplication in an unfamiliar place

b p w s ill it ink

---

br tr st ick ack ing

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What can you do before this particular curriculum is out?

Make the Familiar Strange

- Look for surprises in familiar content
- Look for surprises in students’ responses
- Look for connections among “unrelated” ideas

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Thank you!

Bye!

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Contact Information

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www2.edc.org/mathworkshop