SETTING AND SHARING GOALS

The essential act leading to a successful learning experience is the establishment, and hewing to, of coherent and important learning goals. While this statement may seem obvious, too many courses (e.g., United States History [1492–Present] or Algebra II) are little more than a title and a grab bag of poorly connected topics. Figuring out what activities will aid students in achieving the sought-after goals is an exciting and challenging task.

The habits of problem-posing, creating representations, explaining connections, and testing and checking are central to the development of interesting new mathematics. Students need to see these habits as worthwhile activities. Mathematics investigations can involve multiple variables and solution methods, appealing questions with no obvious path to understanding, and answers that vary according to the assumptions made. Encounters with such settings dispel students' notion that the sole trademark of mathematics is the exactness and uniqueness of results. Rather, creativity and the recognition of underlying structure and abstraction become dominant features of the discipline. We must help our students become comfortable with the roles that analysis, creativity, and clear communication play in active learning and discovery. Students must also be curious and willing to take risks. Successful students in traditional mathematics courses are rewarded for speed and technical accuracy. A different type of confidence is required when they begin posing problems with no immediately clear method of solution and no guarantee that a solution can be found.

Make your goals explicit and share them with your students before starting an activity. The greater students' initial and subsequent understanding of what they are doing and why it is worthwhile, the more readily they will connect each activity to that larger understanding and retain its lessons. A sample handout of goals for a mathematics research strand or course is in Appendix A at the end of this discussion. The point of such a handout is not to detail every learning goal (e.g., note the lack of specific proof techniques that might be studied), but to invite students into a reflection on how they have learned mathematics in the past and what the expectations of a research experience might be. In general, goal-setting is even more effective if the students themselves can generate the objectives and standards. This approach is possible when the students already have some prior experience with the task at hand (e.g., if they have engaged in open-ended mathematics explorations). The Building Collaborative Skills section provides an example of student-generated standards.

Once students have read and discussed the goals in class, a homework assignment (see Figure 1, below) asks students to reflect on both the goals and the challenges the goals pose for them in the coming year. Because doing mathematics research is, for most students, a radical departure in both content and approach from typical math classes, it is crucial that they be aware of, and ultimately value, the changed expectations. A class should periodically discuss the meaning and purpose of the goals in order to develop this appreciation. Some of the goals will make more sense as the activities provide students with a sense of context. When your students encounter research as a non-elective part of a standard mathematics course, you should not assume or try to force approval of, or commitment to, the goals. Rather, you must negotiate, encourage, justify, and inspire that commitment and enthusiasm.

Reread the goals sheet for the course. Respond to **one** of the following questions about the goals listed in the habits and attitudes section (one page, more or less).

1. Choose some goals to which you have a stronger than average response. Are there goals that you particularly like or dislike? Are there goals that you think may be easier or more difficult for you? Please explain **why** these goals are more (or less) appealing to you or why you think they will be more (or less) challenging for you.

OR

2. Some of these goals are more general and some are more tangible than others. Pick a few goals and discuss how you would demonstrate to someone, or measure for yourself, your progress toward each objective.

Figure 1. Goals Reflection Assignment

The three quotes in Figure 2, below, are excerpts from student responses from both mathematical modeling and pure mathematics research courses to Question 1 of the Goals Reflection Assignment. These responses reveal several common reactions: fear, optimism, and skepticism. (Another common student response is to like a goal because the student already feels competent in that area [e.g., being persistent or writing clearly]). These essays are the start of a dialogue. Responses to this assignment should be positive and encouraging comments (such as those in italics) that agree with, or at least acknowledge, the feelings expressed and observations made. When a student cites a particular weakness (e.g., calculator use), offer help in reaching his or her goals.

I noticed three things that I have never had to do for a math course before. These goals may take me a while to get used to and/or get good at. They include defining math problems and asking math questions, knowing and identifying which math tools to use when solving a problem and writing narrations of exploration and problem-solving efforts . . . In all of the math courses I have taken in the past the teacher has taught us exactly what we needed to know to go on to the next year's courses, nothing more, nothing less. We were never allowed to stray too far off topic, thus we didn't pose too many questions of our own. The problems we did do came from the book. As a result of this I am a little apprehensive of this course but hope that I will learn quickly.

Be patient with the process. You are right in noting wholesale changes in expectations. Give yourself time to adjust.

"Articulate your thoughts and discoveries" is a goal that is not 100% to my liking, yet strikes my curiosity. Rarely before have I ever written narrations of my discoveries and problem-solving techniques. I believe that if I am capable of explaining my actions in writing, then I would definitely be able to understand what I was doing since writing and explaining could be considered as my weaknesses.

You are right in noting that it is impossible to write clearly until you fully grasp an idea yourself, but the writing itself can facilitate thinking. Your writing here reveals your thoughtfulness.

When I see the heading "Enjoy Mathematics," I am immediately inclined to think, "Yeah, right." I have never enjoyed math, but that may be because I have never had a chance to really apply it to real life. For example, in Algebra classes and such, the closest we ever got to applications were word problems, which are pretty much laid out [for you].

As the year proceeds, please let me know if you continue to find this goal implausible and we will search for an intersection between your interests and our studies.

Figure 2. Excerpts from Student Responses to the Goals Reflection Assignment

Few students choose to answer Question 2, which is more difficult. For either question, students sometimes give answers without providing much explanation. For example, favorite or feared goals may be noted without mentioning a reason for this reaction, or a student might suggest keeping a diary in order to demonstrate progress toward meeting the goals, without detailing how that record would demonstrate improvement. As with all feedback, it is important to point out the need to elaborate on comments and justify claims.

Several weeks into the course, you can exchange written evaluations with each student on the student's progress. It is helpful to look back on the students' initial reflections and note how they have fared with the goals that particularly concerned them. Students often make the most progress in those areas that stood out enough to write about in the first place. If not, offer advice on how to start improving. Positive feedback on what progress has been made in each area should be detailed (see Class Time in the Assessment section).

APPENDIX A: AN OUTLINE OF THE MAIN MATHEMATICS RESEARCH GOALS

Essential Questions to Explore

- What is mathematics? What is a mathematical system?
- What are the processes of understanding and discovering mathematics?
- What does it mean for a mathematical statement to be true? What constitutes proof of a claim? What is the role of proof in mathematics?

Habits and Attitudes to Be Developed and Extended

Enjoy mathematics.

Bring passion to your study of mathematics. If you enjoy mathematics explorations, you will want to continue studying and using mathematics throughout your life.

Create new problem settings and pose new questions.

Develop new mathematics questions about the topics that you study. The richest mathematical experiences frequently evolve from problems posed by the problem-solver. New mathematics is created through the modification of existent questions, statement of new definitions, or the identification of a new area of exploration.

Search for structures.

- Actively seek to uncover symmetries, relationships, connections, and patterns in the settings you explore.
- Attempt to generalize your observations into conjectures.

Read mathematical works.

Be patient learning how to read primary source mathematics. It is neither a speedy nor a linear process. Strive to read both peer and professional mathematics writings carefully as you identify the assumptions, test the conditions, and verify the conclusions.

Prove.

- Develop logical arguments that prove or disprove the conjectures you investigate.

- Make informed choices about which of your mathematical skills would be helpful in constructing a proof, or whether new mathematics needs to be studied in order to support a claim.

Write mathematics clearly.

- Write narrations of your explorations and problem-solving efforts.
- Use the simplest language possible to express an idea. Abstract content does not require impenetrable prose. The motivation behind your work and your reasoning should be clearly crafted and presented. Technical language and symbols should be used when they enhance communication.

Check your reasoning and solutions.

Only accept conclusions if you can verify or estimate the validity of an answer.

Extend yourself.

Take responsibility for guiding the activities of the class, for responding to one another's ideas, and for persisting in the face of difficult challenges.

Use technology.

Know how and when to use calculators and computer tools. Understand the limitations and power of each.

The above goals have two overarching objectives: that you broaden and refine your own aesthetic and intuition about pure mathematical questions, and that you become mathematicians who create your own pathways for investigation.

