Michelle and Han's proof

In the dialogue that follows, Michelle and Han are discussing a *combinatorial* proof of the binomial theorem. Finish the discussion and proof for them. Be sure to provide a proof of the general result, not just the $n = 5$ case.

Michelle: I have another proof of the binomial theorem.

Han: How's it go?

Michelle: Well, look at $(a + b)$. It's really just $(a + b)(a + b)(a + b)(a + b)(a + b)$, right?

Han: And you want to *do* that?

Michelle: I want to *imagine* doing it. If I multiplied all this out, I'd get a sum of terms, each of which is a sum of some a's multiplied by some b's. I get each term by taking a letter from each parentheses then multiplying them together. For example, I could take a from parentheses 1, 2, and 4 and b from parentheses 3 and 5. That would give me an a^3b^2.

Han: But you could also get an a^3b^2 by taking a from parentheses 1, 2, and 3 and b from parentheses 4 and 5.

Michelle: Right, so the *coefficient* of a^3b^2 will be the number of ways I can pick 3 a's and 2 b's from the 5 parentheses.

Han: And *that* is just the number of ways you can pick three things (the a's) from each of the five parentheses. So it's $\binom{5}{3}$.

Michelle: Or, think of it as the number of ways you can pick two things (two b's) from the five parentheses. That's $\binom{5}{2}$, which is the *same* as $\binom{5}{3}$.

Han: OK, call it $\binom{5}{2}$ if you want. So, we have one part of $(a + b)^5$. It's $\binom{5}{2}a^3b^2$.

Michelle: And the same idea applies to other terms. You can pick no a's and five b's, one a and four b's, . . .

From Connecting with Mathematics (2002). *Combinatorial algebra, session 4*. Newton, MA: EDC.